

$$\mathbf{1} \quad E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 7 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

$\mathbf{2}$   $E_{32}E_{21}b = (1, -5, -35)$  but  $E_{21}E_{32}b = (1, -5, 0)$ . When  $E_{32}$  comes first, row 3 feels no effect from row 1.

$$\mathbf{3} \quad \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \quad M = E_{32}E_{31}E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{bmatrix}.$$

$$\mathbf{18} \quad EF = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}, \quad FE = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b+ac & c & 1 \end{bmatrix}, \quad E^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2a & 1 & 0 \\ 2b & 0 & 1 \end{bmatrix}, \quad F^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3c & 1 \end{bmatrix}.$$

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$$K = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$E_{21} K = \begin{bmatrix} 1 & & & \\ \frac{1}{2} & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} K = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \triangleq K_1$$

$$E_{32} K_1 = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & \frac{3}{2} & 1 & \\ & & & 1 \end{bmatrix} K_1 = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & 0 \\ 0 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \triangleq K_2$$

$$E_{43} K_2 = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & \frac{3}{4} & 1 \end{bmatrix} K_2 = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & 0 \\ 0 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & 0 & \frac{5}{4} \end{bmatrix} = U.$$