MA552. Linear Algebra

Definition of a vector space

Definition 0.1. *A vector space (or linear space) consists of the following:*

1. *a field F of scalars;*

2. *a set V of objects, called vectors;*

3. *a rule (or operation), called vector addition, which associates with each pair of vectors α, β in V a vector $\alpha + \beta$ in V, called the sum of α and β, in such a way that*

    (a) *addition is commutative, $\alpha + \beta = \beta + \alpha$;*

    (b) *addition is associative, $\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$*

    (c) *there is a unique vector 0 in V, called the zero vector, such that $\alpha + 0 = \alpha$ for all α in V;*

    (d) *for each vector α in V there is a unique vector $-\alpha$ in V such that $\alpha + (-\alpha) = 0$;*

4. *a rule (or operation), called scalar multiplication, which associates with each scalar c in F and vector α in V a vector $c\alpha$ in V, called the product of c and α, in such a way that*

    (a) *$1\alpha = \alpha$ for every α in V;*

    (b) *$(c_1 c_2)\alpha = c_1(c_2 \alpha)$;*

    (c) *$c(\alpha + \beta) = c\alpha + c\beta$;*

    (d) *$(c_1 + c_2)\alpha = c_1 \alpha + c_2 \alpha$.*