Exam #1

EEL 3657 (Fall 2004)

Name: Solutions

SS#: ____________

Please show all work for partial credit. (100 points total)

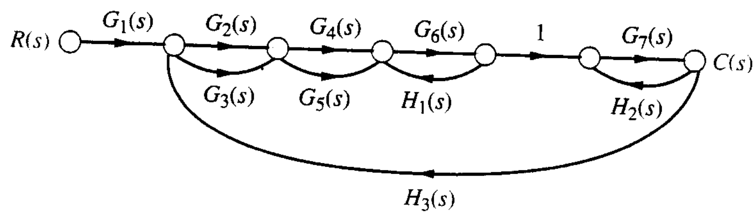
Problem 1. (30 points) Find the transfer function, G(s)=C(s)/R(s), corresponding to the differential equation:

$$\frac{d^3c(t)}{dt^3}+3\frac{d^2c(t)}{dt^2}+7\frac{dc(t)}{dt}+5c(t)=\frac{d^2r(t)}{dt^2}+4\frac{dr(t)}{dt}+3r(t)$$

Taking Laplace transform

$$G(s)=\frac{C(s)}{R(s)}=\frac{s^2+4s+3}{s^3+3s^2+7s+5}.$$

Problem 2. (30 points) Using Mason's rule, find the transfer function, T(s)=C(s)/R(s), for the system shown below:



Closed-loop gains: $G_2G_4G_6G_7H_3$; $G_2G_5G_6G_7H_3$; $G_3G_4G_6G_7H_3$; $G_3G_5G_6G_7H_3$; G_6H_1; G_7H_2

Forward-path gains: $T_1 = G_1G_2G_4G_6G_7$; $T_2 = G_1G_2G_5G_6G_7$; $T_3 = G_1G_3G_4G_6G_7$; $T_4 =$

$G_1G_3G_5G_6G_7$

Nontouching loops 2 at a time: $G_6H_1G_7H_2$

$\Delta = 1 - [H_3G_6G_7(G_2G_4 + G_2G_5 + G_3G_4 + G_3G_5) + G_6H_1 + G_7H_2] + [G_6H_1G_7H_2]$

$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$

$$T(s) = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4}{\Delta}$$

$$= \frac{G_1G_2G_4G_6G_7 + G_1G_2G_5G_6G_7 + G_1G_3G_4G_6G_7 + G_1G_3G_5G_6G_7}{1 - H_3G_6G_7(G_2G_4 + G_2G_5 + G_3G_4 + G_3G_5) - G_6H_1 - G_7H_2 + G_6H_1G_7H_2}$$

Problem 3. (40 points) Sketch the root locus for the following open-loop transfer function

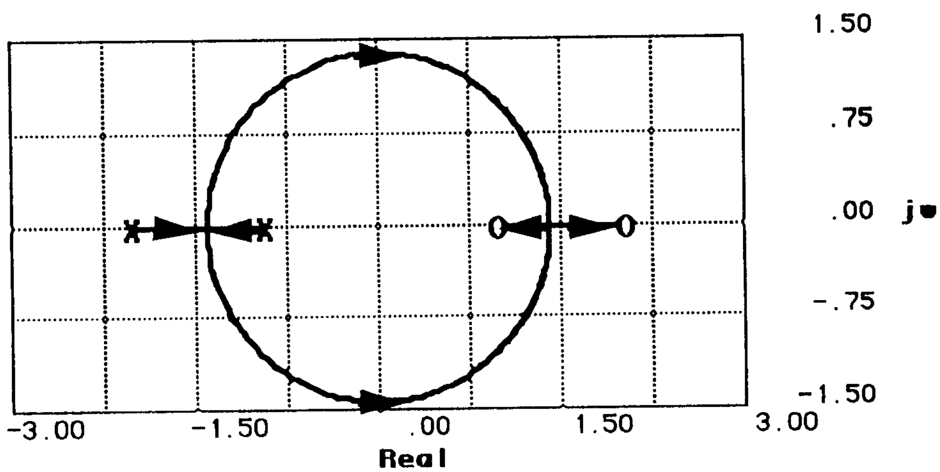
$$\frac{K(s-1)(s-2)}{(s+1)(s+2)},$$

and find:

1). break-out/break-in points;

2). jw axis crossings and the K value at the jw axis crossings;

3). the range of K for which the closed-loop system is stable.



Imaginary axis crossing: j1.41 at K = 1. Stability: K < 1. Breakaway: -1.41 at K = 0.03. Break-in: 1.41 at K = 33.97. Points on root locus: -1.5 ± j0, K = 0.02857; -0.75 ± j1.199, K = 0.33; 0 ± j1.4142, K = 1; 0.75 ± j1.1989, K = 3.