MA552. Homework 7

Homework due October 24, 2006

(1) Let us define a linear mapping f of R^4 into itself by giving the coordinates X, Y, Z, T of the vector $f(u)$ as a function of the coordinates x, y, z, t of the vector u:

$$\begin{array}{ccrrrr} X & = & x & +y & +z & -t \\ Y & = & -x & +y & -z & -t \\ Z & = & x & -y & -z & -t \\ T & = & -x & -y & +z & +3t \end{array}$$

Determine the rank of f and define the space $f(R^4)$. Show that $f(R^4)$ has no points in common with the domain D defined by

$$X > 0, \;\; Y > 0, \;\; Z > 0, \;\; T > 0,$$

i.e. that there do not exist values for x, y, z, and t such that these four inequalities are verified.

(2) Let us define a linear mapping f of R^n into R^p by giving the coordinates of the vector $f(u)$, the image of the vector u:

$$X_i = f_i(u) = \sum_{j=1}^{n} a_{ij} x_j, \quad (i = 1,\ 2, \ldots, p)$$

Assume that the rank of f is at least equal to $p - 1$. Show that a necessary and sufficient condition for the system of inequalities

$$f_i(u) > 0 \quad (i = 1, 2, \ldots, p)$$

to have no solution is that there exist positive numbers λ_i, not all zero, such that

$$\sum_i \lambda_i f_i = 0$$