1. Show that the determinant of an antisymmetric matrix of odd order is zero. (A square matrix $A = (a_{ij})$ is said to be antisymmetric if

$$a_{ij} + a_{ji} = 0$$

for all $i$ and $j$.)

2. We recall that the set of continuous mappings of $\mathbb{R}$ into $\mathbb{R}$ has a vector space structure on $\mathbb{R}$ if we define the sum of two mappings and the product of a mapping by a real number in the usual way.

Are the functions $f_n$ defined by

$$f_n(t) = \sin^n(t)$$

independent in this space?

3. A linear mapping $f$ of $\mathbb{R}^3$ into itself is defined by giving the coordinates $(X, Y, Z)$ of the vector $f(u)$ as a function of the coordinates $(x, y, z)$ of the vector $u$:

$$X = (m - 2)x + 2y - z$$
$$Y = 2x + my + 2z$$
$$Z = 2mx + 2(m + 1)y + (m + 1)z$$

Show that the rank of $f$ is equal to 3 except for particular values of $m$ and determine these particular values. Find the ranks for these values and define the subspace $f(\mathbb{R}^3)$. 