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21 $\begin{bmatrix}1&4\\0&0\end{bmatrix}=\begin{bmatrix}1&4\\0&-1\end{bmatrix}\begin{bmatrix}\mathbf{1}&0\\0&\mathbf{0}\end{bmatrix}\begin{bmatrix}1&4\\0&-1\end{bmatrix};\begin{bmatrix}1&4\\0&-1\end{bmatrix}\begin{bmatrix}e^t&0\\0&\mathbf{1}\end{bmatrix}\begin{bmatrix}1&4\\0&-1\end{bmatrix}=\begin{bmatrix}e^t&4e^t-4\\0&1\end{bmatrix}.$

23 $e^A=\begin{bmatrix}e&4(e-1)\\0&1\end{bmatrix}$ from **21** and $e^B=\begin{bmatrix}1&-4\\0&1\end{bmatrix}$ from **19**. By direct multiplication $e^Ae^B\neq e^Be^A\neq e^{A+B}=\begin{bmatrix}e&0\\0&1\end{bmatrix}.$

24 $A=\begin{bmatrix}1&1\\0&3\end{bmatrix}=\begin{bmatrix}1&1\\0&2\end{bmatrix}\begin{bmatrix}\mathbf{1}&0\\0&\mathbf{3}\end{bmatrix}\begin{bmatrix}1&-\frac{1}{2}\\0&\frac{1}{2}\end{bmatrix}$. Then $e^{At}=\begin{bmatrix}e^t&\frac{1}{2}(e^{3t}-e^t)\\0&e^{3t}\end{bmatrix}.$

26 (a) The inverse of e^{At} is e^{-At} (b) If $Ax=\lambda x$ then $e^{At}x=e^{\lambda t}x$ and $e^{\lambda t}\neq 0$. To see $e^{At}x$, write $(I+At+\frac{1}{2}A^2t^2+\cdots)x=(1+\lambda t+\frac{1}{2}\lambda^2t^2+\cdots)x=e^{\lambda t}x$.