Midterm Exam

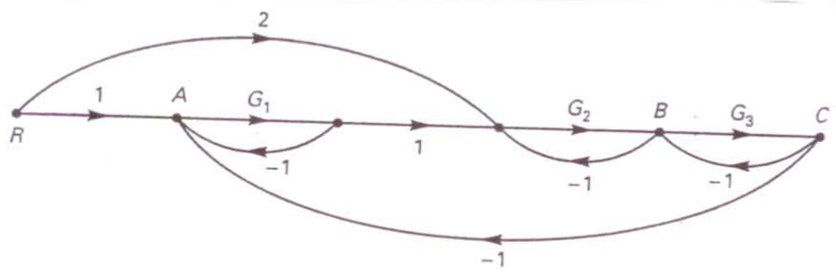
EE 478 Control Systems (Fall 2009)

Name: Solutions

SID: ____________

Do all work in the spaces provided, and you can use the back of the paper if needed. Show all work and organize it for partial credit. Closed-book, closed-notes. 1-page note allowed.

Problem 1. (30 points) For the signal flow graph shown below, use Mason's rule to find the transfer function from R(s) to C(s).



Hint: Mason's rule:

The transfer function, $C(s)/R(s)$, of a system represented by a signal-flow graph is

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta} \qquad (5.28)$$

where

k = number of forward paths

T_k = the kth forward-path gain

$\Delta = 1 - \sum$ loop gains $+ \sum$ nontouching-loop gains taken two at a time $- \sum$ nontouching-loop gains taken three at a time $+ \sum$ nontouching-loop gains taken four at a time $- \cdots$

$\Delta_k = \Delta - \sum$ loop gain terms in Δ that touch the kth forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the kth forward path

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Solution: forward paths: $G_1G_2G_3$, $2G_2G_3$

loops: $-G_1$, $-G_2$, $-G_3$, $-G_1G_2G_3$

Nontouching loops taken two at a time:

G_1G_2, G_1G_3.

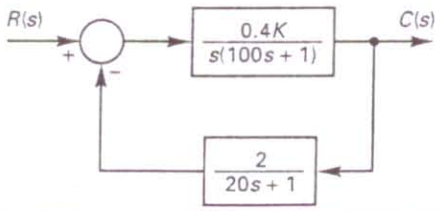
$$\Delta = 1 + G_1 + G_2 + G_3 + G_1G_2G_3 + G_1G_2 + G_1G_3$$

$\Delta_1 = 1$ since all loops touch the first forward path

$\Delta_2 = 1 + G_1$ since the loop $(-G_1)$ does not touch the second forward path.

$$G(s) = \frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + 2G_2G_3(1+G_1)}{1+G_1+G_2+G_3+G_1G_2G_3+G_1G_2+G_1G_3}$$

Problem 2. (35 points) Given a system shown below, find the characteristic equation of the system, and then use Routh stability criterion to determine the range of K for the system to be stable.



Solution: CE: $1 + G(s)H(s) = 0$

$$1 + \frac{0.4k}{S(100s+1)} \cdot \frac{2}{20s+1} = 0$$

$$S(100s+1)\cdot(20s+1) + 0.8k = 0$$

$$2000s^3 + 120s^2 + S + 0.8k = 0$$

Routh ~~table~~:

s^3	2000	1
s^2	120	0.8k
s^1	b_1	b_2
s^0	c_1	c_2

$$b_1 = -\frac{\begin{vmatrix} 2000 & 1 \\ 120 & 0.8k \end{vmatrix}}{120} = \frac{120 - 1600k}{120}$$

$$b_2 = 0$$

$$c_1 = -\frac{\begin{vmatrix} 120 & 0.8k \\ b_1 & 0 \end{vmatrix}}{b_1} = 0.8k$$

$$c_2 = 0$$

$$\left.\begin{matrix} b_1 > 0 \\ c_1 > 0 \end{matrix}\right\} \Rightarrow \begin{cases} 120 - 1600k > 0 \\ 0.8k > 0 \end{cases}$$

$$\Rightarrow \begin{cases} k < \frac{120}{1600} = 0.075 \\ k > 0 \end{cases}$$

$$\Rightarrow 0 < K < 0.075.$$

Problem 3. (35 points)

A unity feedback system has a feedforward transfer function $G(s) = \frac{1}{s(s+1)}$.

1). Determine the damping ratio and undamped natural frequency of the system.
2). Find the closed-loop poles.
3). For the unit step response of the system, calculate the overshoot, peak time, and steady-state error.

Solution: 1). TF: $G_{cl}(s) = \frac{G(s)}{1+G(s)}$

$$= \frac{1}{s^2+s+1}$$

$$\left.\begin{array}{l} 2\xi w_n = 1, \\ w_n^2 = 1 \end{array}\right\} \Rightarrow \begin{array}{l} \xi = 0.5 \\ w_n = 1 \end{array}$$

2). $\sigma = \xi w_n = \frac{1}{2}$, $w_d = w_n\sqrt{1-\xi^2} = \frac{\sqrt{3}}{2}$,

$$\Rightarrow \quad s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}.$$

3). $M_p = e^{-\frac{\sigma}{w_d}\pi} = 0.163 = 16.3\%$.

$$t_p = \frac{\pi}{w_d} = 3.63 \text{ sec.}$$

$$e_{ss} = \frac{1}{1+k_p} = 0.$$

$$\left(k_p = \lim_{s\to 0} G(s) = \lim_{s\to 0}\frac{1}{s(s+1)} = \infty\right)$$