.1 Determine whether each of the systems below is controllable and/or observable

a) $x(k+1)=\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} x(k)+\begin{bmatrix} 1 \\ -1 \end{bmatrix} u(k)$

$y(k)=\begin{bmatrix} 5 & 1 \end{bmatrix} x(k)$

Solution:

$P=\begin{bmatrix} B & AB \end{bmatrix}=\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ rank$(P)=1$ NOT controllable

$Q=\begin{bmatrix} C \\ CA \end{bmatrix}=\begin{bmatrix} 5 & 1 \\ 4.5 & .5 \end{bmatrix}$ rank$(Q)=2$ OBSERVABLE

b) $\dot{x}=\begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix} x+\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} u$

$y=\begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix} x$

Solution:

$\text{rank}(\text{ctrb}(A, B))=2$ => NOT controllable

$\text{rank}(\text{obsv}(A, B))=2$ => NOT observable

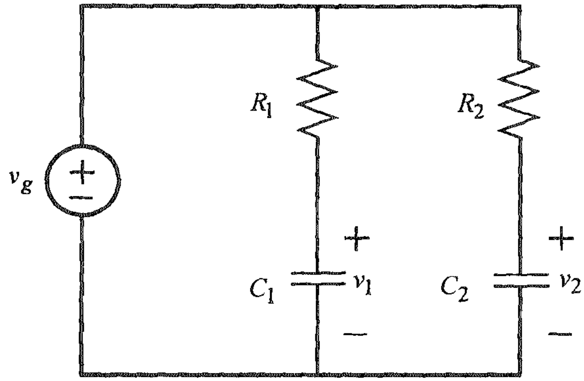
c) $A=\begin{bmatrix} 2 & -5 \\ -4 & 0 \end{bmatrix}$ $b=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $c=\begin{bmatrix} 1 & 1 \end{bmatrix}$

Solution:

$\text{rank}(\text{ctrb}(A, B))=2$ => controllable

$\text{rank}(\text{obsv}(A, B))=2$ => observable

For the circuit shown, find conditions on the system components R_1, R_2, C_1, and C_2 that result in an uncontrollable system. Consider v_g to be the input, and v_1 and v_2 to be the state variables.



Solution: $R_1C_1 = R_2C_2$

Using the KVL,

$$v_g = R_1C_1\frac{dv_1}{dt} + v_1 \qquad v_g = R_2C_2\frac{dv_2}{dt} + v_2$$

$$\dot{v}_1 = -\frac{1}{R_1C_1}v_1 + \frac{1}{R_1C_1}v_g \qquad \dot{v}_2 = -\frac{1}{R_2C_2}v_2 + \frac{1}{R_2C_2}v_g$$

$$\dot{v} = \begin{bmatrix} -\frac{1}{R_1C_1} & 0 \\ 0 & -\frac{1}{R_2C_2} \end{bmatrix} v + \begin{bmatrix} \frac{1}{R_1C_1} \\ \frac{1}{R_2C_2} \end{bmatrix} v_g$$

The state space equation is in the JORDAN form, and obviously, if $\underline{R_1C_1 = R_2C_2}$, the system is UNCONTROLLABLE.