FE570 Financial Markets and Trading

Lecture 6. Volatility Models and Sequential Trade Models
(Ref. Joel Hasbrouck - *Empirical Market Microstructure*)

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Outline

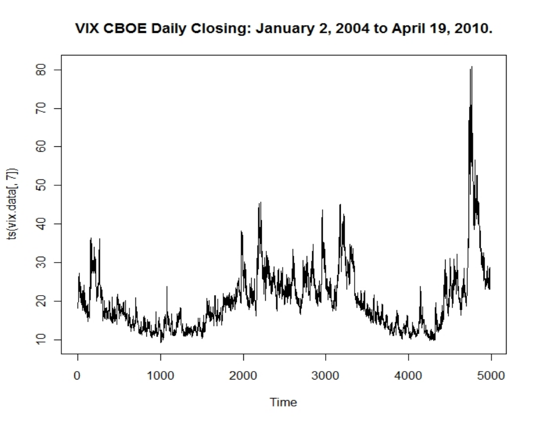
Volatility:

- Definition: *the conditional standard deviation of the underlying asset return.*
- Volatility has many financial applications:
  - Option (derivative) pricing, e.g., Black-Scholes formula.
  - Risk management, e.g. value at risk (VaR).
  - Asset allocation, e.g., minimum-variance portfolio.
- Use high-frequency data: French, Schwert & Stambaugh (1987);
  - Realized volatility of daily returns in recent literature.
  - Use daily high, low, and closing (log) prices, e.g. range.
- Implied volatility of options data, e.g, VIX of CBOE.

  The volatility index of a market has become a financial instrument.

  The VIX volatility index complied by the Chicago Board of Option Exchange (CBOE) started to trade in futures on March 26, 2004.

Characteristics of Volatility



- A special feature of stock volatility is that it is not directly observable.

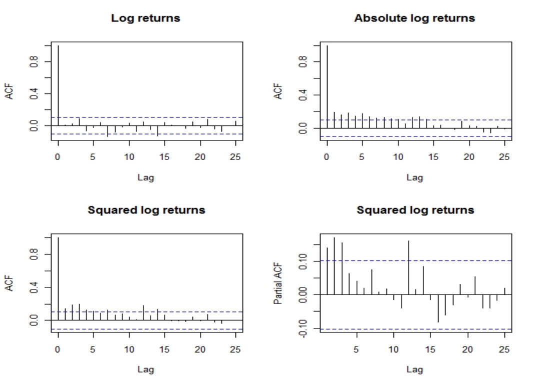
Characteristics of Volatility:

- There exist volatility clusters (i.e., volatility may be high for certain time periods and low for other periods).
- Volatility evolves over time in a continuous manner - that is, volatility jumps are rare.
- Volatility does not diverge to infinity - that is, volatility varies with some fixed range. (this means that volatility is often stationary.)
- Volatility seems to react differently to a big price increase or a big price drop, referred to as the *leverage* effect.

** Important in the development of the volatility models.

Structure of a Model

- Let r_t be the return of an asset at time t. The basic idea behind volatility study is that the series $\{r_t\}$ is either serially uncorrelated or with minor lower order serial correlations, but it is dependent series.



Structure of a Model

- To put the volatility models in proper perspective, it is informative to consider the conditional mean and variance of r_t given F_{t-1}; that is

$$\mu_t = E(r_t|F_{t-1}), \sigma_t^2 = Var(r_t|F_{t-1}) = E[(r_t - \mu_t)^2|F_{t-1}], \quad (1)$$

where F_{t-1} denote the information set, and typically F_{t-1} consists of all linear functions of the past returns.

- We can model r_t as a stationary $ARMA(p,q)$ model with some explanatory variables.

$$r_t = \mu_t + a_t, \mu_t = \phi_0 + \sum_{i=1}^{k} \beta_i x_{it} + \sum_{i=1}^{p} \phi_i r_{t-i} - \sum_{i=1}^{q} \theta_i a_{t-i}, \quad (2)$$

for r_t, where k, p and q are non-negative integers, and x_{it} are explanatory variables.

Structure of a Model

- Combine the last two equations, we have

$$\sigma_t^2 = Var(r_t|F_{t-1}) = Var[(a_t|F_{t-1}],$$

- The conditional heteroscedastic models are concerned with the evolution of σ_t^2. The manner under which σ_t^2 evolves over time distinguishes one volatility model from another. They can classified into two general categories:

  1. Those use an exact function to govern the evolution of σ_t^2;
  2. Those a stochastic equation to describe σ_t^2.

- Autoregressive Conditional Heteroscedastic (ARCH) model of Engle (1982), and the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model of Bollerslev (1986) belong to the first category whereas the stochastic volatility model is in the second category.

Volatility Model Building

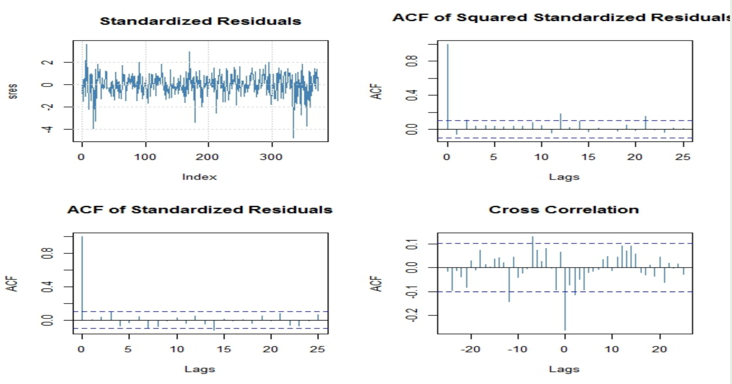
1. Specify a mean equation by testing for serial dependence in the data and, if necessary, building an econometric model (e.g. an ARMA model) for the return series to remove any linear dependence.
2. Use the residuals of the mean equation to test for ARCH effects.

$$a_t = \sigma_t \epsilon_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots + \alpha_m a_{t-m}^2.$$

   where $\{\epsilon\}$ is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1, $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i > 0$.
3. Specify a volatility model if ARCH effects are statistically significant and perform a joint estimation of the mean and volatility equations.
4. Check the fitted model carefully and refine it if necessary.

Example

Volatility ARCH(1) Example



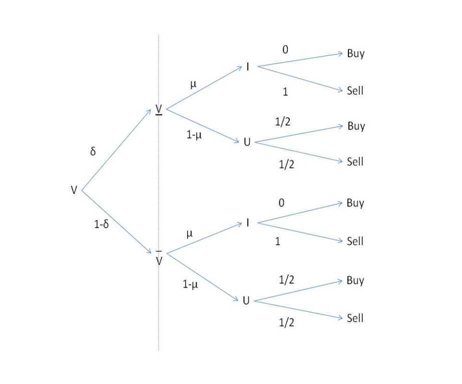
- Fit an ARCH (3) model first based on PACF
- Examine the fitness of the model and further reduce the model to ARCH(1), and residual past Ljung-Box Q(m) test.

A Simple Sequential Trade Model - *A special case of Glosten and Milgrom (1985).*

- There is one security with a value [payoff] V that is either high or low, $\bar{V}$ or $\underline{V}$. The probability of the low outcome is δ. The value is revealed after the market closes - it is not affected by trading.
- The trading population comprises informed and uninformed traders. Informed traders (insiders) know the value outcome. The proportion of informed traders in the population is μ.
- A dealer posts bid and ask quotes, B and A. A trader is drawn at random from the population. If the trader is informed, she buys if $V = \bar{V}$ and sells if $V = \underline{V}$. If the trader is uninformed, he buys or sells randomly and with equal probability. The dealer does not know whether the trader is informed.
- I and U denote the arrivals of informed and uninformed traders.

Event Tree of the Sequential Trade Model

- A buy is a purchase by the customer at the dealer's ask price, A; a sell is a customer sale at the bid.



** The value attached to the arrow is the probability of the indicated transition. Total probabilities are obtained by multiplying along a path.

Event Tree Probabilities

- For example, the probability of a low realization for *V*, followed by the arrival of an uninformed trader who buys is $\delta(1-\mu)/2$.
- The sum of the total probabilities over terminal *Buy* nodes gives the unconditional probability of a buy:

$$Pr(\text{Buy}) = \frac{(1+\mu(1-2\delta))}{2}$$

- Similarly,

$$Pr(\text{Sell}) = \frac{(1-\mu(1-2\delta))}{2}$$

In the case where $\delta = 1/2$ (equal probabilities of good and bad outcomes), the buy and sell probabilities are also equal.

Dealer's Ask Quotes

- If the dealer is a monopolist, expected profits are maximized by setting the bid infinitely low and the ask infinitely high.
- In practice, the dealer's market power is constrained by competition and regulation, and we assume that competition among dealers drives expected profits to zero.
- The dealer's inference given that the first trade is a buy or sell can be summarized by her revised beliefs about the probability of a low outcome:

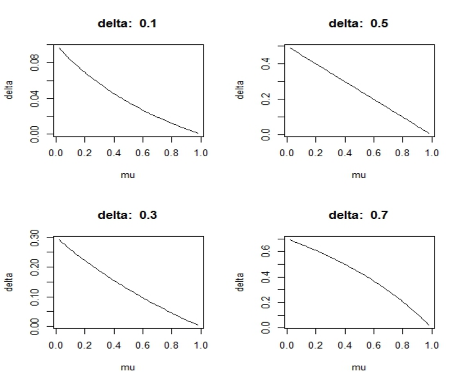
$$\delta_1(\text{Buy}) = Pr(\underline{V}|\text{Buy}) = \frac{Pr(\underline{V}, \text{Buy})}{Pr(\text{Buy})} = \frac{\delta(1-\mu)}{1+\mu(1-2\delta)}. \quad (3)$$

Because μ and δ are between zero and one, $\partial\delta_1(\text{Buy})/\partial\mu < 0$: The revision in beliefs is stronger when there are more informed traders in the population.

Dealer's Ask Quotes

- The dealer's beliefs:

$$\frac{\partial \delta_1(\text{Buy})}{\partial \mu} = -\frac{\delta\mu + \delta\mu^2(1-2\delta) + \delta(1-\mu)(1-2\delta)}{[1+\mu(1-2\delta)]^2}. \quad (4)$$



Dealer's Realized Profit:

- The dealer's realized profit on the transaction is $\pi = A - V$.
- Immediately after the trade, the dealer's expectation of this profit is:

$$E[\pi|\text{Buy}] = A - E[V|\text{Buy}],$$
$$E[V|\text{Buy}] = \delta_1(\text{buy})\underline{V} + (1 - \delta_1(\text{Buy}))\bar{V}$$

- If competition drives this expected profit to zero, then

$$A = \frac{\underline{V}(1-\mu)\delta + \bar{V}(1-\delta)(1+\mu)}{1+\mu(1-2\delta)}. \tag{5}$$

A dealer's quote is essentially a proposal of terms of trade. When the bid is hit or the offer is lifted, this proposal has been accepted. In the present model, the ask is simply what the dealer believes the security to be worth.

Net Wealth Transfer:

- The ask quote strikes a balance between informed and uninformed traders. The conditional expectation of value can be decomposed as

$$E[V|\text{Buy}] = E[V|U,\text{Buy}]Pr(U|\text{Buy}) + E[V|I,\text{Buy}]Pr(I|\text{Buy}).$$

- Substitute this into the zero-expected profit condition $A = E[V|\text{Buy}]$ and rearrange, we have

$$\underbrace{(A - E[V|U,\text{Buy}])}_{\substack{\text{Gain from an}\\ \text{uninformed trader}}} \; Pr(U|\text{Buy}) = - \underbrace{(A - E[V|I,\text{Buy}])}_{\substack{\text{Loss to an}\\ \text{informed trader}}} \; Pr(I|\text{Buy}) \tag{6}$$

The expected gains from uninformed traders are balanced by the losses to informed traders. In this model, therefore, there is a net wealth transfer.

Dealer's Bid Quotes

- Following a sale to the dealer:

$$\delta_1(\text{Sell}) = Pr(\underline{V}|\text{Sell}) = \frac{Pr(\underline{V}, \text{Sell})}{Pr(\text{Sell})} = \frac{\delta(1+\mu)}{1-\mu(1-2\delta)}. \quad (7)$$

Because μ and δ are between zero and one, $\delta_1(\text{Sell}) > \delta_1(\text{Buy})$. $\underline{V}$ is less likely if the customer bought, reasons the dealer, because an informed customer who knew $V = \underline{V}$ would have sold. Further more $\partial \delta_1(\text{Sell})/\partial \mu > 0$

$$B = E[V|\text{Sell}] = \frac{\underline{V}(1+\mu)\delta + \bar{V}(1-\delta)(1-\mu)}{1-\mu(1-2\delta)}. \quad (8)$$

- The bid-ask spread is

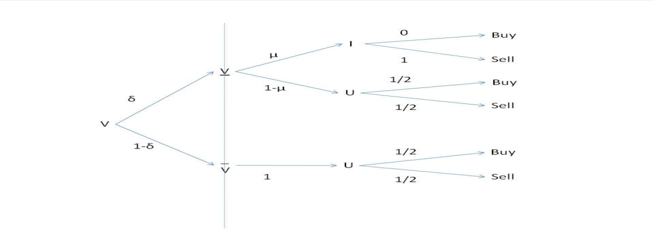
$$A - B = \frac{4(1-\delta)\delta\mu(\bar{V} - \underline{V})}{1-(1-2\delta)^2\mu^2}. \quad (9)$$

Example

Example: consider a variation of the model in which there is informed trading only in the low state (V=$\underline{V}$). Verify that

$$\delta_1(\text{Buy}) = \frac{\delta(1-\mu)}{1-\delta\mu}; \delta_1(\text{Sell}) = \frac{\delta(1+\mu)}{1+\delta\mu}$$

$$A = \frac{\underline{V}(1-\mu)\delta + \bar{V}(1-\delta)}{(1-\mu\delta)}; B = \frac{\underline{V}(1+\mu)\delta + \bar{V}(1-\delta)}{(1-\mu\delta)}.$$



Market Dynamics: Bid and Ask Quotes over Time

- Let δ_k denote the probability of a low outcome given δ_{k-1} and the direction of the kth trade, with the original (unconditional) probability being $\delta_0 \equiv \delta$. Then we have:

$$\delta_k(\mathrm{Buy}_k; \delta_{k-1}) = \frac{\delta_{k-1}(1-\mu)}{1+\mu(1-2\delta_{k-1})}. \tag{10}$$

$$\delta_k(\mathrm{Sell}_k; \delta_{k-1}) = \frac{\delta_{k-1}(1+\mu)}{1-\mu(1-2\delta_{k-1})}. \tag{11}$$

- Market dynamics have the following features:
  - The trade price series is a martingale. (A sequence conditioned on expanding information sets is a martingale)
  - The order flow is not symmetric.
  - Spread declines over time. (estimate more precisely.)
  - The orders are serially correlated (one subset always trade in the same direction).
  - There is a price impact of trades.