MA651 Topology. Homework 1

Homework due January 31, 2006

1. Find the mistakes in the lecture notes (I have intentionally made several mistakes, and you might find even more).

2. Consider the questions given as a homework in the lecture notes (Propositions 3.3, 4.1, 4.2)

3. The symmetric difference of two sets A and B is the set $A \triangle B$ is $(A - B) \bigcup (B - A)$. Let A be a nonempty set, and another set X such that the following statement holds true:

$$A \triangle X = A$$

   Prove or disprove that $X = \emptyset$

4. Prove Theorem 4.1 (a), i.e.

$$A \times (B \bigcup C) = A \times B \bigcup A \times C$$

5. Let Γ denote a set and let $H \subset \Gamma$. The mapping h of Γ onto the set consisting of the two numbers 0 and 1 defined by

$$h(x) = 0 \text{ if } x \notin H$$
$$h(x) = 1 \text{ if } x \in H$$

   is called the *characteristic function* of the subset H.

   (i) Suppose that h and f are the characteristic functions of two subsets H and F of Γ. What are the sets whose characteristic functions are $1 - h$, hf, and $h + f - hf$?

   (ii) Use the result of (i) to show that

$$(H \bigcap F) \bigcap G = H \bigcap (F \bigcap G)$$
$$(H \bigcap F) \bigcup G = (H \bigcup G) \bigcap (F \bigcup G)$$
$$\mathbf{C}(H \bigcap F) = (\mathbf{C}H) \bigcup (\mathbf{C}F)$$