FE670 Algorithmic Trading Strategies

Lecture 10. Investment Management and Algorithmic Trading

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Outline

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Algorithmic Trading Strategies

Algorithmic trading: is commonly defined as the use of computer algorithms to automatically make trading decisions, submit orders, and mange those orders after submission.

Goal: *The main objective of algo trading is not necessarily to maximize profits but rather to control execution costs and market risk.*

- Different strategies may target at different frequencies, and the profitability of a trading strategy is often measured by certain return metric.

The Market in Numbers

- US Equities volumes: 5 and 10 billion shares per day
- 1.2 $\sim$ 2.5 Trillion shares per year
- Annual volume: USD 30 $\sim$ 70 trillion
- At least 30% of the volume is algorithmic: 360 $\sim$ 750 billion shares/year
- Typical large “sell side broker trades between 1 and 5 USD Tri per year using algos
- Each day, between 15,000 and 30,000 orders are processed
- An algorithmic execution strategy can be divided into 500 $\sim$ 1,000 small daughter orders

The Market in Numbers

- Algorithms started as tools for institutional investors in the beginning of the 1990s. Decimalization, direct market access (DMA), 100% electronic exchanges, reduction of commissions and exchange fees, rebates, the creation of new markets aside from NYSE and NASDAQ and Reg NMS led to an explosion of algorithmic trading and the beginning of the decade.

- Today, brokers compete actively for the commission pool associated with algorithmic trading around the globe a business estimated at USD 400 to 600 million per year.

- Orders come from institutional investors, hedge funds and Wall Street trading desks

Why Algorithms?

- Institutional clients need to trade large amounts of stocks. These amounts are often larger than what the market can absorb without impacting the price.
- The demand for a large amount of liquidity will typically affect the cost of the trade in a negative fashion ("slippage")
- Large orders need to be split into smaller orders which will be executed electronically over the course of minutes, hours, day.
- The procedure for executing this order will affect the average cost per share, according to which algorithm is used.
- In order to evaluate an algorithm, we should compare the average price obtained by trading with a market benchmark.

High Frequency Trading

- Being faster the traders can react to changes in the market before everyone else, thereby gaining an advantage.
- Their competitive advantage arise from being able to process and disseminate information sooner and faster than other market participants.
- These sophisticated high-frequency trading firms, representing about 2% of the approximately 20,000 trading firms in the United States, are believed to be responsible for almost three-quarters of all U.S. equity trading volume.
- These businesses include hundreds of the most secretive proprietary trading desks at the major investment banks, and maybe about 100 or so of the most sophisticated hedge funds.

Optimal Execution

Implementation shortfall(IS) is a measure of the total transaction costs. IS represents the difference between the actual portfolio return and the paper estimate of this return at the beginning of trading.

If trading of an order with size X started at price p_0 (*arrival price*) and ended at price p_N, and the order was split into N child orders of size x_k that were filled at price p_k, then

$$\text{IS} = \sum x_k p_k - p_0 \sum x_k + (p_N - p_0)(X - \sum x_k) + C \quad (1)$$

where C is the fixed cost. The first two terms represent execution cost, and the third tells opportunity cost.

Execution Strategies

- Note that not all child orders may be executed during the trading day. For example, submission of child orders may be conditioned on specific price behavior. The unfilled amount, $X - \sum x_k$, determines an *opportunity cost*.
- *Algorithmic trading* is a new field that focuses on making decisions where and how to trade. The professional trading community attributes algorithmic trading primarily to the execution strategies (Johnson 2010).
- The question of *whether* to trade is beyond the scope of our lecture today. It is assumed that the decision to trade a given amount within a given time horizon has been made and we are concerned only with its implementation.
- The decision *where* to trade is important for institutional trading, and modern trading systems often have *liquidity aggregators* that facilitate connections to various sources.

Execution Strategies

Two major families of execution algorithms:

- **Benchmark algorithms** are based on some simple measures of market dynamics rather than on explicit optimization protocols.
- **Cost-driven algorithms** minimize IS and are often named *implementation shortfall algorithms*.

Obviously, any execution algorithm addresses the problem of minimizing execution costs. Market impact due to order execution in conditions of limited liquidity is the main culprit of trading loss. Large orders can move price in the adverse direction, and a general way of reducing trading loss is splitting large orders into smaller child orders and spanning them over a given time interval.

Benchmark-Driven Schedules

- **Time-weighted average price** (TWAP). In this schedule, child orders are spread uniformly over a given time interval. Such a simple protocol has a risk of exposure of the trader's intentions to other market participants.
  For example, some scalpers *may realize that a large order is being traded and start trading the same instrument in expectation that the large trading volume will inevitably move the price.*
  - To prevent information leak, TWAP schedule may be randomized in terms of size and submission time of child orders.
    For example, if the trading interval is four hours, 25% of the trading volume must be executed each hour, and the child order size may be adjusted deterministically for each hour.
  - More sophisticated TWAP schedules may use adaptive algorithms based on short-term price forecast.

- **Volume-weighted average price** (VWAP). Markets often have pronounced intraday trading volume patterns. Therefore, the VWAP schedule may be more appropriate than the TWAP schedule.

  If an asset during some time interval has N trades with price p_k and volume v_k, its VWAP is

$$\text{VWAP} = \sum_{k=1}^{N} v_k p_k / \sum_{k=1}^{N} v_k \qquad (2)$$

  Practical implementation of the VWAP algorithm involves calculation of the percentage of daily trading volume u_k for each trading period k using historical market data:

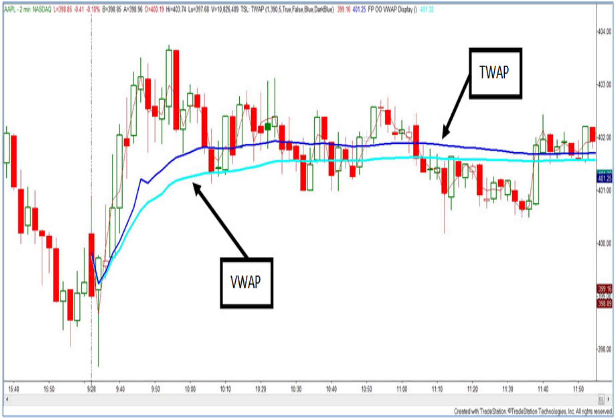
$$u_k = v_k / \sum_{i=1}^{N} v_i, \text{ the size of k-th child order } x_k = X u_k \qquad (3)$$

- Historical estimates of u_k may have significant variation. Therefore, sophisticated VWAP algorithms have adaptive mechanisms accounting for short-term price trend and dynamics of u_k.
- It should be noted that while the VWAP algorithm helps in minimizing the market impact cost, it does not necessarily yield possible price appreciation, which is, in fact, a form of opportunity cost.

  > Indeed, if price grows (falls) on a high volume during a day, the trader might get more price appreciation if the entire buy (sell) order is placed in the morning rather than spread over the day. On average, however, such an opportunity cost is compensated for buy (sell) orders on days when the price falls (grows).

- The VWAP benchmark has become very popular in post-trade analysis. How well an algorithm performs can be checked by comparing the realized trading cost with the true VWAP calculated using available market data.

TWAP vs. VWAP During a slow trading day, the TWAP may be very similar to the VWAP, even to the penny at times. However, in a volatile session, or when volume is higher than usual, the two indicators may start to diverge.



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- **Percent of volume** (POV). In this schedule, the trader submits child orders with sizes equal to a certain percentage of the total trading volume, γ.

  This implies that child orders have acceptable market impact (if any), and execution time is not strictly defined.

  In estimating the size of child order x_k, one should take into account that the child order must be included in the total trading volume X_k at time period k:

$$\gamma = x_k/(X_k + x_k)$$

$$\text{As a result, } x_k = \gamma X_k/(1-\gamma)$$

- **Participation weighted price** (PWP). This benchmark is a combination of VWAP and POV. Namely, if the desirable participation rate is γ and the order volume is N, PWP for this order is VWAP calculated over N/γ shares traded after the order was submitted.

Cost-Driven Schedules / Risk-Neutral Framework

- **Cost-Driven Schedules** While executing a large order, a risk-averse trader faces a dilemma: Fast execution implies larger child orders and hence higher market impact and higher IS. On the other hand, submitting smaller child orders consumes more time and exposes traders to the price volatility risk (market risk).
- Cost-driven schedules can be partitioned into **risk-neutral algorithms** and **risk-averse algorithms**. In the former case, the schedule is derived by minimizing market impact. In the later case, the schedule is derived by minimizing utility function that has two components: market impact and volatility risk.
- **Risk-Neutral Framework** Bertsimas & Lo (1998) introduced the following model for optimal execution. The objective is to minimize the execution cost:

Cost-Driven Schedules / Risk-Neutral Framework

$$\min_{x_k} E\{\sum_{k=1}^{N} x_k p_k\}$$

$$\text{s.t.} \quad \sum_{k=1}^{N} x_k = X$$

It is assumed that price follows the arithmetic random walk in the absence of market impact, and market impact is permanent and linear upon volume:

$$p_k = p_{k-1} + \theta x_k + \epsilon_k$$

where $\theta > 0$ and ϵ_k is an IID process that is uncorrelated with trading and has zero mean. Then, the volume remaining to be bought, w_k can be determined as a dynamic programming problem.

Cost-Driven Schedules / Risk-Neutral Framework

$$w_k = w_{k-1} - x_k, w_1 = X, w_{N+1} = 0$$

The dynamic programming optimization is based on the solution optimal for the entire sequence $\{x_1^*, ..., x_N^*\}$ must be optimal for the subset $\{x_k^*, ..., x_N^*\}, k > 1$. This property is expressed in the Bellman equation in recursive format:

$$V_k(p_{k-1}, w_k) = minE\{p_k x_k + V_{k+1}(p_k, w_{k+1})\}, \text{ and } \{x_k\}$$

It follows from the boundary condition $w_{N+1} = 0$ that $x_T^* = w_T$. Then, the Bellman equation can be solved recursively: first by going backward and retrieving the relationship between x_k^* and w_k, and then by going forward, beginning with the initial condition $w_1 = X$.

It turns out a simple and rather trivial solution: $x_1^* = ... = x_N^*$.

Cost-Driven Schedules / Risk-Neutral Framework

- This result is determined by the model assumption that the permanent impact does not depend on either price or the size of the unexecuted order.

- More complicated models generally do not have an analytical solution. Yet, they can be analyzed using numerical implementation of the dynamic programming technique.

- Obizhaeva & Wang (2005) expanded this approach to account for exponential decay of market impact.

- Gatheral (2009) described the relationship between the shape of the market impact function and the decay of market impact. In particular, Gatheral has shown that the exponential decay of market impact is compatible only with linear market impact.

Cost-Driven Schedules / Risk-Averse Framework

- The **risk-averse framework** for optimal execution was introduced by Grinold & Kahn (2000). Almgren & Chriss (2000) expanded this approach by constructing *the efficient trading frontier* in the space of possible execution strategies.

- Let's apply the Almgren-Chriss model to the selling process (the buying process is assumed to be symmetrical). Our goal is to sell X units within the time interval T.
  Let's divide T into N periods with length $\tau = T/N$ and define discrete times $t_k = k^*\tau$ where $k = 0, 1, ..., N$.

  Another list will also be used: $x = \{x_0, ..., x_N\}$, where x_k is the remaining number of units at time t_k to be sold; $x_0 = X$; $x_N = n_0 = 0$

$$x_k = X - \sum_{i=1}^{i=k} n_i = \sum_{i=k+1}^{i=N} n_i$$

Cost-Driven Schedules / Risk-Averse Framework

- It is assumed that price S follows the arithmetic random walk with no drift. Another assumption is that market impact can be partitioned into the permanent part that lasts the entire trading period T, and the temporary part that affects price only during one time interval τ. Then,

  $$S_k = S_{k-1} + \sigma\tau^{1/2} d\xi_1 - \tau g(n_k/\tau)$$

  where the function $g(n_k/\tau)$ describes the permanent market impact. The temporary market impact contributes only to the sale price of the order k

  $$\hat{S}_k = S_{k-1} + \sigma\tau^{1/2} d\xi_1 - \tau h(n_k/\tau)$$

  but does not affect S_k. And the total trading cost equals:

  $$IS = XS_0 - \sum_{k=1}^{N} n_k \hat{S}_k$$

Cost-Driven Schedules / Risk-Averse Framework

Again the total trading cost equals:

$$IS = XS_0 - \sum_{k=1}^{N} n_k \hat{S}_k$$

$$= -\sum_{k=1}^{N} x_k(\sigma\tau^{1/2} d\xi_k - \tau g(n_k/\tau)) + \sum_{k=1}^{N} n_k h(n_k/\tau)$$

Within these assumptions, the expected IS, $E(x)$ and its variance, $V(x)$, equal

$$E(x) = \sum_{k=1}^{N} \tau x_k g(n_k/\tau) + \sum_{k=1}^{N} n_k h(n_k/\tau)$$

$$V(x) = \sigma^2\tau \sum_{k=1}^{N} x_k^2$$

The Almgren-Chriss framework minimizes: $U = E(x) + \lambda V(x)$

Cost-Driven Schedules / Risk-Averse Framework

- Both permanent and temporary market impacts are assumed to be linear on order size:

$$g(n_k/\tau) = \gamma n_k/\tau$$
$$h(n_k/\tau) = \epsilon \operatorname{sign}(n_k) + \eta n_k/\tau$$

Here, γ and η are constant coefficients, ϵ is fixed cost (fees, etc.), and sign is the sign function. Then,

$$E(x) = \frac{1}{2}\gamma X^2 + \epsilon X + \frac{\tilde{\eta}}{\tau}\sum_{k=1}^{N} n_k^2, \tilde{\eta} = \eta - \gamma\tau/2$$

- Minimization of the utility function is then reduced to equating zero to $\delta U/\delta x_k$, which yields

$$x_{k-1} - 2x_k + x_{k+1} = \tilde{\kappa}^2\tau^2 x_k$$
$$\text{with } \tilde{\kappa}^2 = \lambda\sigma^2/\tilde{\eta}$$

Cost-Driven Schedules / Risk-Averse Framework

- The solution to the above formulation is

$$x_k = X \frac{\sin h(\kappa(T - t_k))}{\sin h(\kappa T)} \ , \ k = 0, 1, ..., N$$

Then, it follows from the definition $n_k = x_k - x_{k-1}$ that

$$n_k = 2X \frac{\sin h(\kappa\tau/2)}{\cos h(\kappa T)} \cos h(\kappa(T - t_{k-1/2})) \ , \ k = 1, ..., N$$

where $t_{k-1/2} = (k - 1/2)/\tau$ and κ satisfies the following relation

$$2(\cos h(\kappa\tau) - 1) = \tilde{\kappa}^2 \tau^2$$

When τ approaches zero, $\tilde{\eta} \to \eta$ and $\tilde{\kappa}^2 \to \kappa^2$. Note that κ is independent of T and characterizes exponential decay of the size of sequential child orders. Obviously, the higher is risk aversion λ, the shorter is the order's half-life.

Cost-Driven Schedules / Risk-Averse Framework

- Almgren & Chriss (2000) define the efficient trading frontier as the family of strategies that have minimal trading cost for a given cost variance, that is , a curve in the space E-V.

- Recent extension of the Almgren-Chriss framework by Huberman & Stahl (2005), Almgren & Lorenz (2007), Jondeau et al.(2008), and Shied & Schöneborn (2009) have led to models that account for time-dependent volatility and liquidity, sometimes within the continuum-time framework.

- All these extended models generally share the assumption that market impact can be represented as a combination of the permanent and short-lived transitory components.

- Bouchaud et al. (2004) and Schmidt (2010) exhibit the power-law decay of market impact.

What is Next?

- The average trade size for IBM, as reported in the Trade and Quote (TAQ) database, declined from 650 shares in 2004 to 240 shares in 2007. Falling trade sizes are evidence of the impact of algorithmic trading. Large, infrequent portfolio rebalancing and trading are being replaced by *small delta continuous trading*.

- The antithesis of the small delta continuous trading approach is embodied in the idea of *lazy portfolios*, in which portfolios are rebalanced infrequently to reduce market impact costs. Argument against lazy portfolios:

  1). As time passes, the weights drift further and further away from optimal target holdings, in both alpha and risk dimensions. 2). Use of an optimzer after long holding periods tends to produce large deviations from current holdings.

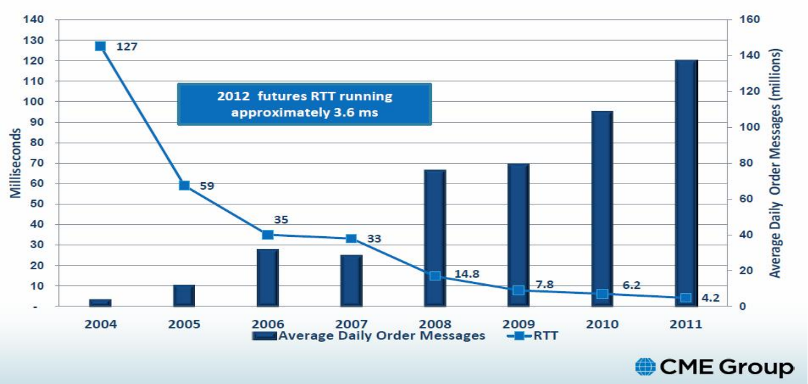
The *dynamic portifolio* or *small delta continuous trading* problem represents the next step in the evolution of institutional money management. The dynamic portolio problem differs in several different ways from the classical multiperiod consumption-investment problem:

1. The return probability distributions change throughout time.
2. The objective functions for active portfolio management do not depend on predicted alpha/risk, but rather on realized return/risk.
3. The dynamics of the model may be far more complex.

Other efforts of ongoing research in algorithmic trading are extending market microstructure and optimal execution models to futures, options, and fixed-income products.

The High-Frequency Arms Race

- Exhibit 11.3 Order Size and Round Trip Time



2012 futures RTT running approximately 3.6 ms

High Frequency Trading

- Algorithmic traders are liquidity providers that profit from the spread and the rebate (also referred to as the *maker taker fee*.
- Liquidity providers that post orders to buy or sell at fixed prices are offered a rebate from the exchange if their quotes result in trades. For example, in July 2009 Direct Edge paid a rebate of 0.25 cents per share to subscribing firms that provide liquidity and charged liquidity takers a fee of 0.28 cents.
- One part of being faster means reducing latency. A definition is to consider the so-called end-to-end latency, also referred to as total latency, which consists of two components: 1. exchange latency, and 2. member latency.

Latency

In the end-to-end latency, we can break it down into the following steps:

1. Price dissemination and distribution at the exchange.
2. Transmission of price information from the exchange to the firm.
3. Preparation of the order at the firm.
4. Distribution of the order to the exchange.
5. Place the order in order book.
6. Order acknowledgment from the exchange.
7. Final report on the order execution from the exchange.

An important part of latency is the remote location data transfer. With the current technology these transfers can be done in about 35 milliseconds between the West and East coasts.

Liquidity

The most critical component of an exchange is to be able to provide market participants with liquidity. We can loosely define liquidity as (1) the ability to trade quickly without significant price changes, and (2) the ability to trade large volumes without significant price changes. However, there are some known HFT strategies:

1. *Market-making*: market makers simultaneously post limit orders on both sides of the electronic limit order book to make the spread. In this way they provide liquidity, and they take the risks to lose to the informed traders.
2. *Relative value and arbitrage trading*: arbitrageurs take advantage of short-term mis-prices of indices's or assets traded on different venues to make profit. On example, would be S&P 5000 futures vs. SPY on other venues.
3. *Directional trading*: some HFT firms electronically parse news release, apply textual analysis, and trade on the inferred news from social media etc.

Obviously, there are physical limitations as to how much latency can be decreased. Standard arguments of economic theory suggest that over time through competition the profit margins of high-frequency trading will decrease.