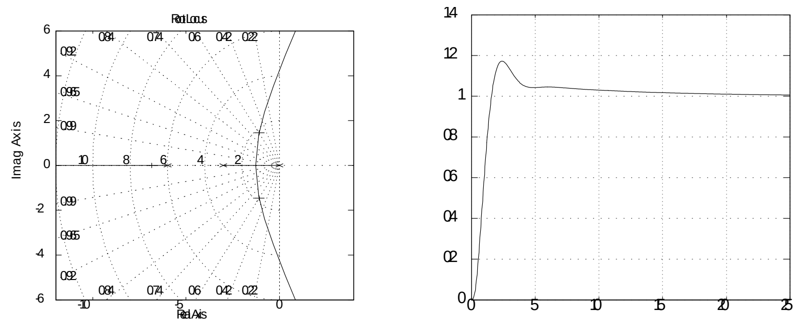


Root locus and step response for Problem 5.27

28. A numerically controlled machine tool positioning servomechanism has a normalized and scaled transfer function given by

$$G(s) = \frac{1}{s(s+1)}.$$

Performance specifications of the system in the unity feedback configuration of Fig. 5.71 are satisfied if the closed-loop poles are located at $s = -1 \pm j\sqrt{3}$.

(a) Show that this specification cannot be achieved by choosing proportional control alone, $D(s) = k_p$.

(b) Design a lead compensator $D(s) = K\frac{s+z}{s+p}$ that will meet the specification.

(a) With proportional control, the poles have real part at $s = -.5$.

(b) To design a lead, we select the pole to be at $p = -10$ and compute the zero and gain to be $z = -3$, $k = 12$.

29. A servomechanism position control has the plant transfer function

$$G(s) = \frac{10}{s(s+1)(s+10)}.$$

You are to design a series compensation transfer function $D(s)$ in the unity feedback configuration to meet the following closed-loop specifications:

- The response to a reference step input is to have no more than 16% overshoot.
- The response to a reference step input is to have a rise time of no more than 0.4 sec.
- The steady-state error to a unit ramp at the reference input must be less than 0.02

(a) Design a lead compensation that will cause the system to meet the dynamic response specifications.

(b) If $D(s)$ is proportional control, $D(s) = k_p$, what is the velocity constant K_v?

(c) Design a lag compensation to be used in series with the lead you have designed to cause the system to meet the steady-state error specification.

(d) Give the MATLAB plot of the root locus of your final design.

(e) Give the MATLAB response of your final design to a reference step .

Solution:

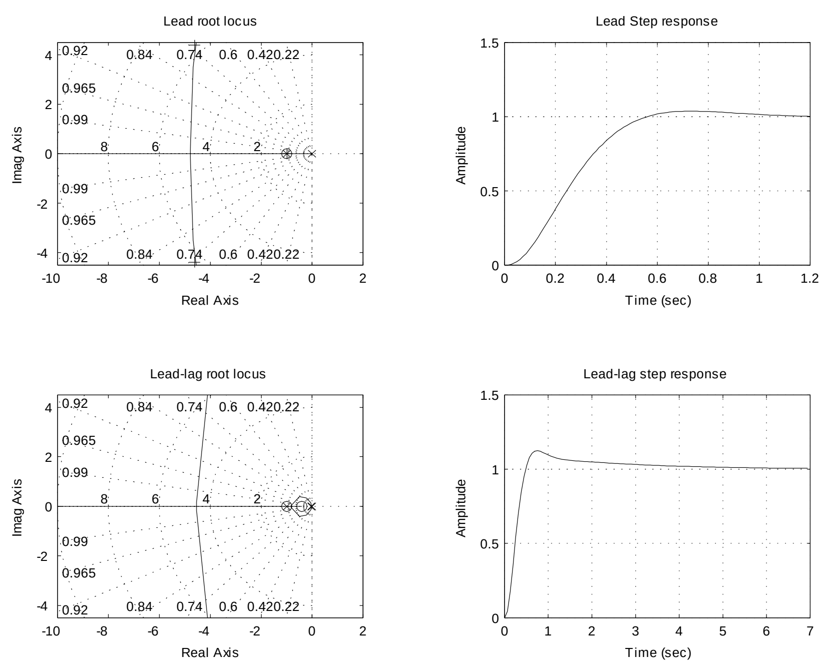
(a) Setting the lead pole at $p = -60$ and the zero at $z = -1$, the dynamic specifications are met with a gain of 245 resulting in a $K_v = 4$.

(b) Proportional control will not meet the dynamic spec. The K_v of the lead is given above.

(c) To meet the steady-state requirement, we need a new $K_v = 50$, which is an increase of 12.5. If we set the lag zero at $z = -.4$, the pole needs to be at $p = -0.032$.

(d) The root locus is plotted below.

(e) The step response is plottted below.



Solution to Problem 29

30. Assume the closed-loop system of Fig. 5.71 has a feed forward transfer function $G(s)$ given by

$$G(s) = \frac{1}{s(s+2)}.$$

Design a lag compensation so that the dominant poles of the closed-loop system are located at $s = -1 \pm j$ and the steady-state error to a unit ramp input is less than 0.2.

Solution:

The poles can be put in the desired location with proportional control alone, with a gain of $k_p = 2$ resulting in a $K_v = 1$. To get a $K_v = 5$, we add a compensation with zero at 0.1 and a pole at 0.02. $D(s) = 2\dfrac{s + 0.1}{s + 0.02}$.

31. An elementary magnetic suspension scheme is depicted in Fig. 5.72. For

34. For the system in Fig. 5.74:

(a) Sketch the locus of closed-loop roots with respect to K.

(b) Find the maximum value of K for which the system is stable. Assume $K = 2$ for the remaining parts of this problem.

(c) What is the steady-state error ($e = r - y$) for a step change in r?

(d) What is the steady-state error in y for a constant disturbance w_1?

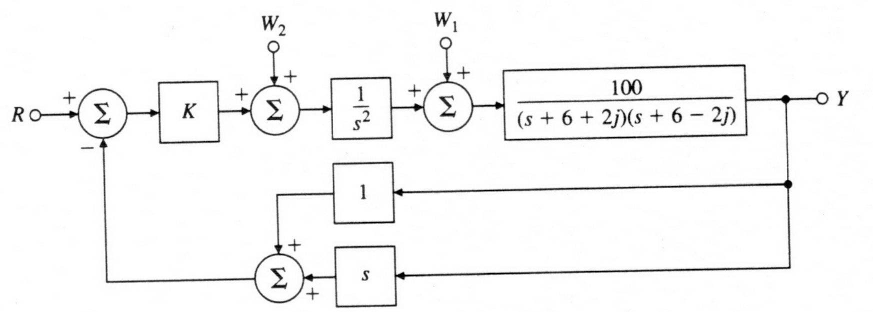


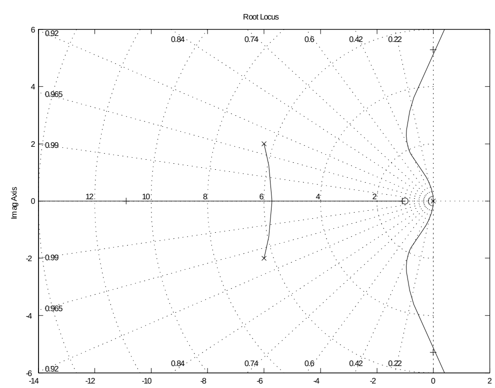
Figure 5.74: Control system for Problem 34

(e) What is the steady-state error in y for a constant disturbance w_2?

(f) If you wished to have more damping, what changes would you make to the system?

Solution:

(a) For the locus, $L(s) = \dfrac{100(s+1)}{s^2(s^2+12s+40)}$. The locus is plotted below.



Locus for Problem 34

(b) The maximum value of K for stability is $K = 3.35$.

(c) The equivalent plant with unity feedback is $G' = \dfrac{200}{s^2(s^2+12+40)+200s}$. Thus the system is **type** 1 with $K_v = 1$. If the velocity feedback were zero, the system would be type 2 with $K_a = \dfrac{200}{40} = 5$.

(d) The transfer function $\dfrac{Y}{W_1} = \dfrac{100s^2}{s^2(s^2+12s+40)+200(s+1)}$. The system is thus **type 2** with $K_a = 100$.

(e) The transfer function $\dfrac{Y}{W_2} = \dfrac{100}{s^2(s^2+12s+40)+200(s+1)}$. The system here is **type 0** with $K_p = 1$.

(f) To get more damping in the closed-loop response, the controller needs to have a lead compensation.

35. Consider the plant transfer function