E246: Electronics & Instrumentation

Lecture 2: Review of Basic Electronics
Plan

- Resistors in Series
- Resistors in Parallel
- Node Voltage Analysis
- Mesh Current Analysis
- Sections in textbook: 2.4~2.8
Introduction

- We reduce more complex circuits into simpler equivalent circuits
- We continue our focus on circuits composed of resistors
- We deal with ideal constant current or constant voltage sources (DC sources)
Resistors in Series

- If two elements connect at a single node, they are said to be in series.
- Series-connected elements carry the same current.

![Diagram of resistors in series](image)
Resistors in Series

From Kirchhoff’s voltage law

\[-v_s + v_1 + v_2 + v_3 = 0\]
Resistors in Series

From Kirchhoff’s current law

\[ i_s = i_1 = i_2 = i_3 \]
Resistors in Series

From Ohm’s law:

\[ v_1 = i_1 R_1 \]
\[ v_2 = i_2 R_2 \]
\[ v_3 = i_3 R_3 \]
Resistors in Series

Solving

\[ \nu_s = \nu_1 + \nu_2 + \nu_3 \]
\[ = i_1 R_1 + i_2 R_2 + i_3 R_3 \]
\[ = i_s (R_1 + R_2 + R_3) \]
\[ = i_s R_{eq} \]
\[ R_{eq} = (R_1 + R_2 + R_3) \]
Resistors in Series

In general,

\[ R_{eq} = \sum_{i=1}^{n} R_i = R_1 + R_2 + \cdots + R_n \]
Resistors in Series
Resistors in Parallel

- If elements connect at both terminals, they are said to be in parallel
- Parallel elements have the same voltage across their terminals

\[ \begin{circuit} 
 \node \hole (ground) at (-5,0) {};
 \node \hole (source) at (0,0) {};
 \node \hole (R1) at (1,0) {};
 \node \hole (R2) at (2,0) {};
 \node \hole (RN) at (3,0) {};
 \node \hole (terminal) at (4,0) {};
 \draw (source) to [R=R1] (R1);
 \draw (R1) to [R=R2] (R2);
 \draw (R2) to [R=\ldots] (RN);
 \draw (RN) to [R=\ldots] (terminal);
 \draw (ground) to (source);
 \end{circuit} \]
Resistors in Parallel

From Kirchhoff’s voltage law

\[ v_s = v_1 = v_2 \]
Resistors in Parallel

From Kirchhoff’s current law

\[ i_s - i_1 - i_2 = 0 \]

From Ohm’s law

\[ i_1 = \frac{v_1}{R_1}, \quad i_2 = \frac{v_2}{R_2} \]
Resistors in Parallel

Combine equations, we get

\[ i_s = v_s \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

Define conductance \( G \) as the inverse of resistance \( R \). Then we get

\[ i_s = v_s G_{eq} \]

where \( G_{eq} = G_1 + G_2 = \frac{1}{R_1} + \frac{1}{R_2} \)
Resistors in Parallel

In general,

\[ G_{eq} = \sum_{i=1}^{n} G_i = G_1 + G_2 + \cdots + G_n \]
Resistors in Parallel

Since \( G_{eq} = \frac{1}{R_{eq}} = G_1 + G_2 + \cdots + G_n \)

Then: \( \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \)

In the case of \( n=2 \),

\[ R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \]
Resistors in Parallel
Notations

- **Node**: A point where two or more circuit elements join.
- **Node voltage**: The voltage at any node of the circuit, relative to the reference node, is called a node voltage.
Notations

- Nodes are drawn as points, or are drawn using horizontal or vertical lines.
- Any node can be selected to be the reference node.
- We will often choose the node at the bottom of the circuit to be the reference node, marked with the symbol used in the right above figure.
Notations

- There are two node voltages:
  - The voltage at node a with respect to the reference node c;
  - The voltage at node b with respect to the reference node c.
Node Voltage Analysis

Analyzing a connected circuit containing $n$ nodes will require $n-1$ equations.

Use Kirchhoff’s current law at each of the circuit’s nodes (except at the reference node), we can get a set of equations, which are called the node equations.
Node Voltage Analysis

- Represent $v_c$ as a function of $v_a$ and $v_b$:
  - Apply KVL to the right-hand mesh, we get
    
    
    $-v_a + v_c + v_b = 0$
Node Voltage Analysis

Now all three resistor voltages are represented as functions of node voltages.

Apply Ohm’s law to each resistor, we get:
Node Voltage Analysis

Apply KCL at nodes a and b, we get node equations:

\[
i_s = \frac{v_a}{R_2} + \frac{v_a - v_b}{R_1}
\]

\[
\frac{v_a - v_b}{R_1} = \frac{v_b}{R_3}
\]
Node Voltage Analysis

If the values of $R_1, R_2, R_3, i_s$ are given, solving the above node equations, we can get node voltage $v_a$ and $v_b$. 
Node Voltage Analysis

- **Summary of procedure:**
  - Mark nodes in the circuit, select reference node;
  - Mark voltages of each resistor, represent them using node voltages;
  - Applying Ohm’s law, represent current in each resistor as a function of node voltage and resistance;
  - Applying KCL to each node, get node equations;
  - Solving node equations, get node voltages.
Example 1:

Q: Determine the value of the resistance $R$ in the above circuit.
Example 2:

Q: Obtain the node equations for the above circuit.
A supernode consists of two nodes connected by an independent or a dependent voltage source.
Kirchhoff’s current law holds for the supernode, i.e., the algebraic sum of the currents entering a supernode is zero.
Node Voltage Analysis

At the supernode, \( \frac{v_a}{R_1} + \frac{v_b}{R_2} = i_s \)

\( v_a = v_s + v_b \),

Solve for the unknown \( v_b \), we get

\( v_b = \frac{R_1 R_2 i_s - R_2 v_s}{R_1 + R_2} \)
Node Voltage Analysis with a Voltage Source

Case 1: the voltage source connects node q and the reference node (ground):
- Set $v_q$ equal to the source voltage accounting for the polarities and proceed to write the KCL at the remaining nodes.

The voltage source lies between two nodes, a and b:
- Create a supernode that incorporates a and b and equate the sum of all the currents into the supernode (both node a and b) to zero.
Mesh Current Analysis

- A closed path or a loop is drawn by starting at a node and tracing a path such that we return to the original node without passing an intermediate node more than once.
- A mesh is a loop that does not contain any other loops within it.
- Mesh current analysis is applicable only to planar networks.
Mesh Current Analysis

- A planar circuit is one that can be drawn on a plane, without crossovers.

A planar circuit with four meshes. A nonplanar circuit.
Mesh Current Analysis

- A mesh current is the current that flows through the elements constituting the mesh.
- The current in an element common to two meshes is the algebraic sum of the mesh currents.
Mesh Current Analysis

Mesh current analysis: *use Kirchhoff’s voltage law around each mesh.*

Convention:
- Move around the mesh in the *clockwise* direction;
- Add the voltage if we encounter the + sign before – sign.
Mesh Current Analysis

Mesh 1: \(-v_s + R_1 i_1 + R_3 (i_1 - i_2) = 0\)

Mesh 2: \(R_3 (i_2 - i_1) + R_2 i_2 = 0\)
Mesh Current Analysis

\[
\begin{align*}
- \nu_s + R_1 i_1 + R_3 (i_1 - i_2) &= 0 \quad \text{⇒} \quad i_1 (R_1 + R_3) - i_2 R_3 = \nu_s \\
R_3 (i_2 - i_1) + R_2 i_2 &= 0 \quad \text{⇒} \quad -i_1 R_3 + i_2 (R_3 + R_2) = 0
\end{align*}
\]

The two equations will enable us to determine the two mesh currents \(i_1\) and \(i_2\).
Mesh Current Analysis

A circuit that contains only independent voltage sources and resistors results in a specific format of equations that can readily be obtained.
Mesh Current Analysis

Mesh 1: \(-v_s + R_1 i_1 + R_4 (i_1 - i_2) = 0\)

Mesh 2: \(R_2 i_2 + R_5 (i_2 - i_3) + R_4 (i_2 - i_1) = 0\)

Mesh 3: \(R_5 (i_3 - i_2) + R_3 i_3 + v_g = 0\)
Mesh Current Analysis

After collecting similar terms:

Mesh 1: \((R_1 + R_4)i_1 - R_4i_2 = v_s\)

Mesh 2: \(-R_4i_1 + (R_2 + R_5 + R_4)i_2 - R_5i_3 = 0\)

Mesh 3: \(R_5i_2 + (R_3 + R_5)i_3 = -v_g\)
Mesh Current Analysis

The general matrix equation can be represented by:

$$\mathbf{R}\mathbf{i} = \mathbf{v}_s$$

$$\mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}, \quad \mathbf{v}_s = \begin{bmatrix} v_{s1} \\ v_{s2} \\ \vdots \\ v_{sN} \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} (R_1 + R_4) & -R_4 & 0 \\ -R_4 & (R_2 + R_4 + R_5) & -R_5 \\ 0 & -R_5 & (R_3 + R_5) \end{bmatrix}$$
Example 1:

Q: Determine the value of the voltage measured by the voltmeter.
Mesh Current Analysis with Current Source

Case 1:

\[ i_2 = -i_s \]

\[ i_1 (R_1 + R_2) - i_2 R_2 = v_s \]
Mesh Current Analysis with Current Source

**Conclusion:**
If a current source appears on the periphery of only one mesh \( n \), then equate the mesh current \( i_n \) to the current source current, accounting for the direction of the current source.
Mesh Current Analysis with Current Source

Case 2:

\[ i_2 - i_1 = i_s \]

Mesh 1: \[ R_1 i_1 + v_{ab} = v_s \]

Mesh 2: \[ (R_2 + R_3) i_2 - v_{ab} = 0 \]
Mesh Current Analysis with Current Source

**Supermesh:** One large mesh created from two meshes that have an independent or dependent current source in common.
Mesh Current Analysis with Current Source

- Write KVL around the periphery of the supermesh,

Supermesh:
\[-10 + 1(i_1 - i_3) + 3(i_2 - i_3) + 2i_2 = 0\]

Mesh 3:
\[1(i_3 - i_1) + 2i_3 + 3(i_3 - i_2) = 0\]

Current source:
\[1i_1 - 1i_2 = 5\]
Conclusion:
If a current source is common to two meshes, there are two solution methods:

- Assume a voltage $v_{ab}$ across the terminals of the current source, write the KVL equations for the two meshes, and add them to eliminate $v_{ab}$;
- Create a supermesh as the periphery of the two meshes and write one KVL equation around the periphery of the supermesh. In addition, write the constraining equation for the two mesh currents in terms of the current source.
Example 2:

Q: Determine the mesh currents.