Ma 635. Real Analysis I. Quiz 1. Colloquium

1. Let X be a complete metric space and let $A: \ X \mapsto X$ satisfy the following condition: $\exists n$ such that A^n is contractive. Prove that $\exists!\ x \in X$ such that $A(x) = x$.

2. Whether two definitions below are equivalent?

(1) Norms $\|.\|_1$ and $\|.\|_2$ are equivalent if

$$\|x_n - x\|_1 \to 0 \iff \|x_n - x\|_2 \to 0 \ \text{ as } n \to \infty$$

(2) Norms $\|.\|_1$ and $\|.\|_2$ are equivalent if

$$\exists\, c_1, c_2 > 0 \ \text{ such that } \forall x,\ c_1\|x\|_1 \le \|x\|_2 \le c_2\|x\|_1$$

3. Whether the sequence $x_n(t) = \frac{1}{n}\sin(n^3 t)$ is pre-compact in space $C[0,1]$?

4. Whether two definitions below are equivalent?

(1) $f: \ X \mapsto Y$ is continuous if $x_n \to x$ implies $f(x_n) \to f(x)$

(2) $f: \ X \mapsto Y$ is continuous if pre-image of any open set in Y is also open in X.

5. Let X be a metric space. Whether three definitions below are equivalent?

(1) Set $A \subset X$ is compact if A is closed and any infinite sequence from A has a converging subsequence

(2) Set $A \subset X$ is compact if A is closed and totally bounded

(3) Set $A \subset X$ is compact if any open cover of A has a finite subcover.