Final Exam

EEL 3657(Spring 2004)

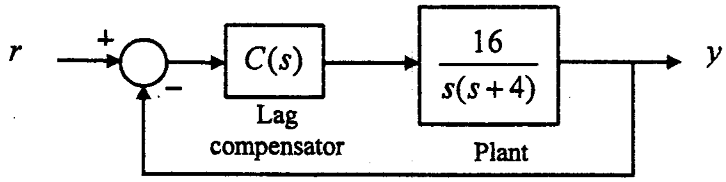
Name: Solutions

Student ID#: ________________

Instructions

1. There are totally 4 problems in this exam.
2. Do all work in the space provided.
3. Show all work for partial credit.
4. Assemble your work for each problem in a logical order.
5. Justify your conclusion. I cannot read minds.

Problem 1. (25 points)



1) Verify that with C(s)=1 the system shown has closed-loop poles at $s = -2 \pm j2\sqrt{3}$.
2) What is the static velocity error constant K_v?
3) The pole locations are considered suitable but it is desired that $K_v = 20$. Consider the use of a lag compensator $C(s) = \dfrac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$. To what value should β be set?

Solution:

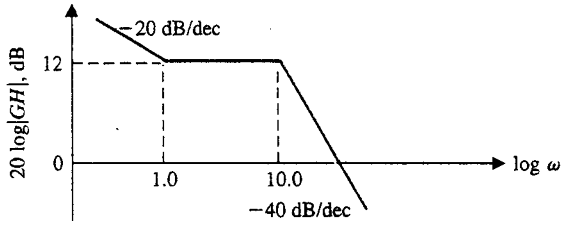
1). $C(s)=1$, char eqn is $s^2+4s+16=0$ $\therefore$ poles at $s = -2 \pm j2\sqrt{3}$

$(s+2)^2 + (\sqrt{12})^2$

2). $K_v = \lim_{s \to 0} s \cdot \dfrac{16}{s(s+4)} = 4$

3). With lag compensator, need $C(0) = 5$, $\therefore$ $\beta = 5$

Problem 2. (20 points) The asymptotic log-magnitude curve for a transfer function is given in the following diagram. Determine the transfer function.



Solution:

Idetify three factors: $\frac{1}{s}$, $1+\frac{s}{1}$, $\frac{1}{1+\frac{s}{10}}$

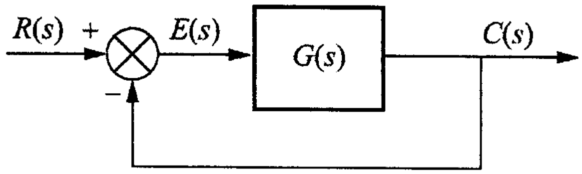
Determine k: $20\log K = 12\text{dB}$

$\Rightarrow k = 4$

So: $$GH(s) = \frac{4(1+s)}{s(1+\frac{s}{10})^2}$$

$$= \frac{400(s+1)}{s(s+10)^2}$$

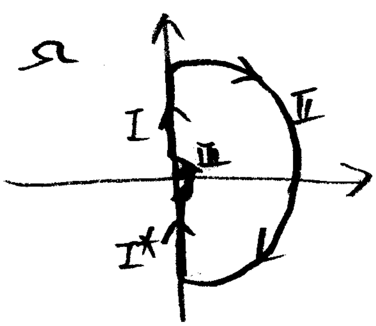
Problem 3. (~~25~~ 40 points) For a unity feedback system shown below,



Where $G(s) = \dfrac{K(s+1)}{s(s-10)}$,

1). Sketch the Nyquist plot for K=1.
2). Using Nyquist criterion, determine if the closed-loop system is stable when K=1, and the range of K for the closed-loop system to be stable.
3). Sketch the root locus, find the value of K at the imaginary-axis crossing points, and determine the range of K for the closed-loop system to be stable from the root locus.

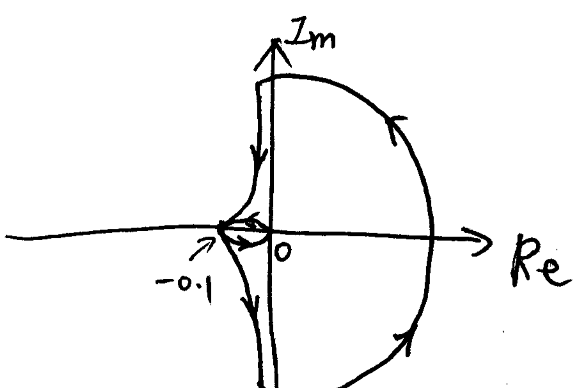
Solution: 1).



$$G(j\varepsilon) = \infty \angle -270^\circ$$

$$G(j\infty) = 0 \angle -90^\circ$$

$$G(-j\varepsilon) = \infty \angle 270^\circ$$



Negative real axis crossing:

$$G(j\omega) = \frac{j\omega+1}{j\omega(j\omega-10)}$$

$$= \frac{j\omega+1}{-\omega^2-j10\omega} \times \frac{-\omega^2+j10\omega}{-\omega^2+j10\omega}$$

$$= \frac{(-\omega^2-10\omega^2)+j(10\omega-\omega^3)}{\omega^4+(10\omega)^2}$$

$$= R(\omega) + jX(\omega)$$

Let $X(\omega)=0 \Rightarrow 10\omega-\omega^3=0 \Rightarrow \omega=0, \pm\sqrt{10}$

Substitute $\omega=\sqrt{10}$ into $R(\omega)$: $R(j\sqrt{10}) = \dfrac{-11}{(\sqrt{10})^2+100} = \dfrac{-11}{110} = -0.1$

2).

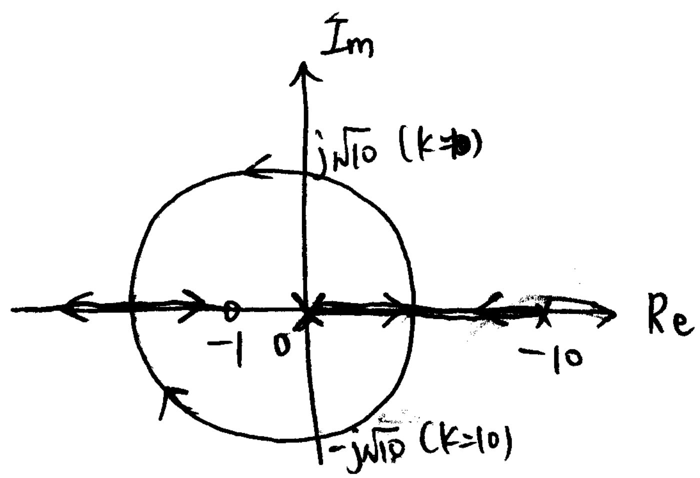
$$Z = N + P$$
$$= 0 + 1 \Rightarrow \text{unstable}$$

To let ~~the~~ system stable, $N = -P = -1$.

So: $K = \frac{-1}{-0.1} = 10$.

The range of K for stability is $K > 10$.

3).



CE: $1 + \frac{K(s+1)}{s(s-10)} = 0 \Rightarrow s^2 + (K-10)s + K = 0$

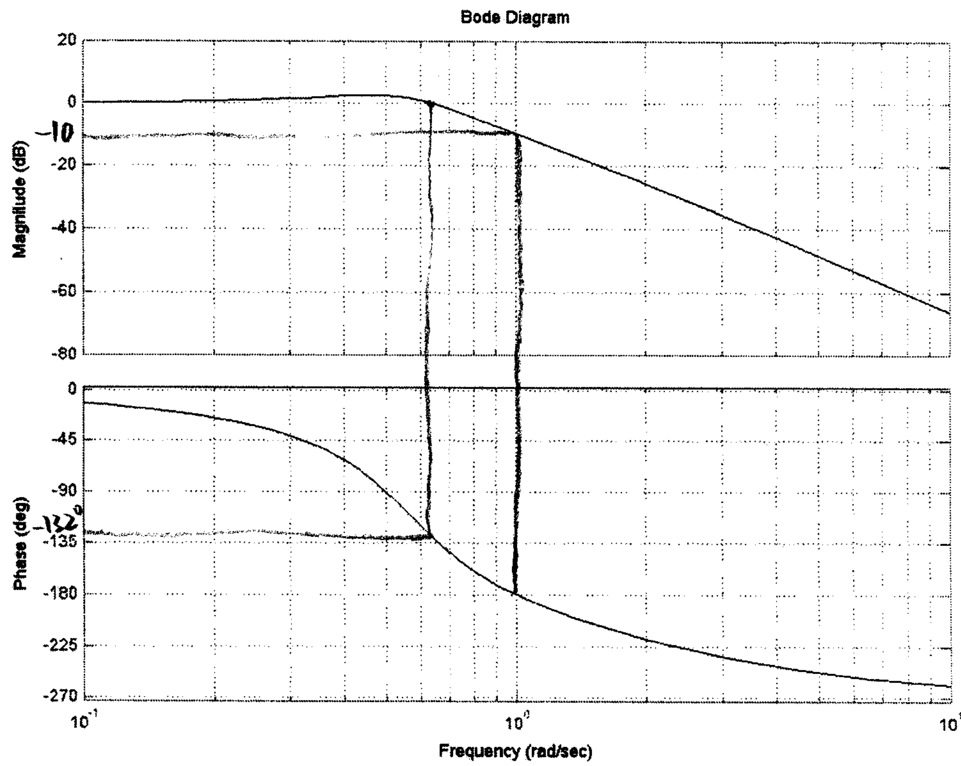
Let $s = j\omega$, $(j\omega)^2 + (K-10)j\omega + K = 0$

$$(K - \omega^2) + j(K-10)\omega = 0$$

$$\Rightarrow \begin{cases} K - \omega^2 = 0 \\ (K-10)\omega = 0 \end{cases} \Rightarrow \begin{cases} \omega = 0, \pm\sqrt{10} \\ K = 10 \end{cases}$$

From the root locus, only when $K > 10$, root locus enters into LHP. So range of K for stability is $K > 10$.

Problem 4. (15 points) Determine the gain margin and phase margin from the Bode diagram.



Solution:

$20\log X = 10\text{dB}$

$\Rightarrow$ Gain margin is 3.16.

phase margin $180^\circ - 132^\circ = 48^\circ$.