Denote by $g$ a symmetric bilinear form defined on $\mathbb{R}^n \times \mathbb{R}^n$ and such that $g(x, x) \geq 0$ for every value of the vector $x$, including the value 0 (this quadratic form is not necessarily strictly positive definite).

1. Show that

$$[g(x, y)]^2 \leq g(x, x)g(y, y)$$

2. A necessary and sufficient condition for $g$ to be nondegenerate (that is, for the linear form on $\mathbb{R}^n$ that maps $y$ into $g(x, x)$, where $x$ is fixed, to be nonzero when $x$ is not the zero vector) is that the quadratic form $g(x, x)$ be positive definite.

3. If $g$ is degenerate, show that the elements $x$ at which $g(x, x)$ vanishes, constitute a vector subspace of $\mathbb{R}^n$.

4. If $g$ is nondegenerate, show that the linear mapping $\alpha$ of $\mathbb{R}^n$ into $\mathbb{R}^n$ that leave $g(x, y)$ invariant (that is, such that $g(\alpha x, \alpha y) = g(x, y)$) constitute a group $G$, where the group operation is the composition of mappings.