

2  $z(t) = -2e^{-t}$ , then  $dy/dt = 4y - 6e^{-t}$  with  $y(0) = 5$  gives  $y(t) = 5e^{-t} + 2e^{-t}$  as in Problem 1.

3 
$$\begin{bmatrix} y' \\ y'' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix}. \text{ Then } \lambda = \frac{1}{2}(5 \pm \sqrt{41}).$$

4 
$$\begin{bmatrix} 6 & -2 \\ 2 & 1 \end{bmatrix} \text{ has } \lambda_1 = 5, \quad \mathbf{x}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \lambda_2 = 2, \quad \mathbf{x}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}; \text{ rabbits } r(t) = 20e^{5t} + 10e^{2t},$$
  
 $w(t) = 10e^{5t} + 20e^{2t}$ . The ratio of rabbits to wolves approaches 20/10;  $e^{5t}$  dominates.

5  $d(v+w)/dt = dv/dt + dw/dt = (w-v) + (v-w) = 0$ , so the total  $v+w$  is constant.  $A =$   
$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \text{ has } \lambda_1 = 0 \text{ and } \lambda_2 = -2 \text{ with } \mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \mathbf{x}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}; \quad \begin{matrix} v(1) = 20 + 10e^{-2} \\ w(1) = 20 - 10e^{-2} \end{matrix}.$$