Ma 635. Real Analysis I. Hw8, due 11/02.

HW 8 (due 11/02):

Read [1] pp. 263–274, 277–289.

Problems:
1. [1] p. 271 # 3
In the definition of the outer measure let's consider finite unions of intervals covering the set. Show that if $Q \cap [0,1]$ is contained in a finite union of open intervals $\bigcup_{i=1}^{n}(a_i, b_i)$ then $\sum_{i=1}^{n}(b_i - a_i) \geq 1$. Thus, $Q \cap [0,1]$ would have measure 1 by this definition.

2. [1] p. 271 # 4
Given any subset E of R and any $h \in R$,show that $m^*(E+h) = m^*(E)$.

3. [1] p. 271 # 9
If $E = \bigcup_{n=1}^{\infty} I_n$ is a countable union of pairwise disjoint intervals, prove that $m^*(E) = \sum_{n=1}^{\infty} |I_n|$.

4. [1] p. 271 # 17
If $m^*(E) = 0$ show that E^c is dense.

5. [1] p. 271 # 18
If E is a compact set with $m^*(E) = 0$, prove that $\forall \varepsilon > 0\ \exists \{I_i\}_{i=1}^{n}$-open intervals, satisfying $\sum_{j=1}^{n} m^*(I_j) < \varepsilon$.

6. [1] p. 272 # 20
If $m^*(E) = 0$, prove that $m^*(E^2) = 0$.

7. [1] p. 273 # 22
Let $E = \bigcup_{n=1}^{\infty} E_n$. Show that $m^*(E) = 0 \iff \forall n\, m^*(E_n) = 0$.

8. [1] p. 273 # 23
Given a bounded open set G and $\varepsilon > 0$, show that there is a compact set $F \subset G$ such that $m^*(F) > m^*(G) - \varepsilon$.

9. [1] p. 273 # 27
For each n, let G_n be an open subset of $[0,1]$ containing the rationals in $[0,1]$ with $m^*(G_n) < 1/n$, and let $H = \bigcap_{n=1}^{\infty} G_n$. Prove that $m^*(H) = 0$ and that $[0,1] \setminus H$ is a first category set in $[0,1]$. Thus, $[0,1]$ is the disjoint union of two "small" sets!

10. (bonus) [1] p. 274 # 29

References

[1] Carothers N.L., *Real Analysis*. Cambridge University Press, 2000.
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[2] Kolmogorov, A.N., and Fomin, S.V., *Introductory Real Analysis*. Dover, 1970.
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[3] Haaser, N.B., and Sullivan, J.A., *Real Analysis.* Dover, 1991.
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[4] Rudin, W., *Real and Complex Analysis*, 3d ed. McGraw-Hills, 1987.

[5] Folland, G.B., *Real Analysis.* Wiley, 1984.

[6] Reed, M. and Simon, B., *Methods of Modern Mathematical Physics. 1. Functional Analysis.* Academic Press 1972.

[7] Oxtoby, J.C., *Measure and Category. A survey of the Analogies between Topological and Measure Spaces.* Springer-Verlag, 1971.