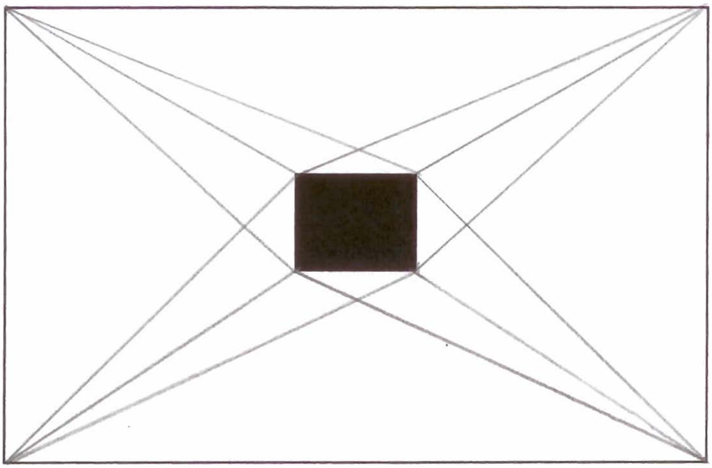
Mid-Term Exam

CpE800C Advanced Mobile Robots (Spring 2006)

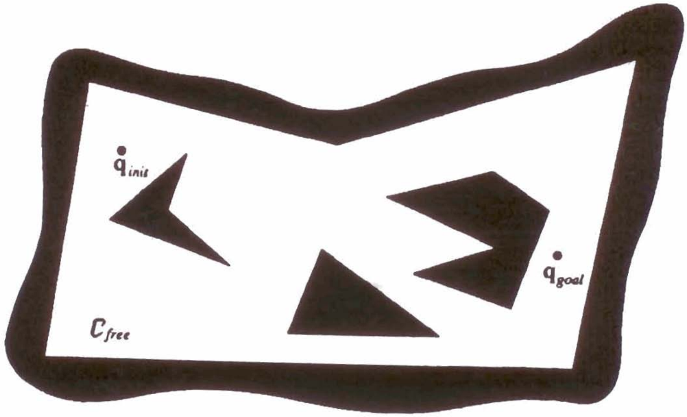
Name: Solutions

Do all work in the spaces provided. Show all work and organize it for partial credit. (100 points total.)

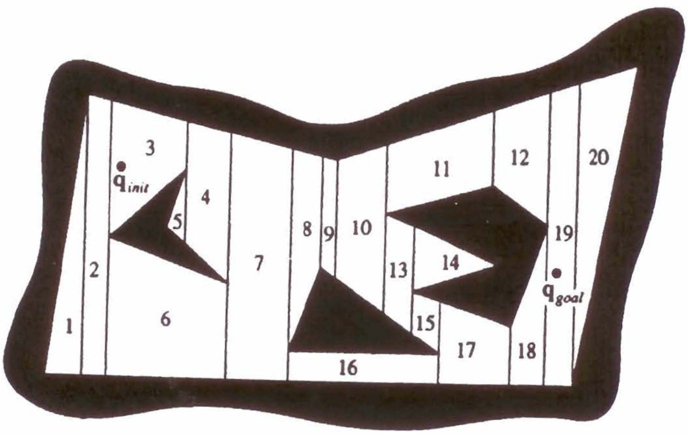
Problem 1. (20 points) Build a visibility graph for the following configuration by hand.



Problem 2. (20 points) The following figures illustrate **an** exact cell decomposition method. The free space is externally bounded by a polygon **and** internally bounded by three polygons (Figure a). Figure b shows the step of decomposing the free space into trapezoidal and triangular cells. Construct the connectivity graph representing the adjacency relation between the cells, and search this graph for a path (denoting the path using thick lines in your connectivity graph).

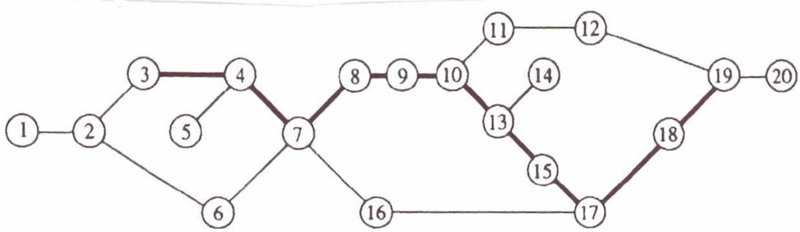


(a)



(b)

dution:



Problem 3. (20 points) Judge if the following statements 1-3 are true or false. Write a "T" or "F" in the space provided. Provide a brief answer to 4.

1. In the potential field path planning method, a potential field **function** is generated by attracting the robot to the goal and repulsing the robot **from** obstacles. ____T____

2. A* returns an optimal path if the h function, the estimated cost of the cheapest solution, over-estimates the distance to the goal. ___F___

3. In an A* search algorithm to return a shortest path, the OPEN list is ordered by **decreasing the** cost, that is, the first element in the list has the biggest cost to the goal. ___F___

4. Compare motion planning with movable objects (dynamic obstacles) to motion planning with multiple robots.

Problem 4. (20 points) For a polynomial trajectory $y = f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6$ where $a_0, \cdots, a_6$ are constants, the initial and final conditions are given below:

Initial configuration: $x_0, y_0, \left.\frac{dy}{dx}\right|_{x=x_0} = l_0, \left.\frac{d^2 y}{dx^2}\right|_{x=x_0} = c_0$;

Final configuration: $x_f, y_f, \left.\frac{dy}{dx}\right|_{x=x_f} = l_f, \left.\frac{d^2 y}{dx^2}\right|_{x=x_f} = c_f$.

Represent coefficients $a_0, \cdots, a_5$ in terms of a_6. (Hint: use vector and matrix expressions.)

Solution:

$$\frac{dy}{dx} = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + 6a_6 x^5$$

$$\frac{d^2y}{dx^2} = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + 30a_6 x^4$$

Six simutanous equations:

$$\begin{cases} y_0 = a_0 + a_1 x_0 + a_2 x_0^2 + a_3 x_0^3 + a_4 x_0^4 + a_5 x_0^5 + a_6 x_0^6 \\ y_f = a_0 + a_1 x_f + a_2 x_f^2 + a_3 x_f^3 + a_4 x_f^4 + a_5 x_f^5 + a_6 x_f^6 \\ l_0 = a_1 + 2a_2 x_0 + 3a_3 x_0^2 + 4a_4 x_0^3 + 5a_5 x_0^4 + 6a_6 x_0^5 \\ l_f = a_1 + 2a_2 x_f + 3a_3 x_f^2 + 4a_4 x_f^3 + 5a_5 x_f^4 + 6a_6 x_f^5 \\ c_0 = 2a_2 + 6a_3 x_0 + 12a_4 x_0^2 + 20a_5 x_0^3 + 30a_6 x_0^4 \\ c_f = 2a_2 + 6a_3 x_f + 12a_4 x_f^2 + 20a_5 x_f^3 + 30a_6 x_f^4 \end{cases}$$

Express it is vector-matrix form:

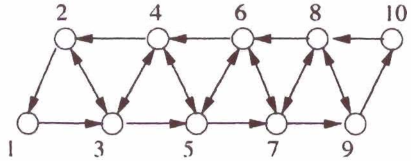
$$Y = XA + a_6 B$$

where $Y = \begin{bmatrix} y_0 \\ y_f \\ l_0 \\ l_f \\ c_0 \\ c_f \end{bmatrix}$, $X = \begin{bmatrix} 1 & x_0 & x_0^2 & x_0^3 & x_0^4 & x_0^5 \\ 1 & x_f & x_f^2 & x_f^3 & x_f^4 & x_f^5 \\ 0 & 1 & 2x_0 & 3x_0^2 & 4x_0^3 & 5x_0^4 \\ 0 & 1 & 2x_f & 3x_f^2 & 4x_f^3 & 5x_f^4 \\ 0 & 0 & 2 & 6x_0 & 12x_0^2 & 20x_0^3 \\ 0 & 0 & 2 & 6x_f & 12x_f^2 & 20x_f^3 \end{bmatrix}$,

$$A = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \qquad B = \begin{bmatrix} x_0^6 \\ x_f^6 \\ 6x_0^5 \\ 6x_f^5 \\ 30x_0^4 \\ 30x_f^4 \end{bmatrix}$$

Therefore, $A = X^{-1}(Y - a_6 \cdot B)$

Problem 5. (20 points) Please write out the Laplacian matrix for the following graph and answer if the graph is balanced or not.



Solution:

$$L_G = \begin{bmatrix} 1 & 0 & -1 & & & & & & & \\ -1 & 2 & -1 & & & & & & & \\ & -1 & 3 & -1 & -1 & & & & & \\ & -1 & -1 & 3 & -1 & & & & & \\ & & & -1 & 3 & -1 & -1 & & & \\ & & & & -1 & 3 & -1 & & & \\ & & & & & -1 & 3 & -1 & -1 & \\ & & & & & -1 & -1 & 3 & -1 & \\ & & & & & & & -1 & 2 & -1 \\ & & & & & & & -1 & 0 & 1 \end{bmatrix}$$

Balanced.