EE631 Cooperating Autonomous Mobile Robots

Lecture: Cooperative Formation Tracking of Multi-Vehicle Systems

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OUTLINE

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Introduction

- Cooperative control: a key issue is the convergence of a common value of the system
- Consensus solution:
  - Necessary and sufficient network condition is that the information exchange topology has a spanning tree
- Need vehicle level control for formation tracking
- Most existing dynamics include single/double integrator, linear system in canonical forms
- We study cooperative control such as formation tracking for general linear dynamic systems

Introduction

- Revealing connection between vehicle level control and cooperation feedback
- Using Lyapunov theory to derive sufficient vehicle and formation stability conditions

Problem Statement

- Vehicle model:

$$\dot{x}_i = A_i x_i + B_i u_i$$

- Control input:

$$u_i = u_i(x_i, x_j), \quad j \in \mathcal{N}_i$$

Problem Statement

- Cooperative stabilization:

For a group of multi-vehicle systems with $i = 1, \ldots, N$, given communication structure of the group forming a connected undirected graph, design decentralized control law

$$u_i = u_i(x_i, x_j) \quad j \in \mathcal{N}_i,$$

such that the vehicles return to their own equilibriums and keep formation. That is,

$$\begin{aligned} x_i &= 0 \quad \text{as} \quad t \to \infty \\ x_i &= x_j \quad \text{as} \quad t \to \infty \end{aligned}$$

for all $i, j = 1, 2, \ldots, N$.

Problem Statement

- Cooperative tracking:

Given a set of formation trajectories described by:

$$\dot{x}_{id} = A_{id}x_{id} + B_{id}u_{id},$$

design decentralized control law

$$u_i = u_i(x_i, x_j) \quad j \in \mathcal{N}_i,$$

such that the ith vehicle asymptotically tracks its reference trajectory and keep formation. That is,

$$\begin{aligned} e_i &= 0 \quad \text{as} \quad t \to \infty \\ e_i &= e_j \quad \text{as} \quad t \to \infty \end{aligned}$$

for all $i, j = 1, 2, \ldots, N$, where $e_i = x_i - x_{id}$.

Preliminaries on Undirected Graph

- To describe the connectivity of the nodes in a graph, use Laplacian matrix, L_G=D-A, where A is the adjacent matrix and D is the degree matrix
- L_G is a zero row sum matrix
- L_G is positive semi-definite

Preliminaries

Definition 1 *A matrix* $E \in \Re^{r \times r}$ *is said to be reducible if the set of its indices,* $\mathcal{I} \triangleq \{1, 2, \ldots, r\}$, *can be divided into two disjoint nonempty set* $\mathcal{S} \triangleq \{i_1, i_2, \ldots, i_\mu\}$ *and* $\mathcal{S}^c \triangleq \mathcal{I} \backslash \mathcal{S} = \{i_1, i_2, \ldots, i_\nu\}$ *(with* $\mu + \nu = r$*) such that* $e_{i_\alpha j_\beta} = 0$*, where* $\alpha = 1, \ldots, \mu$ *and* $\beta = 1, \ldots, \nu$*. A matrix is said to be irreducible if it is not reducible.*

Preliminaries

Lemma 1 (Wu and Chua'95) *Let the set* W *consists all zero row sum matrices which have only non-positive off-diagonal elements. If* A *is a symmetric matrix in* W*, then* A *can be decomposed as* $A = M^T M$ *where* M *is a matrix such that row* i *consists of zeros and exactly one entry* α_i *and one entry* $-\alpha_i$ *for some nonzero* α_i*. Furthermore, if* A *is irreducible, then the graph associated with* M *is connected.*

Cooperative Stabilization--One Dimensional Systems

Vehicle model:

$$\dot{x}_i = -x_i + u_i$$

Consensus protocol:

$$u_i = -\sum_{j \in \mathcal{N}_i} \alpha_{ij}(x_i - x_j)$$

Define Lyapunov candidate

$$V = \sum_{i=1}^{N} \frac{1}{2} x_i^2.$$

Its time derivative along the system dynamics is

$$\dot{V} = -\|x\|^2 - x^T L x$$

$$\dot{V} \;\le\; -x^T L x = -\sum_{i,j\in\mathcal{N}_i} a_{ij}(x_i - x_j)^2$$

From LaSalle's invariance theorem, we know that the system gets to the invariance set $\sum_{i,j} a_{ij}(x_i - x_j)^2 = 0$. Since the graph is connected, we get $x_i = x_j$ for all i, j as $t \to \infty$.

Cooperative Stabilization—General Linear Systems

$$\dot{x} = Ax + Bu$$

$$A = I_N \otimes A_i, \quad B = I_N \otimes B_i.$$

Choose Lyapunov candidate

$$V = x^T P x$$

$$P = I_N \otimes P_i$$

Taking time derivatives of V along the vehicles' dynamics, we get

$$\dot{V} = x^T(PA + A^T P)x + x^T PBu.$$

Let

$$u = I_N \otimes u_i(x_i) + u_j.$$

Let

$$u_i(x_i) = -\epsilon_i B_i^T P_i x_i$$

Then:

$$\dot{V} = \sum_{i=1}^{N} \{x_i^T (P_i A_i + A_i^T P_i - 2\epsilon_i P_i B_i B_i^T P_i) x_i\} + 2x^T P B u_j.$$

Choose P_i to solve the following ARE:

$$P_i A_i + A_i^T P_i - 2\epsilon_i P_i B_i B_i^T P_i + Q_i = 0$$

Then we obtain

$$\dot{V} = -x^T Q x + 2x^T P B u_j$$

$$Q = I_N \otimes Q_i.$$

Choose

$$u_j = -FLx$$

$$F = I_N \otimes F_i, \quad L = L_G \otimes I_m,$$

and $F_i \in \Re^{1\times m}$ is the feedback matrix

$$\begin{aligned}
x^T P B u_j &= -x^T[(C^T C \otimes (P_i B_i F_i)]x \\
&= -x^T (C^T \otimes I_m)[C \otimes (P_i B_i F_i)]x \\
&= -\sum_{i,j} \alpha_{ij}^2 (x_i - x_j)^T (P_i B_i F_i)(x_i - x_j)
\end{aligned}$$

$$\dot{V} = -x^T Q x - \sum_{i,j} 2\alpha_{ij}^2 (x_i - x_j)^T (P_i B_i F_i)(x_i - x_j)$$

From

$$\dot{V} \leq -x^T Q x,$$

we conclude stability of individual vehicle;

From

$$\dot{V} \leq -\sum_{i,j} 2\alpha_{ij}^2 (x_i - x_j)^T (P_i B_i F_i)(x_i - x_j)$$

we conclude consensus.

Cooperative Tracking

Vehicle dynamics:

$$\dot{x}_i = A_i x_i + B_i u_i$$

Reference dynamics:

$$\dot{x}_{id} = A_{id} x_{id} + B_{id} u_{id}$$

Define the error state as follows:

$$e_i = x_i - x_{id}$$

We have the error dynamics

$$\dot{e}_i = A_i e_i + B_i (u_i - u_{id}).$$

Illustrate Examples

- Vehicle model:

$$\begin{aligned} x_{i1} &= x_{i2} + u_{i1} \\ x_{i2} &= u_{i2} \end{aligned}$$

- Communication topology:

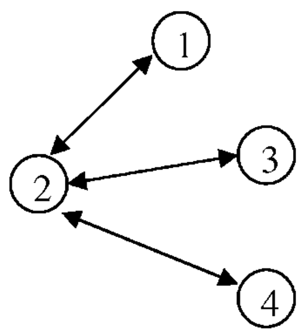


Figure 1: Communication topology of a four-robot team

- Laplacian mat

$$L_G = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

- Decentralized control:

$$u_i = -\epsilon_i B_i^T P_i x_i - (FLx)_i$$

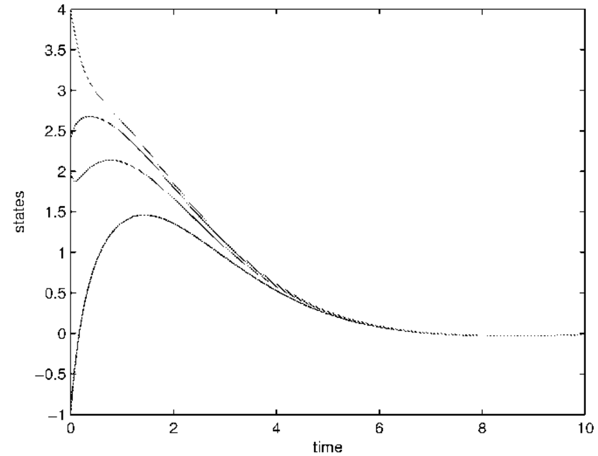


Figure 1: The time history of the first states of four robots

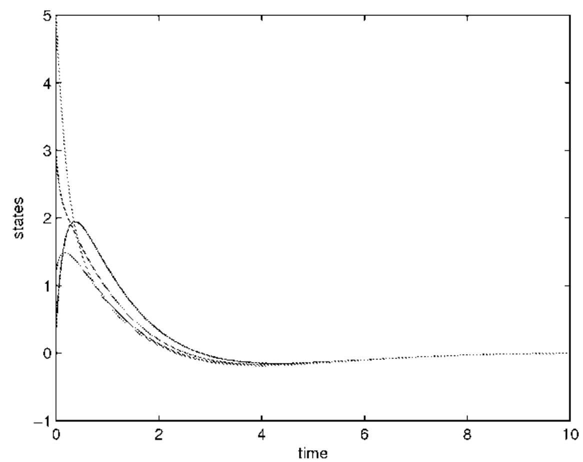


Figure 2: The time history of the second states of four robots

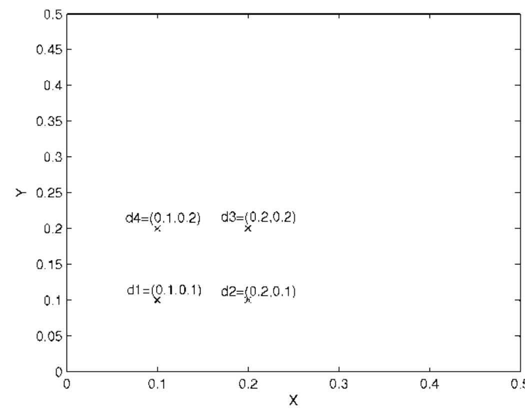


Figure 1: Rigid formation of four robots

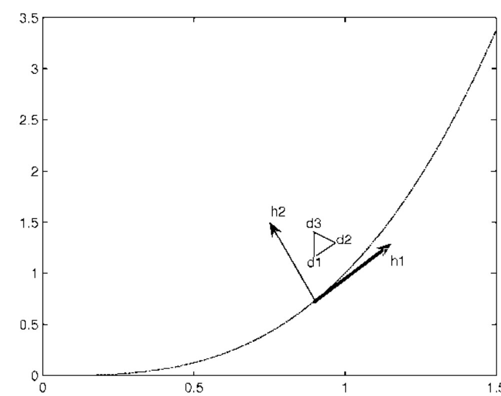
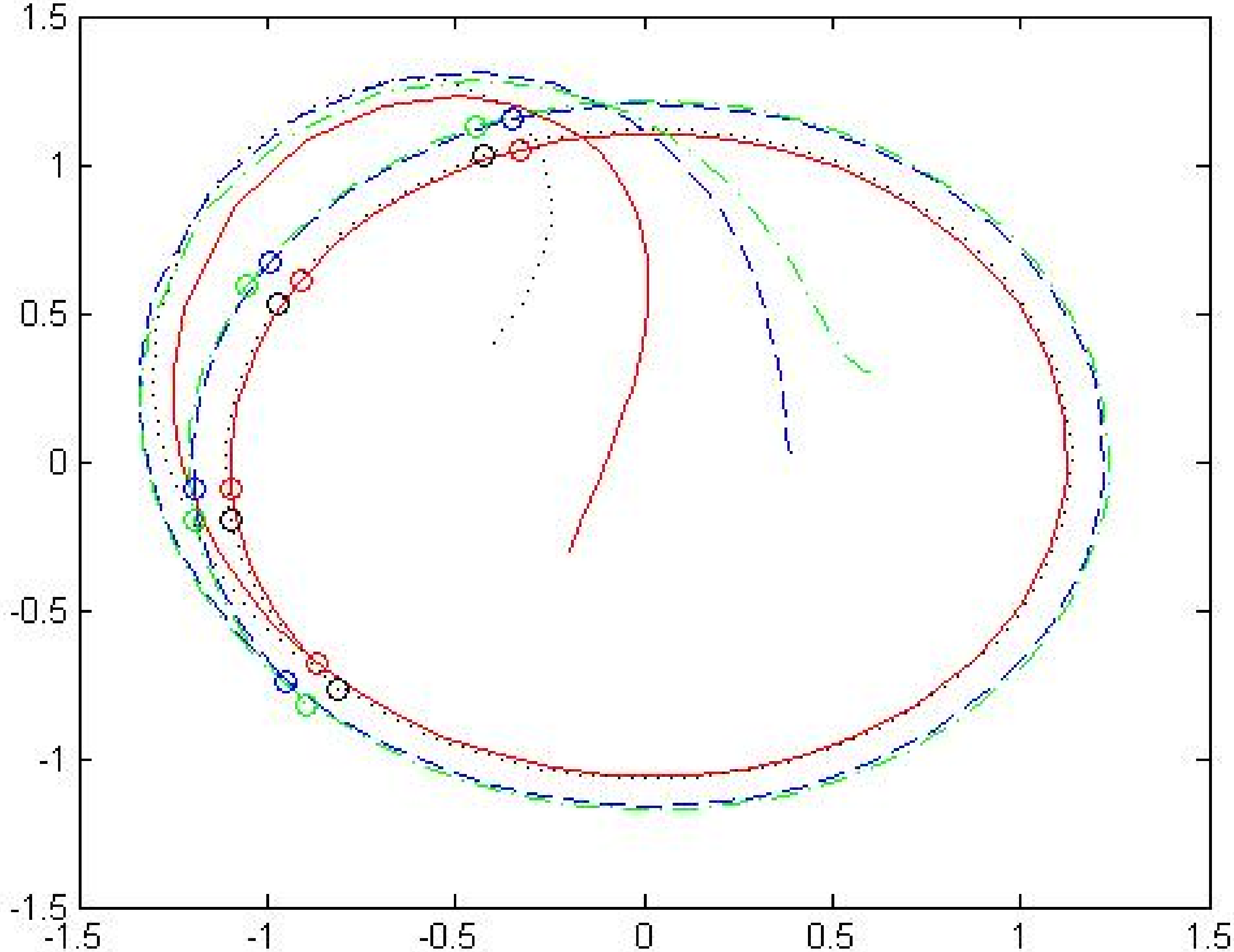


Figure 2: Formation of a three-robot team in a moving coordinate frame


1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5

Conclusions

- We designed cooperative stabilization and cooperative tracking control for a group of linear dynamic systems
- The control includes two parts: individual stabilization and consensus
- Formation tracking is solved as cooperative stabilization of the tracking error systems
- General linear dynamic systems
- Lyapunov based method is used

Reading

Y. Guo, "Cooperative Stabilization and Tracking for Linear Dynamic Systems", book chapter in P. Pardalos, D. Grundel, R. A. Murphey and O. Prokopyev (Eds), Cooperative Networks: Control and Optimization, Edward Elgar Publishing, 2008.