

**8.**

**a.** Pole: -2;  $c(t) = A + Be^{-2t}$ ; first-order response.

**b.** Poles: -3, -6;  $c(t) = A + Be^{-3t} + Ce^{-6t}$ ; overdamped response.

**c.** Poles: -10, -20; Zero: -7;  $c(t) = A + Be^{-10t} + Ce^{-20t}$ ; overdamped response.

d. Poles:  $(-3+j3\sqrt{15})$ ,  $(-3-j3\sqrt{15})$ ;  $c(t) = A + Be^{-3t} \cos(3\sqrt{15} t + \phi)$ ; underdamped.

e. Poles:  $j3$ ,  $-j3$ ; Zero:  $-2$ ;  $c(t) = A + B \cos(3t + \phi)$ ; undamped.

f. Poles:  $-10$ ,  $-10$ ; Zero:  $-5$ ;  $c(t) = A + Be^{-10t} + Cte^{-10t}$ ; critically damped.

9.

**Program:**

```
p=roots([1 6 4 7 2])
```

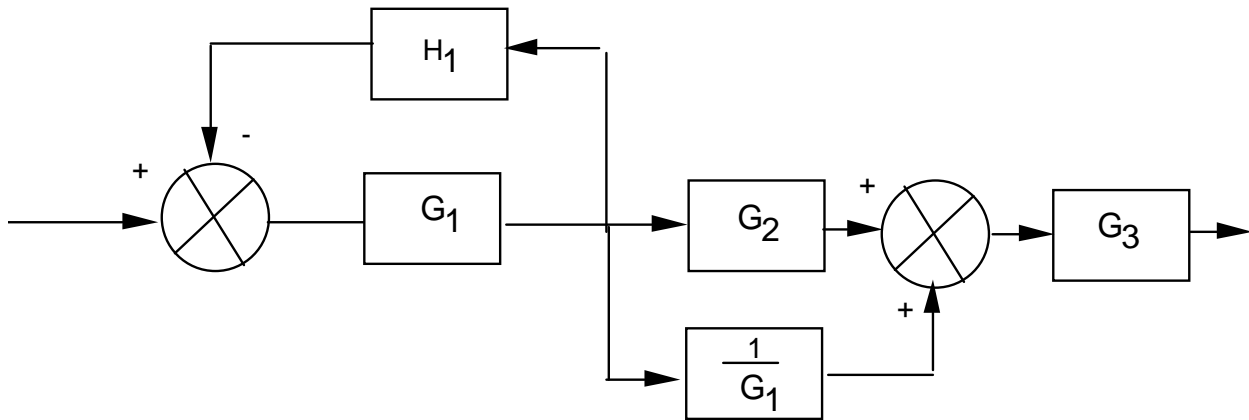
**Computer response:**

```
p =
```

```
-5.4917  
-0.0955 + 1.0671i  
-0.0955 - 1.0671i  
-0.3173
```

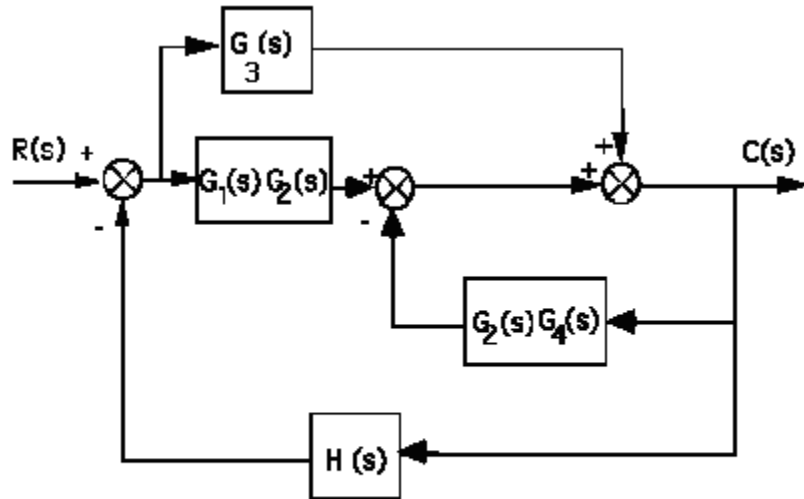
2.

Push  $G_1(s)$  to the left past the pickoff point.

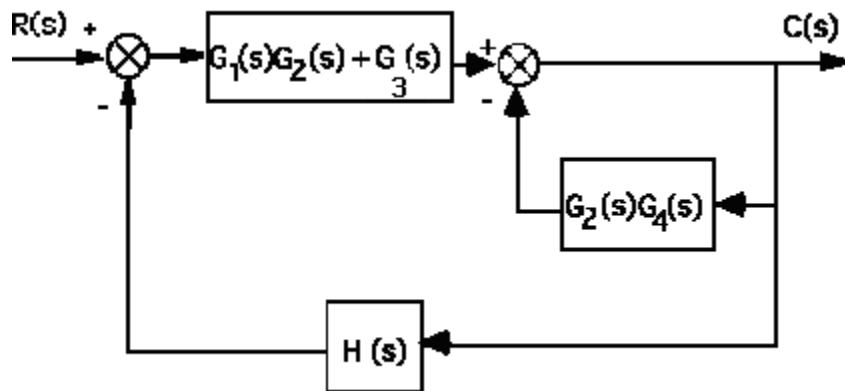


$$\text{Thus, } T(s) = \left( \frac{G_1}{1 + G_1 H_1} \right) \left( G_2 + \frac{1}{G_1} \right) G_3 = \frac{(G_1 G_2 + 1) G_3}{(1 + G_1 H_1)}$$

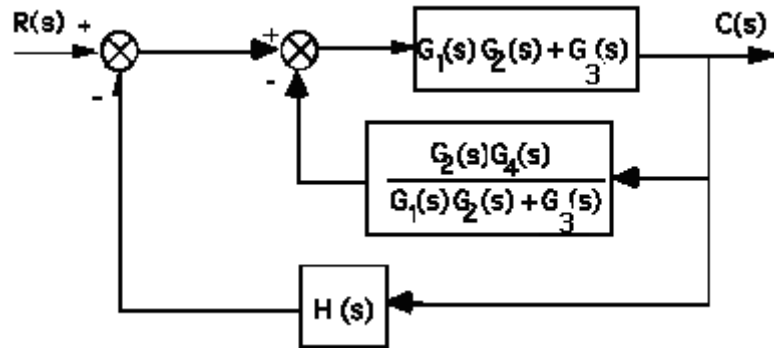
4. Push  $G_2(s)$  to the left past the summing junction.



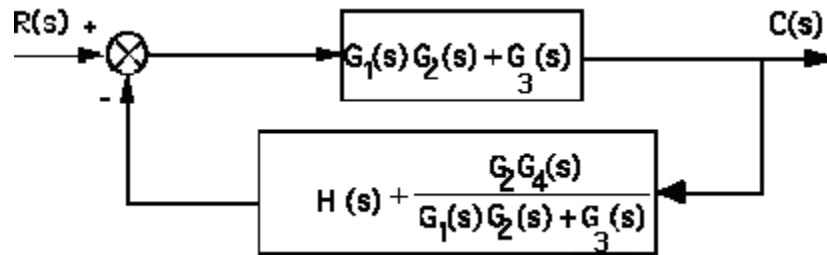
Collapse the summing junctions and add the parallel transfer functions.



Push  $G_1(s)G_2(s) + G_3(s)$  to the right past the summing junction.



Collapse summing junctions and add feedback paths.



Applying the feedback formula,

$$\begin{aligned}
 T(s) &= \frac{G_3(s) + G_1(s)G_2(s)}{1 + [G_3(s) + G_1(s)G_2(s)] \left[ H + \frac{G_2(s)G_4(s)}{G_3(s) + G_1(s)G_2(s)} \right]} \\
 &= \frac{G_3(s) + G_1(s)G_2(s)}{1 + H[G_3(s) + G_1(s)G_2(s)] + G_2(s)G_4(s)}
 \end{aligned}$$

26.

$\Delta = 1 + [G_2G_3G_4 + G_3G_4 + G_4 + 1] + [G_3G_4 + G_4]$ ;  $T_1 = G_1G_2G_3G_4$ ;  $\Delta_1 = 1$ . Therefore,

$$T(s) = \frac{T_1\Delta_1}{\Delta} = \frac{G_1G_2G_3G_4}{2 + G_2G_3G_4 + 2G_3G_4 + 2G_4}$$

27.

Closed-loop gains:  $G_2G_4G_6G_7H_3$ ;  $G_2G_5G_6G_7H_3$ ;  $G_3G_4G_6G_7H_3$ ;  $G_3G_5G_6G_7H_3$ ;  $G_6H_1$ ;  $G_7H_2$

Forward-path gains:  $T_1 = G_1G_2G_4G_6G_7$ ;  $T_2 = G_1G_2G_5G_6G_7$ ;  $T_3 = G_1G_3G_4G_6G_7$ ;  $T_4 =$

$G_1G_3G_5G_6G_7$

Nontouching loops 2 at a time:  $G_6H_1G_7H_2$

$\Delta = 1 - [H_3G_6G_7(G_2G_4 + G_2G_5 + G_3G_4 + G_3G_5) + G_6H_1 + G_7H_2] + [G_6H_1G_7H_2]$

$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$

$$\begin{aligned}
 T(s) &= \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4}{\Delta} \\
 &= \frac{G_1G_2G_4G_6G_7 + G_1G_2G_5G_6G_7 + G_1G_3G_4G_6G_7 + G_1G_3G_5G_6G_7}{1 - H_3G_6G_7(G_2G_4 + G_2G_5 + G_3G_4 + G_3G_5) - G_6H_1 - G_7H_2 + G_6H_1G_7H_2}
 \end{aligned}$$

**28.**

Closed-loop gains:  $-s^2$ ;  $-\frac{1}{s}$ ;  $-\frac{1}{s}$ ;  $-s^2$

Forward-path gains:  $T_1 = s$ ;  $T_2 = \frac{1}{s^2}$

Nontouching loops: None

$$\Delta = 1 - (-s^2 - \frac{1}{s} - \frac{1}{s} - s^2)$$

$$\Delta_1 = \Delta_2 = 1$$

$$G(s) = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} = \frac{s + \frac{1}{s^2}}{1 + (s^2 + \frac{1}{s} + \frac{1}{s} + s^2)} = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$