18 The solution at time $t+T$ is also $e^{A(t+T)}\boldsymbol{u}(0)$. Thus e^{At} times e^{AT} equals $e^{A(t+T)}$.

19 $\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}1 & 1\\0 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\0 & -1\end{bmatrix}$; $e^{At} = \begin{bmatrix}1 & 1\\0 & -1\end{bmatrix}\begin{bmatrix}e^t & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & -1\end{bmatrix} = \begin{bmatrix}e^t & e^t-1\\0 & 1\end{bmatrix}$.

20 If $A^2 = A$ then $e^{At} = I + At + \frac{1}{2}At^2 + \frac{1}{6}At^3 + \cdots = I + (e^t - 1)A = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \begin{bmatrix}e^t-1 & e^t-1\\0 & 0\end{bmatrix}$.

21 $e^A = \begin{bmatrix}e & e-1\\0 & 1\end{bmatrix}$, $e^B = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$, $e^Ae^B \neq e^Be^A = \begin{bmatrix}e & e-2\\0 & 1\end{bmatrix} \neq e^{A+B} = \begin{bmatrix}e & 0\\0 & 1\end{bmatrix}$.

22 $A = \begin{bmatrix}1 & 1\\0 & 3\end{bmatrix} = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\begin{bmatrix}3 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0 & \frac{1}{2}\\1 & -\frac{1}{2}\end{bmatrix}$, then $e^{At} = \begin{bmatrix}e^t & \frac{1}{2}(e^{3t}-e^t)\\0 & e^{3t}\end{bmatrix}$.

23 $A^2 = A$ so $A^3 = A$ and by Problem 20 $e^{At} = I + (e^t - 1)A = \begin{bmatrix} e^t & 3(e^t - 1) \\ 0 & 1 \end{bmatrix}$.

24 (a) The inverse of e^{At} is e^{-At} (b) If $A\boldsymbol{x} = \lambda\boldsymbol{x}$ then $e^{At}\boldsymbol{x} = e^{\lambda t}\boldsymbol{x}$ and $e^{\lambda t} \neq 0$.

25 $x(t) = e^{4t}$ and $y(t) = -e^{4t}$ is a growing solution. The correct matrix for the exchanged unknown $\boldsymbol{u} = (y, x)$ is $\begin{bmatrix} 2 & -2 \\ -4 & 0 \end{bmatrix}$ and it *does* have the same eigenvalues as the original matrix.