

23.

$$\text{a. } \zeta = \frac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.56, \omega_n = \frac{4}{\zeta T_s} = 11.92. \text{ Therefore, poles} = -\zeta\omega_n \pm j\omega_n \sqrt{1-\zeta^2}$$

$$= -6.67 \pm j9.88.$$

$$\text{b. } \zeta = \frac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.591, \omega_n = \frac{\pi}{T_P \sqrt{1-\zeta^2}} = 0.779.$$

$$\text{Therefore, poles} = -\zeta\omega_n \pm j\omega_n \sqrt{1-\zeta^2} = -0.4605 \pm j0.6283.$$

$$\text{c. } \zeta\omega_n = \frac{4}{T_s} = 0.571, \omega_n \sqrt{1-\zeta^2} = \frac{\pi}{T_n} = 1.047. \text{ Therefore, poles} = -0.571 \pm j1.047.$$

25.

a. Writing the equation of motion yields, $(3s^2 + 15s + 33)X(s) = F(s)$

Solving for the transfer function,

$$\frac{X(s)}{F(s)} = \frac{1/3}{s^2 + 5s + 11}$$

b. $\omega_n^2 = 11$ r/s, $2\zeta\omega_n = 5$. Therefore $\zeta = 0.754$, $\omega_n = 3.32$. $T_s = \frac{4}{\zeta\omega_n} = 1.6$ s; $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.44$

s; %OS = $e^{-\zeta\pi} / \sqrt{1-\zeta^2} \times 100 = 2.7$ %; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$; therefore, $T_r = 0.69$ s.

28.

Since the third pole is more than five times the real part of the dominant pole, $s^2+1.204s+2.829$ determines the transient response. Since $2\zeta\omega_n = 1.204$, and $\omega_n = \sqrt{2.829} = \omega_n = 1.682$, $\zeta = 0.358$,

$$\% \text{OS} = e^{-\zeta\pi / \sqrt{1-\zeta^2}} \times 100 = 30\%, \quad T_s = \frac{4}{\zeta\omega_n} = 6.64 \text{ sec}, \quad T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 2 \text{ sec}; \quad \omega_n T_r = 1.4,$$

therefore, $T_r = 0.832$.

1.

$$e(\infty) = \lim_{s \rightarrow 0} s E(s) = \lim_{s \rightarrow 0} \frac{s R(s)}{1+G(s)}$$

where

$$G(s) = \frac{450(s+12)(s+8)(s+15)}{s(s+38)(s^2+2s+28)}$$

For step, $e(\infty) = 0$. For $37tu(t)$, $R(s) = \frac{37}{s^2}$. Thus, $e(\infty) = 6.075 \times 10^{-2}$. For parabolic input,

$e(\infty) = \infty$.

4.

Reduce the system to an equivalent unity feedback system by first moving $1/s$ to the left past the summing junction. This move creates a forward path consisting of a parallel pair, $\left(\frac{1}{s} + 1\right)$ in cascade

with a feedback loop consisting of $G(s) = \frac{2}{s+3}$ and $H(s) = 7$. Thus,

$$G_e(s) = \left(\frac{s+1}{s}\right) \left(\frac{2/(s+3)}{1+14/(s+3)}\right) = \frac{2(s+1)}{s(s+17)}$$

Hence, the system is Type 1 and the steady-state errors are as follows:

Steady-state error for $15u(t) = 0$.

Steady-state error for $15tu(t) = \frac{15}{K_v} = \frac{15}{2/17} = 127.5$.

Steady-state error for $15t^2u(t) = \infty$

15.

Collapsing the inner loop and multiplying by $1000/s$ yields the equivalent forward-path transfer function as,

$$G_e(s) = \frac{10^5(s+2)}{s(s^2 + 1005s + 2000)}$$

Hence, the system is Type 1.

18.

a. $e(\infty) = \frac{1/10}{K_v} = 0.01$; where $K_v = \frac{7K}{5 \times 8 \times 12} = 10$. Thus, $K = 685.71$.

b. $K_v = 10$.

c. The minimum error will occur for the maximum gain before instability. Using the Routh-Hurwitz Criterion along with $T(s) = \frac{K(s+7)}{s^4 + 25s^3 + 196s^2 + (480+K)s + 7K}$:

s^4	1	196	$7K$	For Stability
s^3	25	$480+K$		
s^2	$4420-K$	$175K$		$K < 4420$
s^1	$-K^2 - 435K + 2121600$			$-1690.2 < K < 1255.2$
s^0	$175K$			$K > 0$

Thus, for stability and minimum error $K = 1255.2$. Thus, $K_v = \frac{7K}{5 \times 8 \times 12} = 18.3$ and

$$e(\infty) = \frac{1/10}{K_v} = \frac{1/10}{18.3} = 0.0055.$$