Final Exam

EEL 3657 (Summer 2003)

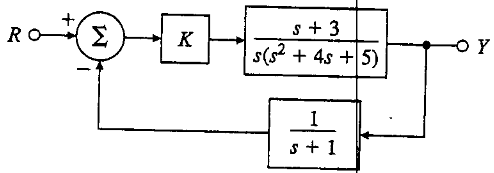
Name: Solutions

Student ID#: ____________

Instructions

1. There are totally 4 problems in this exam.
2. Show all work for partial credit.
3. Assemble your work for each problem in logical order.
4. Justify your conclusion. I cannot read minds.

Problem 1. (25 points) Consider the system shown below,



1). Find the closed-loop characteristic equation.
2). Use Routh's stability criterion, determine all values of K for which the system is stable.

Solution: 1).

$$G_{cl}(s) = \frac{K\cdot\frac{s+3}{s(s^2+4s+5)}}{1+K\cdot\frac{s+3}{s(s^2+4s+5)}\cdot\frac{1}{s+1}}$$

$$= \frac{K(s+1)(s+3)}{s(s^2+4s+5)(s+1)+K(s+3)}$$

$$= \frac{Ks^2+4Ks+3K}{s^4+5s^3+9s^2+(5+K)s+3K}$$

CE: $s^4+5s^3+9s^2+(5+K)s+3K=0$

2). Routh table:

$$\begin{array}{c|ccc} s^4 & 1 & 9 & 3K \\ s^3 & 5 & 5+K & \\ \hline s^2 & b_1 & b_2 & \\ s^1 & C_1 & C_2 & \\ s^0 & d_1 & & \end{array}$$

$b_1 = \frac{40-K}{5}$,

$b_2 = 3K$,

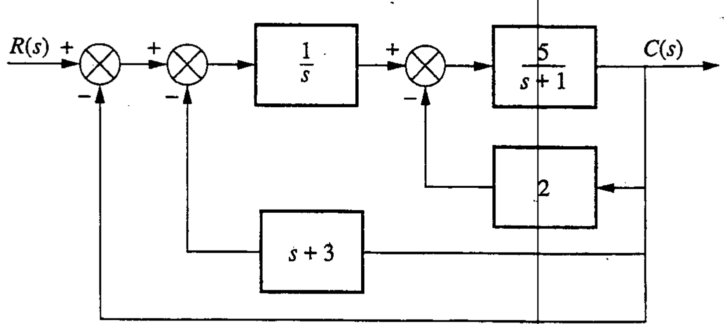
$C_1 = (5+K) - \frac{75K}{40-K}$.

$C_2 = 0$,

$d_1 = 3K$.

$$\left.\begin{array}{l} b_1>0 \\ C_1>0 \\ d_1>0 \end{array}\right\} \Rightarrow \left\{\begin{array}{ll} 40-K>0 & \Rightarrow K<40 \\ (5+K)-\frac{75K}{40-K}>0 & \Rightarrow -44.49<K<4.49 \\ 3K>0 & \Rightarrow K>0 \end{array}\right\} \Rightarrow 0<K<4.49.$$

Problem 2. (25 points) For the system shown below, what steady-state error can be expected for an input of 15u(t)? (u(t) is a unit step function.)



Solution:

Reduce the system to an equivalent unity feedback system.

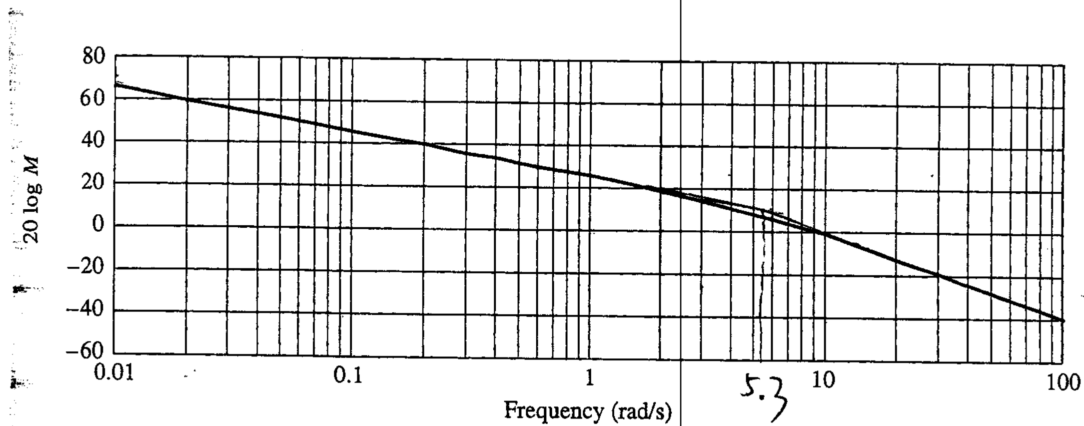
$$G(s) = \frac{\dfrac{5}{s\,(s+11)}}{1+\dfrac{5\,(s+3)}{s\,(s+11)}} = \frac{5}{s\,(s+11)+5\,(s+3)} = \frac{5}{s^2+16s+15}$$

Hence, $e(\infty) = \lim_{s\to 0} G(s) = K_p = \frac{1}{3}$.

Finally,

$$e_{ss} = \frac{15}{1+K_p} = \frac{15}{1+\frac{1}{3}} = \frac{45}{4} = 11.25$$

Problem 3. (20 points) For the Bode magnitude plot (with straight-line segments marked) shown below, determine the transfer function.



Solution: The first (left) segment has a slope −20dB/dec, which indicates a $\frac{1}{s}$ term;

The second (right) segment has a slope −40dB/dec, which indicates a $\frac{1}{1+\frac{s}{5.3}}$ term.

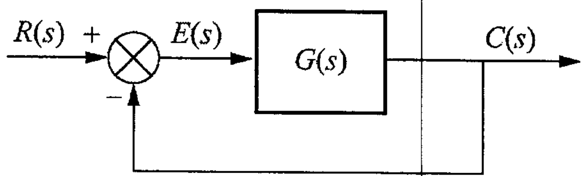
Determine K from low-frequency Bode magnitude plot,

$$67 = 20\log K - 20\log 0.01$$

$$\Rightarrow K = 22.39$$

So $$G(s) = \frac{K}{s\left(1+\frac{s}{5.3}\right)} = \frac{5.3K}{s(s+5.3)} = \frac{118}{s(s+5.3)}.$$

Problem 4. (30 points) For a unity feedback system shown below,



where $G(s) = \dfrac{K}{s(s+2)(s+10)}$,

1). Sketch the Nyquist plot for K=1, and use Nyquist criterion to claim stability for the closed-loop system at K=1.

2). For what range of K is the closed-loop system stable?

3). What is the gain margin?

Solution: 1).

1. Substitute $s = j\omega$ in $G(s)$
Setting $s = j\omega$ we get

$$G(j\omega) = \frac{1}{j\omega(j\omega+2)(j\omega+10)}$$

2. Substituting $\omega = 0$ in the last equation, we get the zero-frequency property of $L(j\omega)$,

$$G(j0) = \infty\angle -90°$$

3. Substituting $\omega = \infty$, the property of the Nyquist plot at infinite frequency is established.

$$G(j\infty) = 0\angle -270°$$

4. To find the intersect(s) of the Nyquist plot with the real axis, if any, we rationalize $G(j\omega)$ by multiplying the numerator and the denominator of the equation by the complex conjugate of the denominator. Thus, Eq. (9-59) becomes

$$G(j\omega) = \frac{[-12\omega^2 - j\omega(20-\omega^2)]}{[-12\omega^2 + j\omega(20-\omega^2)][-12\omega^2 - j\omega(20-\omega^2)]} = \frac{[-12\omega - j(20-\omega^2)]}{\omega[144\omega^2 + (20-\omega^2)]}$$

5. To find the possible intersects on the real axis, we set the imaginary part of $G(j\omega)$ to zero. The result is

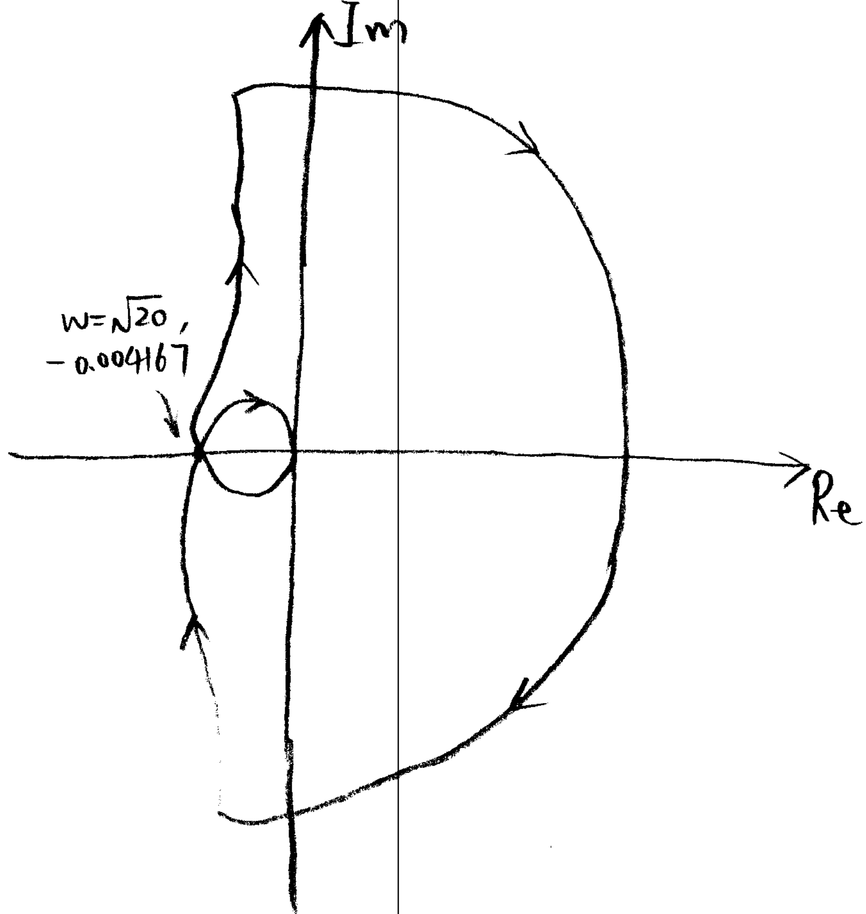
$$\mathrm{Im}[G(j\omega)] = \frac{-(20-\omega^2)}{\omega[144\omega^2 + (20-\omega^2)]} = 0$$

The solutions of the last equation are: $\omega = \infty$, which is known to be a solution at $G(j\omega) = 0$, and

$$\omega = \pm\sqrt{20} \quad \text{rad/sec}$$

Since ω is positive, the correct answer is $\omega = \sqrt{20}$ rad/sec. Substituting this frequency, we have the intersect on the real axis of the $G(j\omega)$-plane at

$$G(j\sqrt{20}) = -\frac{12}{2880} = -0.004167$$



2). $k = \frac{1}{0.004167} = 240$.

3). gain margin = 240 .