

Problem Set 6.5, page 350

5 $x^2 + 4xy + 3y^2 = (x + 2y)^2 - y^2 = \text{difference of squares}$ is negative at $x = 2$, $y = -1$, where the first square is zero.

9 $A = \begin{bmatrix} 4 & -4 & 8 \\ -4 & 4 & -8 \\ 8 & -8 & 16 \end{bmatrix}$ has only one pivot = 4, rank $A = 1$, eigenvalues are 24, 0, 0, $\det A = 0$.

11 Corner determinants $|A_1| = 2$, $|A_2| = 6$, $|A_3| = 30$. The pivots are $2/1$, $6/2$, $30/6$.

12 A is positive definite for $c > 1$; determinants c , $c^2 - 1$, and $(c - 1)^2(c + 2) > 0$. B is *never* positive definite (determinants $d - 4$ and $-4d + 12$ are never both positive).