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Decentralized Coordination Control for Formation Stability of Autonomous Robotic Systems

Yi Guo

***Abstract*— Formation stability of a group of autonomous robotic systems is considered in the paper. We design cooperative control scheme in the framework of Lyapunov theorem for general linear dynamic systems. Decentralized control laws are explicitly constructed for individual systems with inter-system communications. Simulations show asymptotically tracking of a time-varying trajectory with pre-designated formation for a group of dynamic agents.**

***Index Terms*— Formation control, consensus protocol, Lyapunov theory, decentralized control.**

I. INTRODUCTION

Cooperative control has been an active research area due to its application importance in robotics, networked systems, and biological systems. A key problem in cooperative control is the convergence to a common value of the systems, which is called the consensus or agreement problem. Since the authors of [1] provided the first analytic results for analyzing cooperative behaviors of networks of agents, many excellent work appeared in the literature, see [8] for a survey on the consensus problem. Among early work, [2], [4], [9] revealed that the sufficient and necessary condition for a networked system to achieve consensus is that the information exchange topology has a spanning tree. In [5], information flow and communication structure is related to the stability property of the group of agents. In [10], [3], local control laws are presented to achieve formation stability. Later in [6], [7], a comprehensive study of consensus problem using matrix-theory-based framework was presented, and cooperative control for dynamic vehicles in their canonical forms are solved explicitly. Among existing cooperative control results, most focuses on single or double integrator dynamics or a particular vehicle dynamics, except that [6], [7] provide solutions to general, higher dimensional linear systems which are in their canonical forms.

In this paper, we study the problem of constructing decentralized coordination control for a group of autonomous systems to achieve formation stability. Communication topology is fixed and assumed to be connected. The system is in their general linear state space model. We first formulate the formation stabilization problem in an error coordinate of a moving frame. Then Lyapunov theory based methods are used to construct decentralized coordination control. Simulation results are shown for a four vehicle group to asymptotically track a unit circle.

Yi Guo is with the Department of Electrical and Computer Engineering, Stevens Institute of Technology, Hoboken, NJ 07030. yguo1@stevens.edu

II. PROBLEM STATEMENT

We assume the ith robotic system has the following linear model:

$$\dot{x}_i(t) \quad = \quad A_i x_i(t) + B_i u_i(t) \tag{1}$$

where $x_i \in \Re^m, u_i \in \Re^k$, $A_i \in \Re^{m\times m}$, and $B_i \in \Re^{m\times 1}$.

For a group of robots $i = 1, \dots, N$ to achieve cooperative behaviors such as formation, we need to design control law

$$u_i \quad = \quad u_i(x_i, x_j), \quad j \in \mathcal{N}_i \tag{2}$$

where x_j is the state of the jth robot, and $\mathcal{N}_i$ denotes the set of the neighbors of the ith robot. It can be seen that the control law is decentralized with coordination in the sense that it uses feedback from its own and its neighbors' states only.

The control problem is defined in a coordinate frame, which moves with a desired trajectory and defined to be $z_d(t) \in \Re^m$. Let $h_1 \in \Re^m, h_2 \in \Re^m, \dots, h_m \in \Re^m$ be the orthonormal vectors forming the moving frame. Then a formation of a group of n robots is achieved by a set of trajectories:

$$x_i^d(t) \quad = \quad z_d(t) + d_{i1}h_1 + d_{i2}h_2 + \dots + d_{im}h_m \tag{3}$$

In the case of formation control in a two-dimensional space (*i.e.*, $m = 2$), Figure 1) illustrates the coordinate frame of a three robot team.

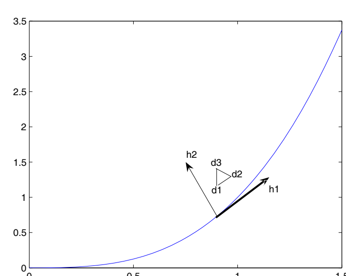

Fig. 1. Formation of a three-robot team in a moving coordinate frame

Define the error states between each robot and its desired trajectory:

$$e_i \quad = \quad x_i - x_i^d. \tag{4}$$

The error dynamics can be obtained as follows:

$$\begin{aligned} \dot{e}_i \quad &= \quad A_i e_i + A_i x_i^d + B_i u_i - \dot{x}_i^d \\ &\triangleq \quad A_i e_i + v_i \end{aligned} \tag{5}$$

where a new control variable, v_i, is defined such that

$$v_i = A_i x_i^d + B_i u_i - \dot{x}_i^d \quad (6)$$

Assume that the new control v_i is feasible (*i.e.*, there exists a u_i such that (6) holds), then the formation control problem is defined as follows:

Given communication structure of the group forming a connected undirected graph, find decentralized control law

$$v_i = v_i(e_i, e_j) \quad j \in \mathcal{N}_i, \quad (7)$$

for (5), such that the ith vehicle asymptotically tracks its reference trajectory and keep formation. That is,

$$\begin{aligned} e_i &= 0 \quad as \quad t \to \infty \\ e_i &= e_j \quad as \quad t \to \infty \end{aligned} \quad (8)$$

for all $i, j = 1, 2, \ldots, N$.

III. Preliminaries from Graph and Matrix Theory

We use some notations from graph theory to describe the communications structure of the group. A graph consists of a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is a nonempty set of nodes and $\mathcal{E} \subset \mathcal{V}^2$ is a set of pairs of nodes, called edges. $\mathcal{E}$ is unordered (or ordered) for a undirect (or direct) graph. A (undirect or direct) path is a sequence of (unordered or ordered) edges connecting two distinct vertices. A graph is called connected if there is a path between any distinct pair of nodes. A tree is a graph where every node, except the root, has exactly one parent node. A spanning tree is a tree formed by graph edges that connect all the nodes of the graph. A graph has a spanning tree if there exists a spanning tree that is a subset of the graph.

We use the Laplacian matrix, L_G, to describe the connectivity of the nodes in a graph. $L_G = D - A$, where A is the adjacent matrix with diagonal entries 0 and off-diagonal entries $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$; D is the degree matrix with diagonal entries $d_{ii} = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$ and off-diagonal entries 0. By definition, L_G is a zero row sum matrix. L_G is positive semi-definite for an undirected graph.

Definition 1: A matrix $E \in \Re^{r\times r}$ is said to be reducible if the set of its indices, $\mathcal{I} \stackrel{\triangle}{=} \{1, 2, \ldots, r\}$, can be divided into two disjoint nonempty set $\mathcal{S} \stackrel{\triangle}{=} \{i_1, i_2, \ldots, i_\mu\}$ and $\mathcal{S}^c \stackrel{\triangle}{=} \mathcal{I}\backslash\mathcal{S} = \{i_1, i_2, \ldots, i_\nu\}$ (with $\mu + \nu = r$) such that $e_{i_\alpha j_\beta} = 0$, where $\alpha = 1, \ldots, \mu$ and $\beta = 1, \ldots, \nu$. A matrix is said to be irreducible if it is not reducible.

It is easy to see that the Laplacian matrix of a connected undirected graph is irreducible since its adjacent matrix does not have two disjoint vertex sets.

We'll need the following properties for zero row sum matrices.

Lemma 1 ([12], [11]): Let the set W consists all zero row sum matrices which have only non-positive off-diagonal elements. If A is a symmetric matrix in W, then A can be decomposed as $A = M^T M$ where M is a matrix such that row i consists of zeros and exactly one entry α_i and one entry $-\alpha_i$ for some nonzero α_i. Furthermore, if A is irreducible, then the graph associated with M is connected.

The matrix M can be constructed as follows ([11]): For each nonzero row of A, we generate several rows of M if the same length: for the ith row of A, and for each $i < j$ such that $A_{ij} = -\alpha$ for some $\alpha > 0$, we add a row to M with the ith element being $\sqrt{\alpha}$, and the jth element being $-\sqrt{\alpha}$. This matrix M satisfies $A = M^T M$.

IV. Decentralized Coordination Control Design

Rewrite the error system dynamics (5) in the following compact form:

$$\dot{E} = AE + V \quad (9)$$

where

$$E = [e_1^T, e_2^T, \ldots, e_N^T]^T, \quad A = I_N \otimes A_i,$$
$$V = [v_1^T, v_2^T, \ldots, v_N^T]^T$$

and I_N is the N-dimensional identity matrix.

Choose Lyapunov candidate

$$W = E^T P E + E^T (L_G \otimes P_i) E \quad (10)$$

where $P_i \in \Re^{m\times m}$ is a positive definite matrix to be chosen later, $P = I_N \otimes P_i$, and L is the Laplacian matrix of the group. The first term is for stabilization of error states in individual systems, and the second term is to accommodate coordinations. Accordingly, let the control be divided into two separate parts: one for stabilizing its own states, η_1, called stabilizing control law; the other for the group coordination, η_2, called coordination control law. That is, $V = \eta_1 + \eta_2$.

Taking time derivatives of W along the vehicles' dynamics, we get

$$\begin{aligned} \dot{W} &= 2E^T (I_N \otimes P_i + L_G \otimes P_i)(AE + V) \\ &= 2E^T (I_N \otimes P_i + L_G \otimes P_i)[(I_N \otimes A_i)E + V] \\ &= E^T[(I_N + L_G) \otimes (P_i A_i + A_i^T P_i)]E \\ &\quad +2E^T (I_N \otimes P_i + L_G \otimes P_i) V \end{aligned} \quad (11)$$

Choose the stabilizing control

$$\eta_1 = -[I_N \otimes (\epsilon_i P_i)]E.$$

Let P_i to solve the following algebraic Riccati equation (ARE)

$$P_i A_i + A_i^T P_i - 2\epsilon_i P_i P_i + Q_i = 0 \quad (12)$$

where Q_i is a positive definite matrix. We can rewrite (11) as

$$\begin{aligned} \dot{W} &= E^T[(I_N + L_G) \otimes (P_i A_i + A_i^T P_i - 2\epsilon_i P_i P_i)]E \\ &\quad +2E^T[(I_N \otimes P_i) + (L_G \otimes P_i)]\eta_2 \\ &= -E^T QE - E^T(L_G \otimes Q_i)E \\ &\quad +2E^T[(I_N \otimes P_i) + (L_G \otimes P_i)]\eta_2. \end{aligned} \quad (13)$$

where $Q = I_N \otimes Q_i$.

If we choose the coordination control as

$$\eta_2 = -FLE \tag{14}$$

where

$$F = I_N \otimes F_i, \quad L = L_G \otimes I_m,$$

and $F_i \in \Re^{m \times m}$ to be chosen later. Recall that m is the dimension of the vehicle state e_i. Simplify FL as follows:

$$\begin{aligned} FL &= (I_N \otimes F_i)(L_G \otimes I_m) \\ &= L_G \otimes F_i. \end{aligned} \tag{15}$$

Substituting (14) into (13), we have

$$\begin{aligned} \dot{W} &= -E^T QE - E^T(L_G \otimes Q_i)E \\ &\quad -2E^T\{[(I_N \otimes P_i) + (L_G \otimes P_i)](L_G \otimes F_i)\}E \\ &= -E^T QE - E^T(L_G \otimes Q_i)E \\ &\quad -2E^T[(L_G + L_G L_G) \otimes (P_i F_i)] \end{aligned} \tag{16}$$

Since both $L_G, L_G L_G$ are positive semi-definite, if F_i is chosen such that $P_i F_i$ is positive definite, then the last term of the above equation is negative semi-definite.

From Lemma 1, we know that L_G can be decomposed as $L_G = C^T C$ where C is a matrix such that row i consists of zeros and exactly one entry α_i and one entry $-\alpha_i$ for some nonzero α_i. Therefore we have the first term in (16) be:

$$\begin{aligned} -E^T(L_G \otimes Q_i)E &= -E^T[(C^T C \otimes Q_i]E \\ &= -E^T(C^T \otimes I_m)[C \otimes Q_i]x \\ &= -\sum_{i,j} \alpha_{ij}^2 (e_i - e_j)^T Q_i (e_i - e_j) \end{aligned} \tag{17}$$

Since the graph associated with C is connected, all $i, j = 1, 2, \ldots, N$ are included in the above equation.

Now we summarize the main result in the following theorem:

Theorem 1: Formation control stability is achieved for a group of robotic systems (1) using the following decentralized coordination control

$$V = -[I_N \otimes (\epsilon_i P_i)]E - FLE \tag{18}$$

if there exist positive definite matrices P_i and F_i such that P_i solves the ARE (12) and $P_i F_i$ is positive definite.

Proof: If the condition in Theorem 1 is satisfied, from (16) and (17), we have

$$\begin{aligned} \dot{W} &\leq -\sum_{i=1}^{N}(e_i Q_i e_i) - \sum_{i,j} \alpha_{ij}^2 (e_i - e_j)^T Q_i (e_i - e_j) \\ &\leq 0. \end{aligned} \tag{19}$$

The last equal sign holds only when $e_i = e_j = 0$ and $e_i - e_j = 0$. (19) indicates the Lyapunov function, W, decreases along the closed-loop dynamics. Since $E(0)$ is bounded, $W(0)$ is bounded. From the LaSalle's invariant theorem, E tends to the invariant set $e_i = 0$ and $e_i = e_j$ for all $i, j = 1, 2, \ldots, N$.

V. Simulation Examples

Consider the following two-dimensional vehicle model:

$$\begin{aligned} x_{i1} &= x_{i2} + u_{i1} \\ x_{i2} &= u_{i2} \end{aligned} \tag{20}$$

where $i = 1, 2, 3, 4$. Correspondingly,

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

The communication structure of the vehicles is shown as in Figure 2. Its Laplacian matrix is

$$L_G = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \tag{21}$$

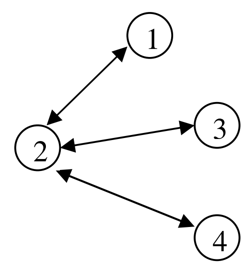

Fig. 2. Communication topology of a four-robot team

We show the states of the four-robot team following a unit circle in a rectangular formation. The geometry of the formation is illustrated in Figure 3. The origin of the formation frame tracks a unit circle. To set up the reference trajectories appropriately, we define a set of coordinations, $d_i, i = 1, 2, \ldots, N$, on a moving frame (h_1, h_2), see Figure 1. The reference trajectory is:

$$x_i^d = P_i + d_{i1} h_1 + d_{i2} h_2 \tag{22}$$

where $P_i = [\cos t \;\; \sin t]^T$, $\quad h_1 = [-\sin t \;\; \cos t]^T$, $\quad h_2 = [\cos t \;\; \sin t]^T$, and $d_i, i = 1, 2, 3, 4$ are given in Figure 3.

Define the error states $e_i = x_i - x_i^d$. Then

$$\begin{aligned} \dot{e}_{i1} &= e_{i2} + v_{i1} \\ e_{i2} &= v_{i2} \end{aligned} \tag{23}$$

where

$$\begin{aligned} v_{i1} &= u_{i1} + x_{i2}^d - \dot{x}_{i1}^d \\ v_{i2} &= u_{i2} - \dot{x}_{i2}^d. \end{aligned} \tag{24}$$

Choose $Q_i = I_2$, *i.e.*, a 2-dimensional identity matrix, and $\epsilon_i = 0.5$. The solution to the Riccati equation (12) is

$$P_i = \begin{bmatrix} 0.9102 & 0.4142 \\ 0.4142 & 1.28720 \end{bmatrix}.$$

Following the control design procedure described in the previous section, the simulation results for time histories of the error states are show in Figures 4-5. The trajectory of circle tracking is shown in Figure 6. It can be seen that cooperative formation is achieved while asymptotically tracking the reference trajectory.

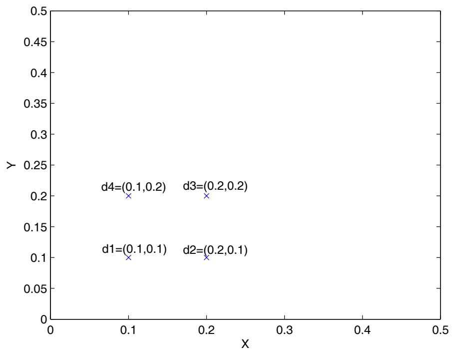

Fig. 3. Rigid formation of four robots

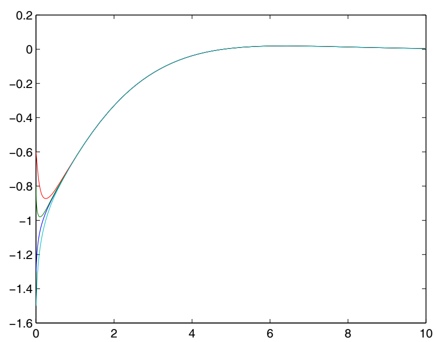

Fig. 4. The time history of the first states of four robots

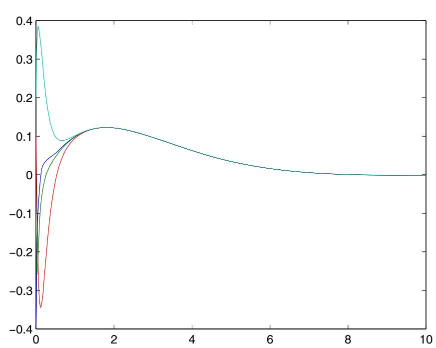

Fig. 5. The time history of the second states of four robots

VI. CONCLUSIONS

We have designed a coordination control scheme for formation stability of a group of linear dynamic systems in their general state space representations. Decentralized control laws are explicitly constructed for each system with information exchange. Making use of the existing results on consensus protocol, we combined the vehicle-level control with consensus protocol, and rendered the group tracking a desired time-varying trajectory with pre-designate formation. Lyapunov theorem based method is used to construct the control laws. Simulation results are illustrated for a four-robot group circling a unit circle. Future research includes the study of cooperative control for uncertain dynamic systems and systems with uncertain communication channels under the present framework.

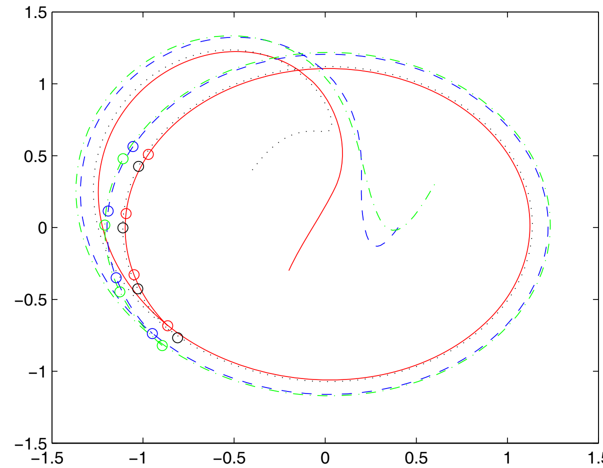

Fig. 6. Circle tracking trajectory

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