Ma 635. Real Analysis I. Hw5, due 10/05.

HW 5 (due 10/05):
1. [1] p. 92 # 9
2. [1] p. 110 # 12
3. [1] p. 110 # 14
4. [1] p. 110 # 17
5. [1] p. 110 # 18
6. [1] p. 111 # 23
7. [1] p. 114 # 34
8. [1] p. 114 # 35
9. [1] p. 114 # 36

10 [Kolmogorov, p. 81, # 4] Show that $E = \{1/n : \ n \text{ a positive integer } \}$ is not compact in $\mathbf{R}$ but $E \cup \{0\}$ is compact.

11 [Kolmogorov, p. 81, # 7] Prove that every finite subset of a metric space is compact.

12 [Kolmogorov, p. 81, # 6] Show that a discrete metric space X is not compact unless X is finite.

13 [Kolmogorov, p. 81, # 9] If X is compact prove that $C(X, \mathbf{R})$ is a complete metric space.

14 [Kolmogorov, p. 81, # 10] Is $C[0,1]$ compact?

15. [Kolmogorov, p. 84, # 4] Prove that any compact metric space has a dense countable subset.

16. [Haaser, p. 115, # 5] Let X be a metric compactum and $A: \ X \mapsto X$ such that $d(Ax,\ Ay) < d(x,y)$ if $x \neq y$. Prove that A has a unique fixed point.

17. Prove that a uniformly bounded set of functions in $C[a,b]$, which satisfy the Lipshitz condition with the same common constant, is compact in $C[a,b]$.
$x(t)$ satisfies the Lipshitz condition with constant L if $\forall t, s: \ |x(t) - x(s)| \leq C|t-s|$.

18. Determine whether the following sets in $C[0,1]$ are relatively compact:
(a) $x_n(t) = \sin(nt)$
(b) $x_n(t) = \sin(t+n)$
(c) $x_\alpha(t) = \arctan(\alpha t)$, $\alpha \in \mathbf{R}$
(d) $x_\alpha(t) = e^{t-\alpha}$, $\alpha \in \mathbf{R}$, $\alpha \geq 0$.

Bonus 2: Prove that the condition $d(f(x), f(y)) < d(x,y), \ x \neq y$, is insufficient for the existence of a fixed point of function f.

Bonus 3; [1] p. 111 # 24.

References

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[3] Haaser, N.B., and Sullivan, J.A., *Real Analysis*. Dover, 1991.
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