

Problem Set 6.3, page 325

- 4 $d(v+w)/dt = (w-v) + (v-w) = 0$, so the total $v+w$ is constant. $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$
has $\lambda_1 = 0$ with $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; $v(1) = 20 + 10e^{-2}$ $v(\infty) = 20$
 $w(1) = 20 - 10e^{-2}$ $w(\infty) = 20$
- 5 $\frac{d}{dt} \begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ has $\lambda = 0$ and $+2$: $v(t) = 20 + 10e^{2t} - \infty$ as $t \rightarrow \infty$.
- 8 $\begin{bmatrix} 6 & -2 \\ 2 & 1 \end{bmatrix}$ has $\lambda_1 = 5$, $x_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\lambda_2 = 2$, $x_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$; rabbits $r(t) = 20e^{5t} + 10e^{2t}$,
 $w(t) = 10e^{5t} + 20e^{2t}$. The ratio of rabbits to wolves approaches $20/10$; e^{5t} dominates.