MA552. Homework 3

Homework due September 19, 2006

1. Let H_2^2 be the linear space of matrices 2x2 over the field of real numbers.

   1) Find the basis and dimension of the $L(A_1, A_2, A_3, A_4)$ subset of H_2^2 spanned by the following matrices:

$$A_1 = \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix},\ A_2 = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix},\ A_3 = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix},\ A_4 = \begin{pmatrix} -2 & 0 \\ 3 & 2 \end{pmatrix}$$

   2) Extend this basis to the basis of the space H_2^2

2. Assume that all numbers are from the field F. Let M(s), N(t), and P(u) be matrices defined by

$$M(s) = \begin{pmatrix} s & 0 \\ 0 & \frac{1}{s} \end{pmatrix},$$

$$N(t) = \begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix},$$

$$P(u) = \begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix},$$

   where the number $s \neq 0$.

   Show that the necessary and sufficient condition for a matrix

$$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

   to be the product $M(s)N(t)P(u)$ is that

$$a \neq 0, \qquad ad - bc = 1$$

   If this condition is satisfied, show that the numbers s, t, and u are uniquely determined.

3. Let a, b and c denote three elements in a vector space E over the field of complex numbers. Define

$$u = b + a, \quad v = c + a, \quad w = a + b$$

1) Show that the vector subspaces generated by a, b, and c on the one hand and u, v, and w on the other are identical

2) Show that the vectors u, v, and w are independent if and only if a, b, and c are independent.

4. Let P_3 be the space of polynomial functions degree ≤ 3 over the field of real numbers. Find coordinates of the element

$p(x) = x^3 + x + 1$ of the space P_3 in the following basis $p_0 = 1$, $p_1 = x + a$, $p_2 = (x + a)^2$, $p_3 = (x + a)^3$