

- 3 Find A to change the scalar equation $y'' = 5y' + 4y$ into a vector equation for $\mathbf{u} = (y, y')$:

$$\frac{d\mathbf{u}}{dt} = \begin{bmatrix} y' \\ y'' \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix} = A\mathbf{u}.$$

What are the eigenvalues of A ? Find them also by substituting $y = e^{\lambda t}$ into $y'' = 5y' + 4y$.

- 4 The rabbit and wolf populations show fast growth of rabbits (from $6r$) but loss to wolves (from $-2w$):

$$\frac{dr}{dt} = 6r - 2w \quad \text{and} \quad \frac{dw}{dt} = 2r + w.$$

Find the eigenvalues and eigenvectors. If $r(0) = w(0) = 30$ what are the populations at time t ? After a long time, is the ratio of rabbits to wolves 1 to 2 or is it 2 to 1?

- 5 A door is opened between rooms that hold $v(0) = 30$ people and $w(0) = 10$ people. The movement between rooms is proportional to the difference $v - w$:

$$\frac{dv}{dt} = w - v \quad \text{and} \quad \frac{dw}{dt} = v - w.$$

Show that the total $v + w$ is constant (40 people). Find the matrix in $d\mathbf{u}/dt = A\mathbf{u}$ and its eigenvalues and eigenvectors. What are v and w at $t = 1$?