Problem

1. Convert $\dddot{y}+\alpha_1\ddot{y}+\alpha_2\dot{y}+\alpha_3 y=\beta_0\dddot{u}+\beta_1\ddot{u}+\beta_2\dot{u}+\beta_3 u$ to the following matrix form:

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -\alpha_3 & -\alpha_2 & -\alpha_1\end{bmatrix},\quad B=\begin{bmatrix}0\\0\\1\end{bmatrix},\quad C=\left[(\beta_3-\beta_0\alpha_3)\ \ (\beta_2-\beta_0\alpha_2)\ \ (\beta_1-\beta_0\alpha_1)\right],\ D=[\beta_0]$$

Solution

1. $$\frac{Y(s)}{U(s)}=\frac{\beta_0 s^3+\beta_1 s^2+\beta_2 s+\beta_3}{s^3+\alpha_1 s^2+\alpha_2 s+\alpha_3}=\beta_0+\frac{(\beta_1-\alpha_1\beta_0)s^2+(\beta_2-\alpha_2\beta_0)s+(\beta_3-\alpha_3\beta_0)}{s^3+\alpha_1 s^2+\alpha_2 s+\alpha_3}$$

$$X_1(s)=\frac{U(s)}{s^3+\alpha_1 s^2+\alpha_2 s+\alpha_3}$$

$$X_2(s)=sX_1(s)$$

$$X_3(s)=sX_2(s)=s^2X_1(s)$$

$$Y(s)=\beta_0U(s)+(\beta_1-\alpha_1\beta_0)X_3(s)+(\beta_2-\alpha_2\beta_0)X_2(s)+(\beta_3-\alpha_3\beta_0)X_1(s)$$

$$sX_3(s)=s^3X_1(s)=\frac{s^3U(s)}{s^3+\alpha_1 s^2+\alpha_2 s+\alpha_3}=U(s)+\frac{-\alpha_1 s^2-\alpha_2 s-\alpha_3}{s^3+\alpha_1 s^2+\alpha_2 s+\alpha_3}U(s)$$

$$=U(s)-\alpha_1X_3(s)-\alpha_2X_2(s)-\alpha_3X_1(s)$$