MA552. Homework 12

Homework due December 5, 2006

Denote by g a symmetric bilinear form defined on $R^n \times R^n$ and such that $g(x,x) \geqslant 0$ for every value of the vector x, including the value 0 (this quadratic form is not necessarily strictly positive definite).

(1) Show that

$$[g(x,y)]^2 \leqslant g(x,x)g(y,y)$$

(2) A necessary and sufficient condition for g to be nondegenerate (that is, for the linear form on R^n that maps y into $g(x,x)$, where x is fixed, to be nonzero when x is not the zero vector) is that the quadratic form $g(x,x)$ be positive definite.

(3) If g is degenerate, show that the elements x at which $g(x,x)$ vanishes, constitute a vector subspace of R^n.

(4) If g is nondegenerate, show that the linear mapping α of R^n into R^n that leave $g(x,y)$ invariant (that is, such that $g(\alpha x, \alpha y) = g(x,y)$) constitute a group G, where the group operation is the composition of mappings.