MA552. Homework 12

Homework due November 28, 2006

Let $\widetilde{A}$ denote a linear mapping of an $n-$dimensional Euclidean space E into itself. Let A be its matrix relative to a given basis in E. Show that a necessary and sufficient condition for the transforms by $\widetilde{A}$ of n independent vectors $u_1, u_2, \ldots, u_n$ to be mutually orthogonal is that, relative to the basis $u_1, \ldots, u_n$, the quadratic form

$$x \to ||\widetilde{A}(x)||^2$$

reduce to a sum of squares. From this, show that there exists at least one system of n unit vectors $u_1, \ldots, u_n$ that are mutually orthogonal and whose transforms by $\widetilde{A}$ are mutually orthogonal.