9. Consider the system shown in Fig. 4.33. Show that the system is type 1 and compute the K_v.

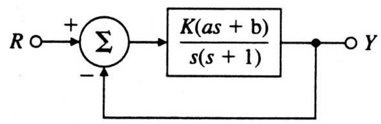


Figure 4.33: Control system for Problem 9

Solution:

The system has unity feedback with one pole at $s = 0$ and is thus Type 1 with $K_v = \lim_{s \to 0} sG(s) = Kb$.

23. A position control system has the closed-loop transfer function (meter/meter) given by

$$\frac{Y(s)}{R(s)} = \frac{b_0 s + b_1}{s^2 + a_1 s + a_2}.$$

(a) Choose the parameters (a_1, a_2, b_0, b_1) so that the following specifications are satisfied simultaneously:

i. The rise time $t_r < 0.1$ sec.

ii. The overshoot $M_p < 20\%$.

iii. The settling time $t_s < 0.5$ sec.

iv. The steady-state error to a step reference is zero.

v. The steady-state error to a ramp reference input of 0.1 m/sec. is not more than 1 mm.

(b) Verify your answer via MATLAB simulation.

Solution:

(a)

$$\frac{Y}{R} = \frac{b_0 s + b_1}{s^2 + a_1 s + a_2} = \frac{b_1}{a_2} \cdot \frac{\frac{s}{b_1/b_0} + 1}{(\frac{s}{\sqrt{a_2}})^2 + \frac{a_1}{\sqrt{a_2}} \cdot \frac{s}{\sqrt{a_2}} + 1} = K \frac{\frac{s}{\alpha \zeta \omega_n} + 1}{(\frac{s}{\omega_n})^2 + \frac{2\zeta s}{\omega_n} + 1}$$

Comparing to standard form:

$$\omega_n = \sqrt{a_2}\ , \ \zeta = \frac{a_1}{2\sqrt{a_2}}\ , \ \alpha = \frac{2b_1}{a_1 b_0}$$

i. $t_r \leq 0.1$ sec

$$\Longrightarrow \omega_n \geq \frac{1.8}{0.1} \quad \Longrightarrow b_1 = a_2 \geq 18^2$$

ii. $M_p \leq 0.20$ and from Chapter 3,

$$\alpha \geq 2 \quad \text{for } \zeta = 0.5$$

$$\alpha \geq 0.7 \quad \text{for } \zeta = 0.7$$

iii. $t_s \leq 0.5$ sec

$$\Longrightarrow \sigma = \zeta \omega_n \geq \frac{4.6}{t_s} \quad \Longrightarrow a_1 \geq 18.4$$

iv.

$$E(s) = R(s) - Y(s) = (1 - \frac{b_0 s + b_1}{s^2 + a_1 s + a_2}) R(s)$$

$$e_{ss}(\text{step}) = \lim_{s \to 0} s \frac{s^2 + (a_1 - b_0)s + (a_2 - b_1)}{(s^2 + a_1 s + a_2)} \frac{r_0}{s} = 0$$

v.

$$e_{ss}(\text{ramp}) = \lim_{s\to 0} s \frac{s^2 + (a_1 - b_0)}{s^2 + a_1 s + a_2} \times \frac{0.1}{s^2} = 0.1 \frac{a_1 - b_0}{a_2} \le 0.001$$

$$\Longrightarrow \; a_1 - b_o \le 0.01 a_2$$

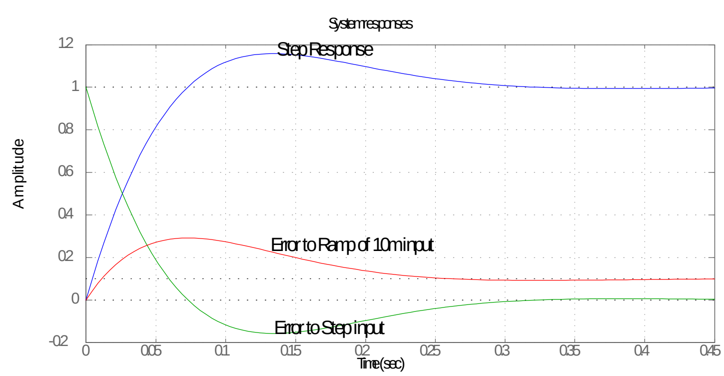
(b) A solution :

$$a_2 = 324 \quad b_1 = 324 \qquad \text{satisfies (i) and (iv)}$$

$$\zeta = 0.7 \implies a_1 = 2\zeta\sqrt{a_2} = 25.7 \qquad \text{satisfies (iii)}$$

$$b_0 = 22.5 \qquad \text{satisfies (v)} \;\; b_0 \ge 21.96$$

Check (b): $\alpha = \dfrac{2 * 324}{25.7 \times 22.5} = 1.12 > 1.1$. If (b) were not satisfied more iterations would be needed.



36. Consider the system shown in Fig. 4.52, which consists of a prefilter and a unity feedback system.

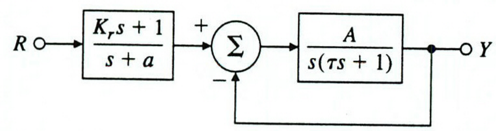


Figure 4.52: Unity feedback system with prefilter for Problem 36

(a) Determine the transfer function from R to Y.

(b) Determine the steady-state error due to a step input.

(c) Discuss the effect of different values of (K_r, a) on the system's response.

(d) For each of the following three cases,

$$(1)\ A = 1,\ \tau = 1, \quad (2)\ A = 10,\ \tau = 1, \quad (3)\ A = 1,\ \tau = 2,$$

use MATLAB to find values for K_r and a so that (if possible)

i. the rise time is less than 1.5 sec.,
ii. the overshoot is less than 20%, ,
iii. the settling time is less than 10 sec. and
iv. the steady-state error is less than 5%.

In cases where the specifications are easily met, try to make the rise time as small as possible. If the specifications cannot be met, find the design to meet as many of the specifications as possible, in the order given.

Solution:

(a)

$$Y = \frac{A}{s(\tau s + 1)} E = \frac{A}{s(\tau s + 1)} \left(\frac{K_r s + 1}{s + a} R - Y\right)$$

or

$$\left(1 + \frac{A}{s(\tau s + 1)}\right) Y = \frac{A(K_r s + 1)}{s(s + a)(\tau s + 1)} R$$

$$Y = \frac{A(K_r s + 1)}{s(s + a)(\tau s + 1)} \frac{s(\tau s + 1)}{s(\tau s + 1) + A} R$$

$$Y = \frac{A(K_r s + 1)}{(s + a)\left[s(\tau s + 1) + A\right]} R = \frac{A K_r s + A}{\tau s^3 + (1 + a\tau)s^2 + (a + A)s + Aa} R$$

(b)

$$y(\infty) = \lim_{s \to 0} s \frac{1}{s} \frac{AK_r s + A}{\tau s^3 + (1 + a\tau)s^2 + (a + A)s + Aa} = \frac{1}{a}$$

$$e_{ss,step} = \frac{1}{a} - 1 = \frac{1 - a}{a}$$

(c) K_r determines the prefilter zero location and can have a significant effect on overshoot. The prefilter pole location a will affect the speed of the transient response and the size of the steady-state error to a step.

- i. $A = 1, \tau = 1 : K_r = 1.1, a = 1$ yields $t_r = 1.51$ sec, $M_p = 18\%$, $t_s = 9$ sec.

  ii. $A = 10, \tau = 1 : K_r = 0.555$, $a = 1$ yields response well within specs, with min. rise time of 0.53 sec.

  iii. $A = 1, \tau = 2$: specs cannot be met. $K_r = 0.1, a = 1$ results in 23% overshoot and $t_s = 17$ sec and $t_r = 2.4$ sec.

See the following step responses.