MA552. Homework 8

Homework due October 31, 2006

We define the following four linear forms on R^4:

$$
\begin{aligned}
X &= \quad\ \ ay + bz + ct \\
Y &= \ ax \quad\ + cz + bt \\
Z &= \ bx + cy \quad\ + at \\
T &= \ cx + by + az
\end{aligned}
$$

(1) Find the rank of the system of forms X, Y, Z, T (that is, the rank of the mapping of R^4 into R^4 that they define).

(2) What is the condition under which these forms will be independent? (Evaluate the determinant of the system.)

(3) Discuss the existence of solutions of the system

$$
X = a + b + c, \quad Y = a, \quad Z = b, \quad T = c
$$