Ma 635. Real Analysis I. Hw4

HW 4 (due 09/28):

1. [1] p. 66 # 25

A function $f:\ (M,d)\mapsto(N,\rho)$ is called Lipshitz if $\exists K,\ \forall x,y\in M,\ \rho(f(x),f(y))\le Kd(x,y)$. Prove that Lipshitz mapping is continuous.

2. [1] p. 66 # 28

Let $g: l_2\mapsto R$, $g(x)=\sum\limits_{n=1}^{\infty}\frac{x_n}{n}$. Is g continuous?

3. [1] p. 66 # 29

Fix $y\in l_\infty$ and define $h:\ l_1\mapsto l_1$ by $h(x)=(x_n y_n)_{n=1}^{\infty}$. Show that h is continuous.

4. [1] p. 71 # 43

5. [1] p. 71 # 45

Prove that N with usual metric is homeomorphic to $\{\frac{1}{n}\}_{n=1}^{\infty}$ with usual metric.

6. [1] p. 71 # 49

Let V be a normed vector space. Given a fixed vector $y\in V$, show that the map $f(x)=x+y$ (translation ny y) is an isometry on V. Given a nonzero scalar $\alpha\in R$, show that the map $g(x)=\alpha x$ (dilation by α) is a homeomorphism on V.

7. [1] prove theorem 5.10 (p. 75)

8. [1] p. 75 # 62

9. [1] p. 94 # 13

Show that R endowed with the metric $\rho(x,y)=|\arctan x-\arctan y|$ is not complete. How about $\tau(x,y)=\left|x^3-y^3\right|$?

10. [1] p. 94 # 14

If we define $d(m,n)=\left|\frac{1}{m}-\frac{1}{n}\right|$ for $m,n\in N$, show that d is equivalent to the usual metric on N but (N,d) is not complete.

11. [1] p. 94 # 19

Show that c_0 is closed in l_∞.

12. [1] p. 94 # 20

13. [1] p. 94 # 21

14. [1] p. 102 # 42

Define $T:\ C[0,1]\mapsto C[0,1]$ by $Tf(x)=\int_0^x f(t)dt$. Show that T^2 is a contraction. What is its fixed point?

15. To solve equation $f(x)=\lambda\int_0^2(x+y)f(y)dy$, we consider $f\in C[0,2]$ and $F(f)(x)=1+\lambda\int_0^2(x+y)f(y)dy$. Find λ at which $F:\ C[0,2]\mapsto C[0,2]$ is contractive. For initial value $f(x)=0$ perform 3 iterations to approach the solution $f(x)$.

(!) Please read the corresponding sections of the textbook.

References

[1] Carothers N.L., *Real Analysis.* Cambridge University Press, 2000.
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[4] Rudin, W., *Real and Complex Analysis*, 3d ed. McGraw-Hills, 1987.

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[6] Reed, M. and Simon, B., *Methods of Modern Mathematical Physics. 1. Functional Analysis.* Academic Press 1972.

[7] Oxtoby, J.C., *Measure and Category. A survey of the Analogies between Topological and Measure Spaces.* Springer-Verlag, 1971.