Problems and solutions for Section 5.2

2. Roughly sketch the root loci for the pole-zero maps as shown in Fig. 5.62. Show your estimates of the center and angles of the asymptotes, a rough evaluation of arrival and departure angles for complex poles and zeros, and the loci for positive values of the parameter K. Each pole-zero map is from a characteristic equation of the form

$$1 + K\frac{b(s)}{a(s)} = 0,$$

where the roots of the numerator $b(s)$ are shown as small circles o and the roots of the denominator $a(s)$ are shown as $\times's$ on the s-plane. Note that in Fig. 5.62(c), there are two poles at the origin.

Solution:

(a)$a(s) = s^2 + s$; $b(s) = s + 1$

Breakin(s) -3.43; Breakaway(s) -0.586

(b) $a(s) = s^2 + 0.2s + 1$; $b(s) = s + 1$

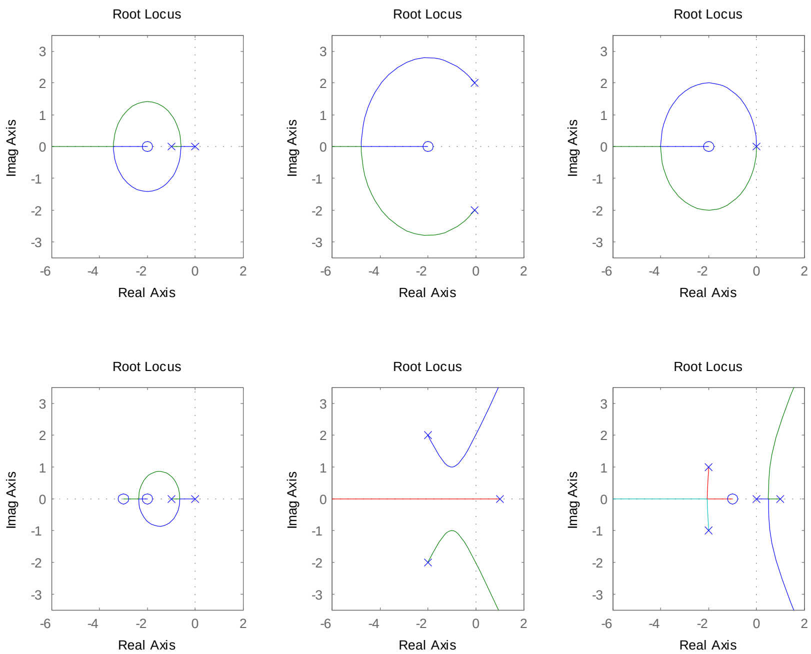


Figure 5.62: Pole-zero maps from Figure 5.62

Angle of departure: 135.7

Breakin(s) -4.97

(c) $a(s) = s^2$; $b(s) = (s+1)$

Breakin(s) -2

(d) $a(s) = s^2 + 5s + 6$; $b(s) = s^2 + s$

Breakin(s) -2.37

Breakaway(s) -0.634

(e) $a(s) = s^3 + 3s^2 + 4s - 8$

Center of asymptotes -1

Angles of asymptotes ± 60, 180

Angle of departure: -56.3

(f) $a(s) = s^3 + 3s^2 + s - 5$; $b(s) = s + 1$

Center of asymptotes -.667

Angles of asymptotes ± 60, -180

Angle of departure: -90

Breakin(s) -2.06

Breakaway(s) 0.503

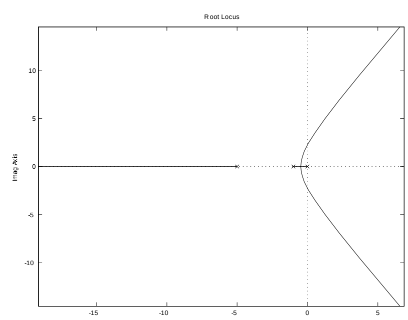
3. For the characteristic equation

$$1 + \frac{K}{s(s+1)(s+5)} = 0:$$

   (a) Draw the real-axis segments of the corresponding root locus.
   (b) Sketch the asymptotes of the locus for $K \to \infty$.
   (c) For what value of K are the roots on the imaginary axis?
   (d) Verify your sketch with a MATLAB plot.

Solution:

   (a) The real axis segments are $0 > \sigma > -1$; $-5 > \sigma$
   (b) $\alpha = -6/3 = -2$; $\phi_i = \pm 60, 180$
   (c) $K_o = 30$



Solution for Problem 5.3

15. For the feedback configuration of Fig. 5.65, use asymptotes, center of asymptotes, angles of departure and arrival, and the Routh array to sketch root loci for the characteristic equations of the following feedback control systems versus the parameter K. Use MATLAB to verify your results.

(a) $G(s) = \dfrac{1}{s(s+1+3j)(s+1-3j)}$, $\qquad H(s) = \dfrac{s+2}{s+8}$

(b) $G(s) = \dfrac{1}{s^2}$, $\qquad H(s) = \dfrac{s+1}{s+3}$

(c) $G(s) = \dfrac{(s+5)}{(s+1)}$, $\qquad H(s) = \dfrac{s+7}{s+3}$

(d) $G(s) = \dfrac{(s+3+4j)(s+3-4j)}{s(s+1+2j)(s+1-2j)}$, $\qquad H(s) = 1+3s$

Solution:

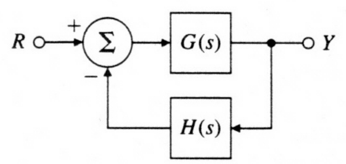
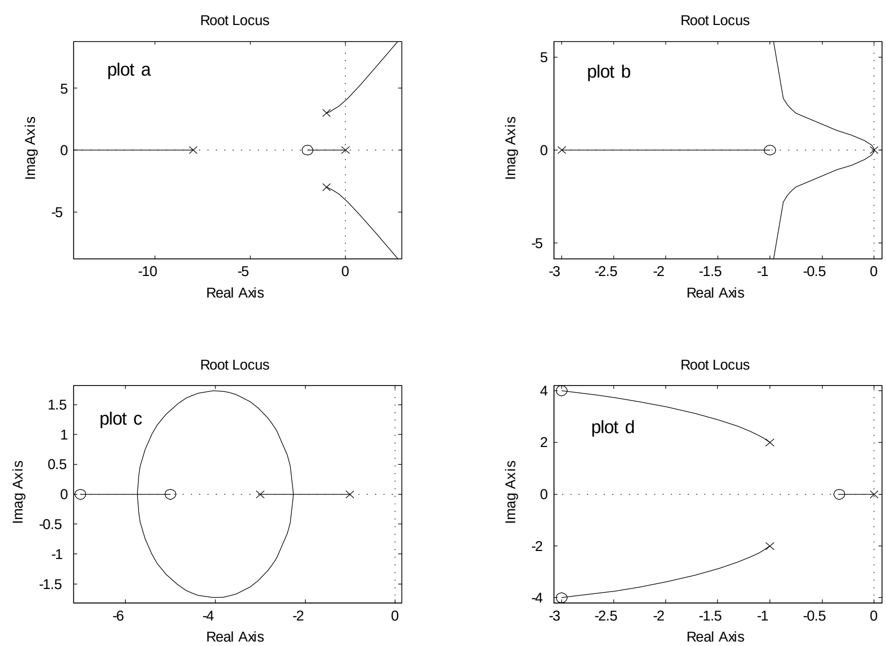


Figure 5.65: Feedback system for problem 15



Solution for problem 5.15

16. Consider the system in Fig. 5.66.

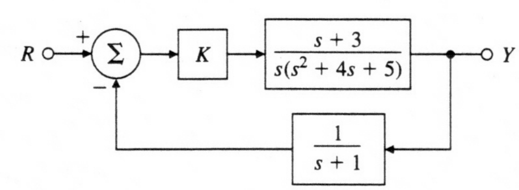


Figure 5.66: Feedback system for problem 5.16

(a) Using Routh's stability criterion, determine all values of K for which the system is stable.

(b) Sketch the root locus of the characteristic equation versus K. Include angles of departure and arrival, and find the values for K and s at all breakaway points, break-in points and imaginary-axis crossings.

Solution:

(a) a. $0 \leq K \leq 40$

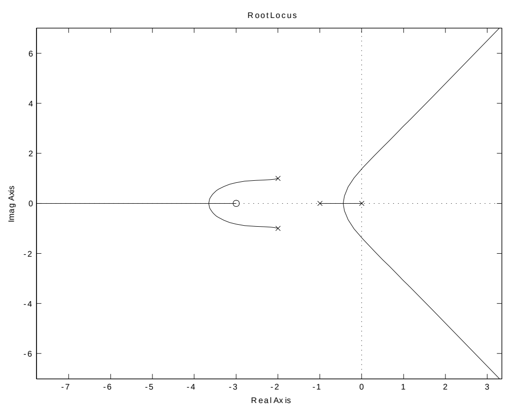
(b) $\theta_d = \pm 161.6°$ $\theta_a = 0°$

At imaginary axis crossing s=$\pm j1.8186$ $k = 6.2758$

$S_{breakaway} = -0.4363$ $k = 0.331$

$S_{breakin} = -3.6503$ $k = 55.4$

Root locus is attched for reference.



Root locus for problem 16