Problems and Solutions for Section 7.8: Compensator Design: Combined Control Law and Estimator

46. A certain process has the transfer function $G(s) = 4/(s^2 - 4)$.
a) Find **F, G**, and **H** for this system in observer canonical form.
b) If $u = -\mathbf{K}\mathbf{x}$, compute **K** so that the closed-loop control poles are located at $s = -2 \pm 2j$.
c) Compute **L** so that the estimator-error poles are located at $s = -10 \pm 10j$.
d) Give the transfer function of the resulting controller (for example, using Eq. (7.177)).
e) What are the gain and phase margins of the controller and the given open-loop system?

Solution:

(a) From the transfer function, we can read off the elements that will give observer canonical form,

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{F}_o\mathbf{x} + \mathbf{G}_o u, \\ y &= \mathbf{H}_o\mathbf{x}, \\ \mathbf{F}_o &= \begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix}, \ \mathbf{G}_o = \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \ \mathbf{H}_o = \begin{bmatrix} 1 & 0 \end{bmatrix}. \end{aligned}$$

(b) With $u = -[k_1 \ k_2][x_1 \ x_2]^T$, we want to achieve the following closed-loop characteristic equation:

$$\alpha_c(s) = (s + 2 + 2j)(s + 2 - 2j) = s^2 + 4s + 8 = 0.$$

From $\det(s\mathbf{I} - \mathbf{F} + \mathbf{G}\mathbf{K}) = 0$, we obtain,

$$s^2 + 4k_2 s + 4k_1 - 4 = 0.$$

Comparing the coefficients yields $k_1 = 3$, and $k_2 = 1$. This result can be verified using MATLAB's place command.

(c) The estimator roots are determined by the equation $\alpha_e(s) = 0$. We want to find l_1 and l_2 such that,

$$\alpha_e(s) = (s + 10 + 10j)(s + 10 - 10j) = s^2 + 20s + 200.$$

$$\begin{aligned} \alpha_e(s) &= \det(s\mathbf{I} - \mathbf{F} + \mathbf{L}\mathbf{H}) \\ &= \det\left(\begin{bmatrix} s & -1 \\ -4 & s \end{bmatrix} + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix}\right) \\ &= \det\begin{bmatrix} s + l_1 & -1 \\ -4 + l_2 & s \end{bmatrix} = s^2 + l_1 s + l_2 - 4. \end{aligned}$$

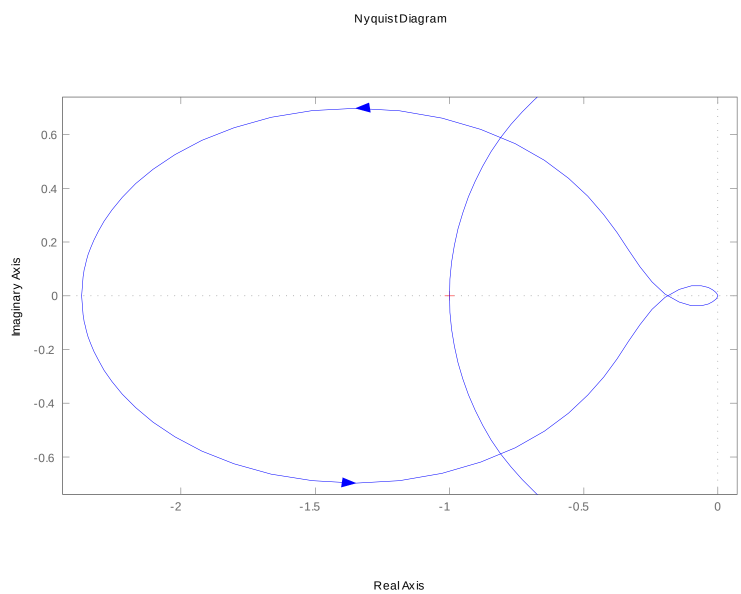
Comparing the coefficients yields $l_1 = 20$, $l_2 = 204$. This result can be verified using MATLAB's place command.

(d) The transfer function of the resulting compensator is,

$$\begin{aligned} D(s) &= \frac{U(s)}{Y(s)} = -\mathbf{K}(s\mathbf{I} - \mathbf{F} + \mathbf{G}\mathbf{K} + \mathbf{L}\mathbf{H})^{-1}\mathbf{L}, \\ &= -\begin{bmatrix} 3 & 1 \end{bmatrix}\begin{bmatrix} s + 20 & -1 \\ 212 & s + 4 \end{bmatrix}^{-1}\begin{bmatrix} 20 \\ 204 \end{bmatrix} = \frac{-264s - 692}{s^2 + 24s + 292}. \end{aligned}$$

This result can be verified using MATLAB's ss2tf command.

(e) The next figure shows the Nyquist plot generated by MATLAB (using the nyquist command), note that there is both a positive and negative gain margin. The Nyquist plot has a positive gain margin of 0.4220 (i.e., the gain can be increased by $1/0.422 = 2.37$) and a negative margin of 5.46 (i.e., the gain can be decreased by $1/5.46 = 0.183$) before the number of encirclements of the -1 point changes.



Nyquist plot for Problem 7.46.

49. Consider the control of

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10}{s(s+1)}.$$

a) Let $y = x_1$ and $\dot{x}_1 = x_2$, and write state equations for the system.
b) Find K_1 and K_2 so that $u = -K_1x_1 - K_2x_2$ yields closed-loop poles with a natural frequency $\omega_n = 3$ and a damping ratio $\zeta = 0.5$.
c) Design a state estimator for the system that yields estimator error poles with $\omega_{n1} = 15$ and $\zeta_1 = 0.5$.
d) What is the transfer function of the controller obtained by combining parts (a) through (c)?
e) Sketch the root locus of the resulting closed-loop system as plant gain (nominally 10) is varied.

Solution:

The state equations are,

$$\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} &= \begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 10\end{bmatrix}u,\\ y &= \begin{bmatrix}1 & 0\end{bmatrix}\mathbf{x}.\end{aligned}$$

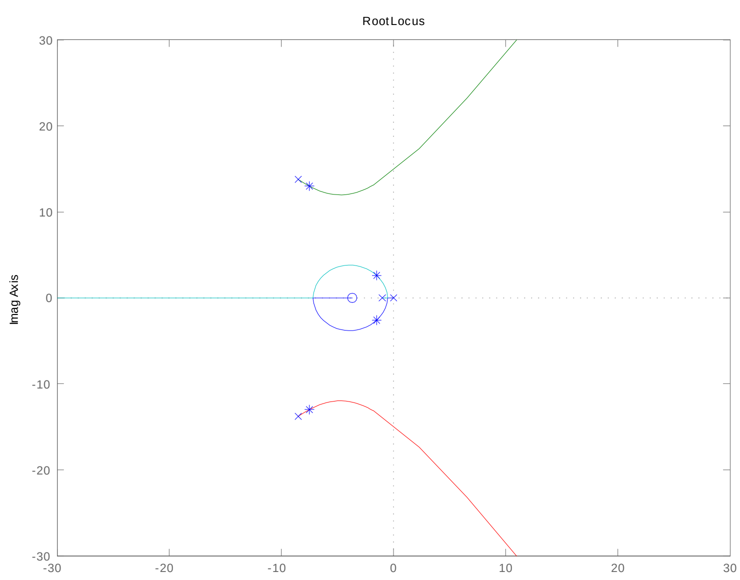
(b) $K = \mathsf{place(F, G, roots([1\ 2*zeta*wn\ wn\hat{\ }2]))} = [0.9\ 0.2]$.

(c) $L = \mathsf{place(F', H', roots([1\ 2*zeta*wn\ wn\hat{\ }2]))'} = [14\ 211]^T$.

(d) The transfer function for the controller is,

$$\begin{aligned}D_c(s) &= -\mathbf{K}(s\mathbf{I} - \mathbf{F} + \mathbf{GK} + \mathbf{LH})^{-1}\mathbf{L}\\ &= \frac{-(54.8s + 202.5)}{s^2 + 17s + 262}.\end{aligned}$$

(e) The figure below shows the root locus around a nominal gain of 10, which is indicated by asterisk.



Problem 7.49: Root locus of the closed-loop system as plant gain is varied.