2.

a. Using the frequency shift theorem and the Laplace transform of sin ωt, $F(s) = \frac{\omega}{(s+a)^2+\omega^2}$.

b. Using the frequency shift theorem and the Laplace transform of cos ωt, $F(s) = \frac{(s+a)}{(s+a)^2+\omega^2}$.

c. Using the integration theorem, and successively integrating u(t) three times, $\int dt = t$; $\int t\,dt = \frac{t^2}{2}$;

$\int \frac{t^2}{2} dt = \frac{t^3}{6}$, the Laplace transform of $t^3u(t)$, $F(s) = \frac{6}{s^4}$.

3.

a. The Laplace transform of the differential equation, assuming zero initial conditions,

is, $(s+7)X(s) = \frac{5s}{s^2+2^2}$. Solving for X(s) and expanding by partial fractions,

$$\frac{5s}{(s+7)(s^2+4)} = -\frac{35}{53}\frac{1}{s+7} + \frac{5}{53}\frac{7s+4}{s^2+4}$$

Or,

$$\frac{5s}{(s+7)(s^2+4)} = -\frac{35}{53}\frac{1}{s+7} + \frac{5}{53}\frac{7s+2\sqrt{4}}{s^2+4}$$

Taking the inverse Laplace transform, $x(t) = -\frac{35}{53} e^{-7t} + (\frac{35}{53} \cos 2t + \frac{10}{53} \sin 2t)$.

b. The Laplace transform of the differential equation, assuming zero initial conditions, is,

$(s^2+6s+8)X(s) = \frac{15}{s^2+9}$.

Solving for X(s)

$$X(s) = \frac{15}{(s^2+9)(s^2+6s+8)}$$

and expanding by partial fractions,

$$X(s) = -\frac{3}{65}\frac{6s + \frac{1}{\sqrt{9}}\sqrt{9}}{s^2+9} - \frac{3}{10}\frac{1}{s+4} + \frac{15}{26}\frac{1}{s+2}$$

Taking the inverse Laplace transform,

$$x(t) = -\frac{18}{65}\cos(3t) - \frac{1}{65}\sin(3t) - \frac{3}{10}e^{-4t} + \frac{15}{26}e^{-2t}$$

c. The Laplace transform of the differential equation is, assuming zero initial conditions, $(s^2+8s+25)x(s) = \frac{10}{s}$. Solving for X(s)

$$X(s) = \frac{10}{s\left(s^2 + 8s + 25\right)}$$

and expanding by partial fractions,

$$X(s) = \frac{2}{5}\,\frac{1}{s} - \frac{2}{5}\,\frac{1\,(s+4) + \frac{4}{\sqrt{9}}\sqrt{9}}{s + 4^2 + 9}$$

Taking the inverse Laplace transform,

$$x(t) = \frac{2}{5} - e^{-4t}\left(\frac{8}{15}\sin(3t) + \frac{2}{5}\cos(3t)\right)$$

4.

a. Taking the Laplace transform with initial conditions, $s^2X(s)-2s+3+2sX(s)-4+2X(s) = \frac{2}{s^2+2^2}$.

Solving for X(s),

$$X(s) = \frac{2s^3 + s^2 + 8s + 6}{(s^2+4)(s^2+2s+2)}.$$

Expanding by partial fractions

$$X(s) = -\left(\frac{1}{5}\right)\frac{s + \frac{1}{\sqrt{4}}\sqrt{4}}{s^2 + 4} + \left(\frac{1}{5}\right)\frac{11(s+1) - \frac{3}{\sqrt{1}}\sqrt{1}}{(s+1)^2 + 1}$$

Therefore, $x(t) = -0.2\cos 2t - 0.1\sin 2t + e^{-t}(2.2\cos t - 0.6\sin t)$.

b. Taking the Laplace transform with initial conditions, $s^2X(s)-2s-1+2sX(s)-4+X(s) = \frac{5}{s+2} + \frac{1}{s^2}$.

Solving for X(s),

$$X(s) = \frac{2s^4 + 9s^3 + 15s^2 + s + 2}{(s+2)(s+1)^2 s^2}$$

$$X(s) = 5\frac{1}{s+2} - \frac{1}{s+1} + 9\frac{1}{(s+1)^2} - 2\frac{1}{s} + \frac{1}{s^2}$$

Therefore, $x(t) = 5e^{-2t} - e^{-t} + 9te^{-t} - 2 + t$.

c. Taking the Laplace transform with initial conditions, $s^2X(s)-s-2+4X(s) = \frac{2}{s^3}$. Solving for X(s),

$$X(s) = \frac{s^4 + 2s^3 + 2}{(s^2+4)s^3}$$

$$X(s) = \frac{1}{8}\,\frac{9s + 8.2}{s^2 + 4} - \frac{1}{8}\,\frac{1}{s} + \frac{1}{2}\,\frac{1}{s^3}$$

Therefore, $x(t) = \frac{9}{8}\cos 2t + \sin 2t - \frac{1}{8} + \frac{1}{4}t^2$.

7.

The Laplace transform of the differential equation, assuming zero initial conditions, is,

$(s^3+3s^2+5s+1)Y(s) = (s^3+4s^2+6s+8)X(s)$.

Solving for the transfer function, $\dfrac{Y(s)}{X(s)} = \dfrac{s^3+4s^2+6s+8}{s^3+3s^2+5s+1}$.