MA615 Numerical analysis I

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MA615 is a first part of a one-year graduate course in numerical analysis for students majoring in pure and applied mathematics. This course covers numerical methods of linear algebra (solution of linear algebraic equations, eigenvalues and eigenvectors), solutions of non-linear equations, interpolation and approximation methods and extreme problems. The second part of the course, MA616 Numerical Analysis II, will be offered sequentially in the spring semester and covers numerical integration, solutions of differential (both, ODEs and PDEs) and integral equations, and statistical methods.

The course will be very intense; it will include three components theoretical, computational and applications. All presented numerical methods will be considered theoretically in substantial depth including derivation, convergence, error analysis, and limitations. Then the code in C++ for every method will be presented, some methods also will be illustrated with the Mathematica program. Also we will apply numerical methods to real-life problems and they will be used in concert with analytical results; examples will be taken mostly from Mathematical Biology, which is my research area.

In addition to the standard topics such as, computational linear algebra, interpolation, extrapolation, integration, and solution of algebraic, differential and integral equations, some other topics may be considered provided that the mandatory material is covered. In the first semester such an additional topic may be computational geometry with applications in individual-based modeling of spatial populations. In the second semester, in addition to the usual topics we may consider Markov chain Monte Carlo methods and Bayesian inference techniques in application to the ecology, in particular, to apply these methods to the forest inventory (FIA) data.

Prerequisites

The major prerequisites for MA615 are graduate courses in Linear Algebra (MA551 or equivalent) and Advanced Calculus (MA448 and Ma449). It is strongly advised to students to take MA615 only if they have solid background in these disciplines. This course has absolutely no room to deeply review linear algebra and advanced calculus. The course is designed in such a way that no preliminary experience in programming is required; C++ language will be introduced on examples in parallel with the theory development. However, a preliminary background in numerical methods and programming such as an undergraduate course in numerical methods or an introductory course in C++ will certainly be an asset.

Course program:

Lecture 1. – Algorithms, Numbers, C++ and Mathematica.

1. Memory, binary numbers, sources of error.
2. Propagation of errors.
3. Algorithms and C syntax.
4. Stability in numerical analysis.
5. Introduction to Mathematica.

Lecture 2. – Nonlinear algebraic equations. Rootfinding.

6. The bisection method.
7. The secant method.
8. Newton's method.
9. Combined methods.

Lecture 3. –Multiple roots. Roots of Polynomials.

10. The Newton's method and multiple roots.
11. Brent's method.
12. Roots of polynomials.

Lecture 4. –.Matrices in C++

13. Pointers.
14. Dynamic memory allocation.
15. Classes and objects.
16. The class "vector".
17. The class "matrix".

Lecture 5. – Linear algebraic equations I.

18. Gaussian elimination.
19. Pivoting.
20. Modifications of Gaussian elimination.
21. Error analysis.

Lecture 6. – Linear algebraic equations II.

22. LU Decomposition.
23. Iteration methods.
24. Singular Value Decomposition.
25. Sparse Linear Systems.

Lecture 7. – Eigenvalues and eigenvectors I.

26. The Gershgorin circle theorem.

27. Error and stability.
28. Symmetric matrices.
29. Nonsymmetric matrices.

Lecture 8. – Eigenvalues and eigenvectors II.

30. Orthogonal transformations.
31. Householder method.

Lecture 9. – Eigenvalues and eigenvectors III.

32. The QR method.
33. Finding Eigenvectors by Inverse Iteration.

Lecture 10. – Polynomial Interpolation and Extrapolation.

34. Searching an ordered table.
35. Polynomial interpolation theory.
36. Newton divided differences.
37. Forward and backward differences.

Lecture 11. – Piecewise Polynomial Interpolation.

38. Local interpolation problems.
39. Spline functions.
40. Cubic spline interpolation.

Lecture 12. – Orthogonal polynomials.

41. Orthogonal polynomials.
42. The least squares approximation.
43. Chebyshev Approximation.
44. Rational Chebyshev Approximation

Lecture 13. – Computational Geometry I.

45. Points and boxes.
46. Nearest-neighbor finding.
47. Triangles in two and three dimensions.
48. Polygons.
49. Spheres and Rotations.

Lecture 14. – Triangulations.

50. Voronoi diagrams.
51. Application to individual-based spatial population models.