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2 v_1, v_2, v_3 are independent (the -1's are in different positions). All six vectors are on the plane $(1,1,1,1) \cdot v = 0$ so no four of these six vectors can be independent.

5 (a) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -7 \\ 0 & -1 & -5 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -7 \\ 0 & 0 & -18/5 \end{bmatrix}$: invertible $\Rightarrow$ independent columns.

(b) $\begin{bmatrix} 1 & 2 & -3 \\ -3 & 1 & 2 \\ 2 & -3 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -3 \\ 0 & 7 & -7 \\ 0 & -7 & 7 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -3 \\ 0 & 7 & -7 \\ 0 & 0 & 0 \end{bmatrix}$; $A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, columns add to $\mathbf{0}$.

13 The column space and row space of A and U all have the same dimension = 2. *The row spaces of A and U are the same*, because the rows of U are combinations of the rows of A (and vice versa!).

17 The column space of $U = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ is $\mathbf{R}^2$ so take any bases for $\mathbf{R}^2$; (row 1 and row 2) or (row 1 and row 1 + row 2) and (row 1 and $-$ row 2) are bases for the row spaces of U.

25 A has rank 2 if $c = 0$ and $d = 2$; $B = \begin{bmatrix} c & d \\ d & c \end{bmatrix}$ has rank 2 except when $c = d$ or $c = -d$.

38 (a) No, 2 vectors don't span $\mathbf{R}^3$ (b) No, 4 vectors in $\mathbf{R}^3$ are dependent (c) Yes, a basis (d) No, these three vectors are dependent