Dynamic Tracking Control of Uncertain Nonholonomic Mobile Robots

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***Abstract*— We consider the tracking control of a nonholonomic mobile robot with parameter uncertainty and unknown dynamics. A new robust adaptive controller is proposed with the aid of adaptive backstepping and robust control techniques. The proposed controller guarantees that the tracking error converges to a small ball containing the origin. The ball's radius can be adjusted by control parameters. Uncertainties in both of kinematics and dynamics of mobile robots are considered of the first time in the frame of robust and adaptive control in this paper. Simulation results show effectiveness of the proposed controller.**

I. Introduction

There has been a growing interest in the design of feedback control laws for mechanical systems with nonholonomic constraints in recent years. A well-known fact is that a nonholonomic system cannot be asymptotically stabilized to a rest configuration by continuous pure-state feedback laws due to Brockett's necessary condition for the asymptotic feedback stabilization [1]. To overcome the limitation imposed by the Brocket's necessary condition, a number of approaches have been proposed in the last decade. Among these results, on the stabilization problem of the nonholonomic kinematic systems without uncertainty, there are time-varying feedback laws, discontinuous feedback laws, hybrid feedback laws, etc. For details, refer to [12] and the references therein.

Besides the stabilization problem of the nonholonomic systems, the tracking control problem is more interesting in practice. Based on whether the system is described by a kinematic model or a dynamic model, the tracking control problem of the system is classified as either a kinematic or a dynamic tracking control problem. The kinematic tracking control problem has been widely studied in recent years. With the aid of the linearization technique, a local tracking controller was proposed in [11] for a nonholonomic wheeled mobile robot. In [18], Walsh *et al.* proposed a continuous linear local exponential controller with the aid of the linearized model. In [6], local controllers were also proposed with the aid of the linearization technique. Based on the dynamic feedback linearization and the differential flatness concept, the dynamic controllers with singular points were proposed in [3][7]. In [9], global tracking controllers were proposed for nonholonomic wheeled mobile robots. With the aid of the backstepping technique, semi-global tracking controllers were proposed in [10] for a more general nonholonomic system in chained form.

The dynamic tracking control problem of the nonholonomic system has received more attention in recent years. One of the reasons is that most of practical nonholonomic mechanical systems are dynamic systems. The dynamics of the systems usually cannot be neglected in the control when high performance of the closed system is required. In addition, control laws using velocities based on kinematic models only cannot be directly applied to practical dynamic systems which require forces as inputs. Usually, the control laws of the nonholonomic dynamic systems are obtained by simply integrating the control laws of the nonholonomic kinematic systems. However, simple integration requires exact dynamics of the systems which is hard to obtain. Considering practical applications of nonholonomic systems, the difficulty in modeling practical systems exactly, and the unavoidable disturbances in control, effective tracking control design of uncertain nonholonomic systems needs be studied. In [16], Su and Stepanenko studied the tracking control problem of the dynamic nonholonomic systems with unknown inertia parameters, and an adaptive controller was proposed. Chen *et al.* discussed the dynamic tracking problem of the uncertain nonholonomic systems in [2], and a robust H_∞ controller was proposed. However, the proposed controllers in [16][2] can only guarantee partial states of the system to track the desired states. In [6][8], the dynamic tracking problem of a wheeled mobile robot was studied, and a neural network based controller was proposed. In [4][5][17], the dynamic tracking problem of the nonholonomic systems with uncertainty in the dynamics was discussed. Robust and adaptive controllers were proposed with the aid of suitably defined errors and the Barbalat's lemma.

From a review of the literature, most of the results on the dynamic tracking problem of the nonholonomic systems are proposed based on the assumption that the kinematics of the system is exactly known and there are only uncertainties in the dynamics of the system. However, in practice, there is uncertainty in the kinematics because some geometric parameters may not be known exactly. In this paper, we will study the tracking control problem of a wheeled robot with uncertainties in both kinematics and dynamics. It is assumed that there are parameter uncertainties in the kinematics and both parameter and non-parameter uncertainties in the dynamics of the robot. To solve the tracking control problem with parameter and non-parameter uncertainties, adaptive backstepping, robust control techniques and the passivity property of the system are used to design the controller. The novelty of the results is that a systematic controller design procedure is proposed for the tracking control problem of the nonholonomic mobile robot with uncertainties in both kinematics and dynamics. The proposed controller design method can be applied to tracking control of more general dynamic nonholonomic systems with uncertainties.

II. Problem Statement

Consider a wheeled mobile robot moving on a horizontal plane (Figure 1). The robot is constituted of a rigid body, two fixed rear wheels and one steering wheel. Two torque motors are equipped in the front wheel for driving and steering. Given a differentiable simple curve (C) defined by one of its point, the unitary tangent vector at this point, and its curvature $curv(s)$ with s being the curvilinear coordinate along the curve, for a point Q in the curve (C), assume that the curvilinear coordinate at Q is s. Let $\{Q, T(s), N(s)\}$ be the Frenet frame on the curve at point Q, assume $curv(s)$ is bounded and R be a maximum positive constant such that $|curv(s)| < 1/R(\forall s)$ (choose R to be a very large constant if $curv(s) \equiv 0$). If the distance between P and the curve (C) is smaller than R, the position of P is parameterized by (s, d), where d is the coordinate of P along $N(s)$. The robot's configuration is parameterized by $q = [q_1, q_2, q_3, q_4]^T = [s, d, \theta, \beta]^T$, where θ is the angle between PF and $T(s)$ and β is the steering angle of the front wheel with respect to the robot body. By the classic law of Mechanics and also the results in [15], one has

$$\begin{cases} \dot{q}_1 = \dfrac{v_1 \cos q_3}{1 - curv(q_1) q_2}, \\ \dot{q}_2 = v_1 \sin q_3, \\ \dot{q}_3 = \dfrac{v_1 \tan q_4}{l} - \dfrac{v_1 curv(q_1) \cos q_3}{1 - curv(q_1) q_2}, \\ \dot{q}_4 = v_2, \end{cases} \tag{1}$$

$$M(q)\dot{v} + C(q,\dot{q})v + G(q) = B(q)\tau \tag{2}$$

where l is the distance between the two points P and F, v_1 is the velocity of the point P, v_2 is the angular velocity of the steering wheel, $M(q)$ is a bounded positive definite symmetric inertia matrix, $C(q,\dot{q})\dot{q}$ is centripetal and Coriolis torques, $G(q)$ is the gravitational torque, $B(q)$ is the input matrix, τ is the control input, and the superscript T denotes the transpose.

For (1)-(2), it is assumed that

1. In (1), l is not exactly known, i.e., $l \in [l_{min}, l_{max}]$ where $l_{max}(> 0)$ and $l_{min}(> 0)$ are known;

2. In (2), the matrices $M(q)$, $C(q,\dot{q})$, and $G(q)$ are unknown but are bounded by known functions $f_M(q)$, $f_C(q,\dot{q})$ and $f_G(q)$, respectively, i.e.,

$$\|M(q)\| \le f_M(q), \|C(q,\dot{q})\| \le f_C(q,\dot{q}), \|G(q)\| \le f_G(q).$$

3. In (2), the expression of $B(q)$ is known. In fact, it can be easily derived that

$$B(q) = \mathrm{diag}[1/(r\cos q_4), 1]$$

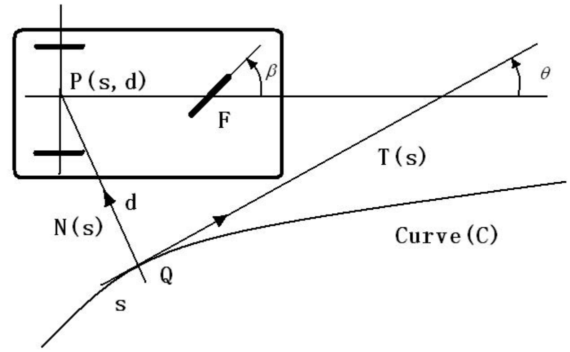

Figure 1. Configuration of a wheeled mobile robot.

where r is radius of the driving wheel. In $B(q)$, it is assumed that r is not exactly known, i.e., $r \in [r_{min}, r_{max}]$ where $r_{min}(> 0)$ and $r_{max}(> 0)$ are known.

Control Problem: Given a desired simple curve (C) and a desired velocity $v_1^*(t)$ of the robot, for (1)-(2), the control problem in this paper is defined as finding a controller τ such that

$$\lim_{t\to\infty} q_2(t) = 0, \lim_{t\to\infty} q_3(t) = 0, \lim_{t\to\infty} (v_1(t) - v_1^*(t)) = 0.$$

Remark 1: Noting that the assumption $|curv(q_1)| < 1/R$, (1) is well-defined if $|q_2| < R$ and $|q_4| < \pi/2$. In the controller design, these conditions will be guaranteed if $|q_2(0)| < R$ and $|q_4(0)| < \pi/2$.

In (2), the following well-known property is satisfied [16].

Property 1: For a suitably defined $C(q,\dot{q})$, $(\dot{M} - 2C)$ is skew-symmetric.

III. Backstepping Design Procedure

To deal with parameter uncertainties and non-parameter uncertainties in (1)-(2), the adaptive backstepping technique [13] and the robust techniques are used to design the controller.

Assume $|q_2(0)| < R$, $|q_3(0)| < \pi/2$, $|q_4(0)| < \pi/2$, let

$$\begin{aligned} & b = 1/l, u_1 = v_1 \cos q_3/(1 - curv(q_1)q_2), \\ & u_2 = v_2, g_2 = [0,0,0,1]^T, \\ & g_1 = [1, (1 - curv(q_1)q_2)\tan q_3, -curv(q_1), 0]^T, \\ & g_3 = [0,0,(1 - curv(q_1)q_2)\tan q_4/\cos q_3, 0]^T, \end{aligned}$$

(1)-(2) can be written as

$$\dot{q} = g_1 u_1 + g_2 u_2 + b g_3 u_1, \tag{3}$$

$$M_1(q)\dot{u} + C_1(q,\dot{q})u + G_1(q) = B_1(q)\tau \tag{4}$$

where

$$\begin{aligned} & M_1(q) = \Psi^T(q) M(q) \Psi(q), \\ & C_1(q,\dot{q}) = \Psi^T(q)(M(q)\dot{\Psi}(q) + C(p,\dot{p})\Psi(q)), \\ & G_1(q) = \Psi^T(q) G(q), \\ & B_1(q) = \mathrm{diag}[(1 - curv(q_1)q_2)/(r\cos q_3 \cos q_4), 1], \\ & \Psi(q) = \mathrm{diag}[(1 - curv(q_1)q_2)/\cos q_3, 1]. \end{aligned}$$

Step 1: Introducing $\tilde{u}_1 = u_1 - v_1^*$, if u_1 were the actual control input, one had $\tilde{u}_1 \equiv 0$ and $u_1 \equiv v_1^*$. Let

$$z_2 = h(q_2)$$

where $h(q_2)$ is a smooth monotonic function which maps $(-R, R)$ onto $(-\infty, +\infty)$ with the first derivative (with respect to q_2) strictly larger than a positive real number and such that $h(0) = 0$, then

$$\dot{z}_2 = v_1^* L_{g_1} z_2 + u_2 L_{g_2} z_2 + b v_1^* L_{g_3} z_2 = v_1^* L_{g_1} z_2, \quad (5)$$

where $L_{g_i} z_j$ is Lie derivative of z_j along g_i. Hereafter L means Lie derivative in this paper.

Introducing

$$z_3 = L_{g_1} z_2 - \alpha_3,$$

if $L_{g_1} z_2$ were the actual control input, one had $z_3 \equiv 0$ and $L_{g_1} z_2 \equiv \alpha_3$. Let Lyapunov function

$$V_2 = \frac{1}{2} z_2^2,$$

to make

$$\dot{V}_2 = -k_2 z_2^2 v_1^*,$$

we choose

$$\alpha_3 = -k_2 z_2, \quad (6)$$

where constant $k_2 (> 0)$ is a design parameter.

Since $L_{g_1} z_2$ is not the control, $z_3 \not\equiv 0$. So

$$\dot{V}_2 = -k_2 z_2^2 v_1^* + z_2 z_3 v_1^*.$$

The second term $z_2 z_3 v_1^*$ will be cancelled at the next step. The closed-loop system (5) with (6) is

$$\dot{z}_2 = -k_2 z_2 v_1^* + z_3 v_1^*. \quad (7)$$

And

$$\dot{z}_3 = v_1^* L_{g_1}^2 z_2 - k_2^2 z_2 v_1^* + k_2 z_3 v_1^* + b v_1^* L_{g_3} L_{g_1} z_2. \quad (8)$$

Step 2: Introducing $z_4 = L_{g_1}^2 z_2 - \alpha_4$, let Lyapunov function

$$V_3 = \frac{1}{2}(z_2^2 + z_3^2) + \frac{1}{2}\gamma_1^{-1}(\widehat{b} - b)^2,$$

where constant $\gamma_1 (> 0)$ is a design parameter, then

$$\begin{aligned}\dot{V}_3 = -k_2 z_2^2 v_1^* + z_3(z_4 + z_2 - k_2^2 z_2 + k_2 z_3 + \widehat{b} L_{g_3} L_{g_1} z_2 \\ + \alpha_4) v_1^* + \gamma_1^{-1}(\widehat{b} - b)(\dot{\widehat{b}} - \gamma_1 v_1^* z_3 L_{g_3} L_{g_1} z_2).\end{aligned}$$

If $L_{g_1}^2 z_2$ were the actual control input, one had $z_4 \equiv 0$. To make

$$\dot{V}_3 = -k_2 z_2^2 v_1^* - k_3 z_3^2 v_1^*,$$

we would choose

$$\begin{aligned}\dot{\widehat{b}} &= \zeta_1, \\ \alpha_4 &= -(k_2 + k_3) z_3 - (1 - k_2^2) z_2 - \widehat{b} L_{g_3} L_{g_1} z_2, \quad (9)\end{aligned}$$

where

$$\zeta_1 = \gamma_1 v_1^* z_3 L_{g_3} L_{g_1} z_2,$$

constant $k_3 (> 0)$ is a design parameter.

Since $L_{g_1}^2 z_2$ is not the control, $z_4 \not\equiv 0$ and we do not use $\dot{\widehat{b}} = \zeta_1$ as an update law in the control. Then

$$\dot{V}_3 = -k_2 z_2^2 v_1^* - k_3 z_3^2 v_1^* + z_3 z_4 v_1^* + \gamma_1^{-1}(\widehat{b} - b)(\dot{\widehat{b}} - \zeta_1).$$

The third term $z_3 z_4 v_1^*$ will be cancelled at the next step. The closed-loop system (8) with (9) is

$$\dot{z}_3 = -k_3 z_3 v_1^* + z_4 v_1^* - z_2 v_1^* + (b - \widehat{b}) v_1^* L_{g_3} L_{g_1} z_2.$$

And

$$\begin{aligned}\dot{z}_4 = v_1^* L_{g_1}^3 z_2 + (k_2 + k_3) z_4 v_1^* - (k_2 k_3 + k_3^2 - 1 \\ + k_2^2) z_3 v_1^* - (2k_2 - k_2^3 + k_3) z_2 v_1^* + \dot{\widehat{b}} L_{g_3} L_{g_1} z_2 \\ + \widehat{b} v_1^* L_{g_1} L_{g_3} L_{g_1} z_2 + (k_2 + k_3)(b - \widehat{b}) v_1^* L_{g_3} L_{g_1} z_2 \\ + \widehat{b} b v_1^* L_{g_3}^2 L_{g_1} z_2 + b v_1^* L_{g_3} L_{g_1}^2 z_2 + \widehat{b} u_2 L_{g_2} L_{g_3} L_{g_1} z_2.\end{aligned}$$

Step 3: Introducing $\tilde{u}_2 = u_2 - \alpha_5$, let Lyapunov function

$$V_4 = \frac{1}{2}(z_2^2 + z_3^2 + z_4^2) + \frac{1}{2}\gamma_1^{-1}(\widehat{b} - b)^2,$$

then

$$\begin{aligned}\dot{V}_4 = & -k_2 z_2^2 v_1^* - k_3 z_3^2 v_1^* + z_4[z_3 v_1^* + v_1^* L_{g_1}^3 z_2 + (k_2 \\ & + k_3) z_4 v_1^* - (k_2 k_3 + k_3^2 - 1 + k_2^2) z_3 v_1^* - (2k_2 \\ & - k_2^3 + k_3) z_2 v_1^* + \dot{\widehat{b}} L_{g_3} L_{g_1} z_2 + \widehat{b} v_1^* L_{g_1} L_{g_3} L_{g_1} z_2 \\ & + \widehat{b}^2 v_1^* L_{g_3}^2 L_{g_1} z_2 + \widehat{b} v_1^* L_{g_3} L_{g_1}^2 z_2 + \widehat{b}(\tilde{u}_2 \\ & + \alpha_5) L_{g_2} L_{g_3} L_{g_1} z_2] + \gamma_1^{-1}(\widehat{b} - b)(\dot{\widehat{b}} \\ & - \gamma_1(z_3 v_1^* L_{g_3} L_{g_1} z_2 + z_4[(k_2 + k_3) v_1^* L_{g_3} L_{g_1} z_2 \\ & + \widehat{b} v_1^* L_{g_3}^2 L_{g_1} z_2 + v_1^* L_{g_3} L_{g_1}^2 z_2])).\end{aligned}$$

If u_2 were the actual control input, one had $\tilde{u}_2 \equiv 0$. The update law of $\widehat{b}$ is chosen as

$$\dot{\widehat{b}} = \begin{cases} \zeta_2 - \delta_1(\widehat{b} - b_0), & \text{if } \widehat{b} \in (b_l, b_u), \text{ or } \widehat{b} = b_l, \zeta_2 \geq 0, \\ & \text{or } \widehat{b} = b_u, \zeta_2 \leq 0; \\ -\delta_1(\widehat{b} - b_0), & \text{if } \widehat{b} = b_l, \zeta_2 < 0, \text{ or } \widehat{b} = b_u, \zeta_2 > 0 \end{cases} \quad (10)$$

where $b_l = 1/l_{max}$ and $b_u = 1/l_{min}$, constants $\delta_1 (> 0)$ and $b_0 (\in (b_l, b_u))$ are design parameters, and ζ_2 is defined by

$$\zeta_2 = \zeta_1 + \gamma_1 z_4 v_1^*[(k_2 + k_3) L_{g_3} L_{g_1} z_2 + \widehat{b} L_{g_3}^2 L_{g_1} z_2 + L_{g_3} L_{g_1}^2 z_2].$$

The virtual control α_5 is chosen as

$$\begin{aligned}\alpha_5 = [-k_4 z_4 v_1^* - z_3 v_1^* - v_1^* L_{g_1}^3 z_2 - (k_2 + k_3) z_4 v_1^* \\ + (k_2 k_3 + k_3^2 - 1 + k_2^2) z_3 v_1^* + (2k_2 - k_2^3 + k_3) z_2 v_1^* \\ + \Pi - \widehat{b} v_1^* L_{g_1} L_{g_3} L_{g_1} z_2 - \widehat{b}^2 v_1^* L_{g_3}^2 L_{g_1} z_2 \\ - \widehat{b} v_1^* L_{g_3} L_{g_1}^2 z_2] / (\widehat{b} L_{g_2} L_{g_3} L_{g_1} z_2)\end{aligned}$$

where

$$\begin{aligned}\Pi = -(\sqrt{1 + \zeta_2^2} + \delta_1 \sqrt{1 + (\widehat{b} - b_0)^2}) L_{g_3} L_{g_1} z_2 \times \\ \tanh(z_4(\sqrt{1 + \zeta_2^2} + \delta_1 \sqrt{1 + (\widehat{b} - b_0)^2}) L_{g_3} L_{g_1} z_2)/\delta_2),\end{aligned}$$

constants $k_4 (> 0)$ and $\delta_2 (> 0)$ are design parameters. α_5 is the third stabilizing function. Then

$$\begin{aligned}\dot{V}_4 \leq & -k_2 z_2^2 v_1^* - k_3 z_3^2 v_1^* - k_4 z_4^2 v_1^* - \frac{\gamma_1^{-1}\delta_1}{2}(\widehat{b} - b)^2 \\ & - \frac{\gamma_1^{-1}\delta_1}{2}(\widehat{b} - b_0)^2 + \frac{\gamma_1^{-1}\delta_1}{2}(b - b_0)^2 + \rho\delta_2,\end{aligned}$$

where ρ satisfies $\rho = e^{-(\rho+1)} (i.e., \rho = 0.2785)$ [14].

Remark 2: By the relation between z and q, boundedness of z_2, z_3 and z_4 guarantees that $q_2 \in (-R, R)$, $q_3 \in$

$(-\pi/2, \pi/2)$ and $q_4 \in (-\pi/2, \pi/2)$. The update law (10) guarantees $\widehat{b} \in [b_l, b_u]$ all the time. Therefore, α_5 is well-defined.

Since u_2 is not the control input, $\tilde{u}_2 \not\equiv 0$. However, we will choose (10) as the final update law of $\widehat{b}$ in the control. Then

$$\begin{aligned}\dot{V}_4 \leq -k_2 z_2^2 v_1^* - k_3 z_3^2 v_1^* - k_4 z_4^2 v_1^* + z_4 \widehat{b} L_{g_2} L_{g_3} L_{g_1} z_2 \tilde{u}_2 \\ -\tfrac{\gamma_1^{-1}\delta_1}{2}(\widehat{b}-b)^2 - \tfrac{\gamma_1^{-1}\delta_1}{2}(\widehat{b}-b_0)^2 + \tfrac{\gamma_1^{-1}\delta_1}{2}(b-b_0)^2 + \rho\delta_2.\end{aligned}$$

And

$$\begin{aligned}\dot{z}_4 = -k_4 z_4 v_1^* - z_3 v_1^* + \tilde{u}_2 \widehat{b} L_{g_2} L_{g_3} L_{g_1} z_2 \\ + L_{g_3} L_{g_1} z_2 \dot{\widehat{b}} + \Pi_1 + (b - \widehat{b})[\widehat{b} L_{g_3}^2 L_{g_1} z_2 \\ + L_{g_3} L_{g_1}^2 z_2 + (k_2 + k_3) L_{g_3} L_{g_1} z_2] v_1^*.\end{aligned}$$

Step 4: Since u_1 is not the control input, $\tilde{u}_1 \not\equiv 0$. Let $\tilde{u} = [\tilde{u}_1, \tilde{u}_2]^T$ and

$$\eta = [v_1^*, \alpha_5]^T, \tag{11}$$

then

$$M_1 \dot{\tilde{u}} = B_1 \tau - C_1 \tilde{u} - (M_1 \dot{\eta} + C_1 \eta + G_1). \tag{12}$$

In (12), M_1, C_1 and G_1 are unknown but are bounded by known functions, i.e.,

$$\begin{aligned}&\|M_1(q)\| \leq f_M(q)\|\Psi(q)\|^2, \\ &\|C_1(q,\dot{q})\| \leq (f_C(q) + f_M(q)\|\dot{\Psi}(q)\|)\|\Psi(q)\|^2, \\ &\|G_1(q)\| \leq f_G(q)\|\Psi(q)\|.\end{aligned}$$

Also in B_1, r is unknown.

Define $c = 1/r$, let $\widehat{c}$ be the estimates of c, we choose the control law

$$\tau = \widehat{B}_1^{-1}[-K_p \tilde{u} + \Delta + \Lambda] \tag{13}$$

where K_p is a positive definite matrix, and $\widehat{B}_1$ is the value of B_1 corresponding to the estimate $\widehat{c}$, i.e.,

$$\widehat{B}_1 = \mathrm{diag}[\widehat{c}(1 - curv(q_1)q_2)/\cos q_3 \cos q_4, 1].$$

Λ and Δ will be determined next. Then

$$\begin{aligned}M_1 \dot{\tilde{u}} &= -K_p \tilde{u} - C_1 \tilde{u} + \Delta - (M_1 \dot{\eta} + C_1 \eta + G_1) \\ &\quad + \Lambda + (B_1 - \widehat{B}_1)\tau.\end{aligned}$$

Let

$$\begin{aligned}V_5 &= \tfrac{1}{2}[z_2^2 + z_3^2 + z_4^2 + \tilde{u}^T M_1 \tilde{u} + \gamma_1^{-1}(\widehat{b} - b)^2 \\ &\quad + \gamma_2^{-1}(\widehat{c} - c)^2],\end{aligned}$$

then

$$\begin{aligned}\dot{V}_5 = -k_2 z_2^2 v_1^* - k_3 z_3^2 v_1^* - k_4 z_4^2 v_1^* + z_4 L_{g_3} L_{g_1} z_2 \dot{\widehat{b}} + z_4 \Pi \\ + \gamma_1^{-1}(\widehat{b} - b)[\dot{\widehat{b}} - z_3 u_1 L_{g_3} L_{g_1} z_2 - z_4 u_1 (\widehat{b} L_{g_3}^2 L_{g_1} z_2 \\ + L_{g_3} L_{g_1}^2 z_2 + (k_2 + k_3) L_{g_3} L_{g_1} z_2)] - \tilde{u}^T K_p \tilde{u} + [\tilde{u}^T \Lambda \\ - (k_2 z_2^2 + k_3 z_3^2 + k_4 z_4^2)\tilde{u}_1 + z_4(\widehat{b} L_{g_2} L_{g_3} L_{g_1} z_2 \\ + L_{g_2} L_{g_1}^2 z_2)\tilde{u}_2] + \gamma_2^{-1}(\widehat{c} - c)(\dot{\widehat{c}} - \tfrac{\gamma_2 \tilde{u}_1 \tau_1 (1 - curv(q_1) q_2)}{\cos q_3 \cos q_4}) \\ + \tilde{u}(\Delta - (M_1 \dot{\eta} + C_1 \eta + G_1))\end{aligned}$$

where τ_1 is the first element of τ.

If we choose the update law $\widehat{b}$ as in (10), the update law of $\widehat{c}$ is

$$\dot{\widehat{c}} = \begin{cases} \dfrac{\gamma_2 \tilde{u}_1 \tau_1 (1 - curv(q_1) q_2)}{\cos q_3 \cos q_4} - \delta_1(\widehat{c} - c_0), \\ \quad \text{if } \widehat{c} \in (c_l, c_u), \text{ or } \widehat{c} = c_l, \\ \quad \dfrac{\tilde{u}_1 \tau_1 (1 - curv(q_1) q_2)}{\cos q_3 \cos q_4} > 0, \text{ or } \widehat{c} = c_u, \\ \quad \dfrac{\tilde{u}_1 \tau_1 (1 - curv(q_1) q_2)}{\cos q_3 \cos q_4} < 0; \\ -\delta_1(\widehat{c} - c_0), \text{ if } \widehat{c} = c_l, \dfrac{\tilde{u}_1 \tau_1 (1 - curv(q_1) q_2)}{\cos q_3 \cos q_4} \\ \leq 0, \text{ or } \widehat{c} = c_u, \dfrac{\tilde{u}_1 \tau_1 (1 - curv(q_1) q_2)}{\cos q_3 \cos q_4} \geq 0, \end{cases} \tag{14}$$

and

$$\begin{aligned}\Delta &= -\frac{\chi^2 \tilde{u}}{\chi \|\tilde{u}\| + \delta_2}, \\ \chi &= f_M(q)\|\Psi(q)\|^2 \|\dot{\eta}\| + f_G(q)\|\Psi(q)\| \\ &\quad + (f_C(q) + f_M(q)\|\dot{\Psi}(q)\|)\|\Psi(q)\|^2 \|\eta\| \\ \Lambda &= \begin{bmatrix} k_2 z_2^2 + k_3 z_3^2 + k_4 z_4^2 \\ -z_4 \widehat{b} L_{g_2} L_{g_3} L_{g_1} z_2 \end{bmatrix},\end{aligned}$$

where $\gamma_2 (> 0)$ and $c_0 (\in (c_l, c_u))$ are design parameters, $c_l = 1/r_{max}$, and $c_u = 1/r_{min}$, then

$$\begin{aligned}\dot{V}_5 \quad &\leq \quad -k_2 z_2^2 v_1^* - k_3 z_3^2 v_1^* - k_4 z_4^2 v_1^* - \frac{\gamma_1^{-1}\delta_1}{2}(\widehat{b} - b)^2 \\ &\quad - \tilde{u}^T K_p \tilde{u} - \frac{\gamma_2^{-1}}{2}\delta_1(\widehat{c} - c)^2 + \frac{\gamma_1^{-1}\delta_1}{2}(b - b_0)^2 \\ &\quad + \frac{\gamma_2^{-1}}{2}\delta_1(c - c_0)^2 + (\rho + 1)\delta_2.\end{aligned} \tag{15}$$

IV. Main Results and Discussions

A. Main Results

With the aid of the preceding design procedure, one has the following result.

Theorem 1: With the controller (13) and the update laws of $\widehat{b}$ and $\widehat{c}$ defined in (10) and (14), respectively, if $v_1^*(t) \geq \epsilon_v > 0$, then $z_i (2 \leq i \leq 4)$, $\tilde{u}$, $(\widehat{b} - b)$, and $(\widehat{c} - c)$ are uniformly bounded and exponentially converge to a small ball containing the origin. The radius of the ball can be adjusted by the design parameters.

Proof: It can be proved that the modified projection algorithm (10) guarantees that $\widehat{b} \in [b_l, b_u]$, therefore α_5 is well-defined all the time. With the update law (14), it can be proved that $\widehat{c} \in [c_l, c_u]$. So the control law (13) is well-defined. Therefore, all variables in the system are well-defined. Differentiating V_5 with respect to time along the closed-loop system, one has (15). Therefore,

$$\dot{V}_5 \leq -\mu_1 V_5 + \mu_2 \tag{16}$$

where μ_1 is a positive constant which depends on the control parameters, and

$$\mu_2 = \tfrac{\gamma_1^{-1}\delta_1}{2}(b - b_0)^2 + \tfrac{\gamma_2^{-1}}{2}\delta_1(c - c_0)^2 + (\rho + 1)\delta_2.$$

So

$$V_5(t) \leq (V_5(0) - \frac{\mu_2}{\mu_1})e^{-\mu_1 t} + \frac{\mu_2}{\mu_1}.$$

$z_i(2 \le i \le 4)$, $\tilde{u}$, $(\widehat{b} - b)$, and $(\widehat{c} - c)$ are uniformly bounded and exponentially converge to a small ball. The radius of the ball can be adjusted by the design parameters $\gamma_i(1 \le i \le 2)$, δ_1, b_0, c_0, and δ_2. ■

With the aid of the state transformation and Theorem 1, one has the following result.

Theorem 2: With the controller (13) and the update laws $\widehat{b}$ and $\widehat{c}$ defined in (10) and (14), respectively, if $|q_2(0)| < R$, $|q_3(0)| \neq \pi/2$, $|q_4(0)| \neq \pi/2$, and $v_1^*(t) \ge \epsilon_v > 0$, then $q_i(2 \le i \le 4)$, $(v_1 - v_1^*)$, $(\widehat{b} - b)$, and $(\widehat{c} - c)$ are uniformly bounded and converge to a small ball containing the origin. The radius of the ball can be adjusted by the design parameters.

Proof: By Theorem 1, $z_i(2 \le i \le 4)$, $\tilde{u}$, $(\widehat{b} - b)$, and $(\widehat{c} - c)$ are uniformly bounded and exponentially converge to a small ball. By calculation, it can be proved that $q_i(2 \le i \le 4)$ and $(v_1 - v_1^*)$ are uniformly bounded and converge to a small ball. ■

B. Discussions

If $|q_2(0)| < R$, in order to make $q_2 \in (-R, R)$ all the time, $z_2 = h(q_2)$ is introduced in Step 1. With the condition imposed on $h(q_2)$, if z_2 is bounded, $q_2 \in (-R, R)$. Therefore, the definition of point Q is unique and d is well defined in the control. If $R < \infty$, one choice of $h(q_2)$ is

$$h(q_2) = \frac{2R}{\pi} \tan(\frac{\pi q_2}{2R}).$$

Specially, if $curv(s) = 0$, one can choose $h(q_2) = q_2$. If $|q_3(0)| < \pi/2$ and $|q_4(0)| < \pi/2$, the proposed controller will make $|q_3| < \pi/2$ and $|q_4| < \pi/2$ all the time. If $|q_2(0)| \ge R$ or $|q_3(0)| = \pi/2$ or $|q_4(0)| = \pi/2$, one can first use an open-loop control law to make the robot move into the region that the proposed controller can be applied, then apply the proposed controller.

Unknown parameters $b(= 1/l)$ and $c(= 1/r)$ are updated by the adaptive laws (10) and (14), respectively. They guarantee that $\widehat{b} \in [b_l, b_u]$ and $\widehat{c} \in [c_l, c_u]$.

The control parameters are $k_i(2 \le i \le 4)$, K_p, $\gamma_i(1 \le i \le 2)$, δ_1, δ_2, b_0, and c_0. Large values of $k_i(2 \le i \le 4)$ and K_p make $q_i(2 \le i \le 4)$ and $(v_1 - v_1^*)$ converge quickly to the small ball. Small values of $\gamma_i^{-1}\delta_1(1 \le i \le 2)$ and δ_1 make the radius of the ball small. Parameters b_0 and also affect the radius of the small ball. If b_0 and c_0 are close to b and c, respectively, the tracking error will be small. Therefore, in order to make $q_i(2 \le i \le 4)$ and $(v_1 - v_1^*)$ converge quickly to the origin, one can make $k_i(1 \le i \le 4)$, K_p, $\gamma_i(1 \le i \le 2)$ large and δ_1 small.

V. Simulation

In order to verify the validity of the proposed controller, Simulations were done with MATLAB. We assume the mobile robot has the following real parameters: $m = 1$, $I = 1$, $l = 1.3$, $r = 0.4$ where m is the mass of the robot, I is the inertia moment around point P. In the simulation, m, I, l and r are not known. However, we know $l_{min} = 0.8$, $l_{max} = 1.4$, $r_{min} = 0.2$ and $r_{max} = 0.6$. During the control, the given path is assume to be a circle with radius 5m and $v_1^* = 3m/s$. The initial conditions $q(0) = [0, 1, -0.7328, -0.041]^T$ and $v(0) = [0, 0]^T$. With the proposed control law, we choose the control parameters as follows. $k_1 = 1$, $k_2 = 1$, $k_3 = 1$, $k_4 = 1$, $k_p = diag[1, 1]$, $\delta_1 = 0.1$, $\delta_2 = 0.1$, $\gamma_1 = 1$, $\gamma_2 = 1$, $b_0 = 0.91$, $c_0 = 2$, $\widehat{b}(0) = 0.91$ and $\widehat{c}(0) = 2$. Figs. 2, 3 and 4 show the responses of d, θ and $(v_1 - v_1^*)$. From the results, it is shown that d, θ and $(v_1 - v_1^*)$ converge to a small ball containing the origin. Figs. 5 and 6 show the responses of $\widehat{b}$ and $\widehat{c}$. It is shown that they are bounded. Especially, $\widehat{b}$ and $\widehat{c}$ do not go through zero. Fig. 7 show the desired path and the real path in X-Y plane. The control inputs calculated from the control law are bounded and not large. They can be realized by typical mobile actuators. These simulation results show that the proposed controller is effective.

VI. Conclusion

In this paper, the tracking control of a nonholonomic wheeled robot with parameter uncertainty and non-parameter uncertainty was considered. A robust adaptive controller was proposed with the aid of adaptive backstepping and robust control techniques. Simulation results demonstrated the effectiveness of the proposed controllers.

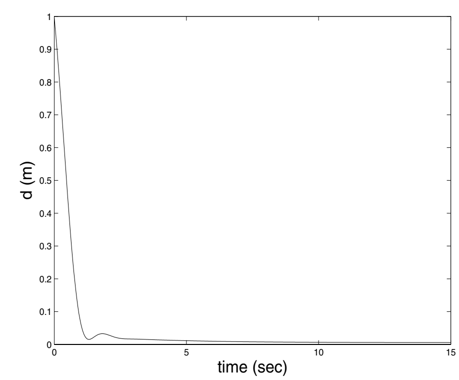

Fig. 2. Response of d.

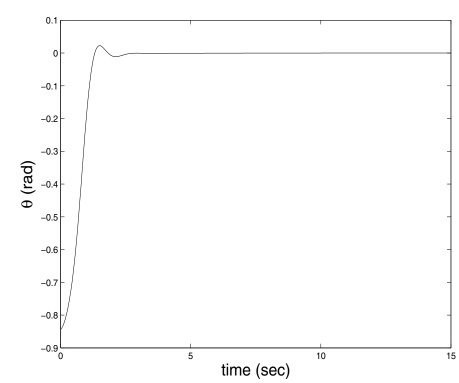

Fig. 3. Response of θ.

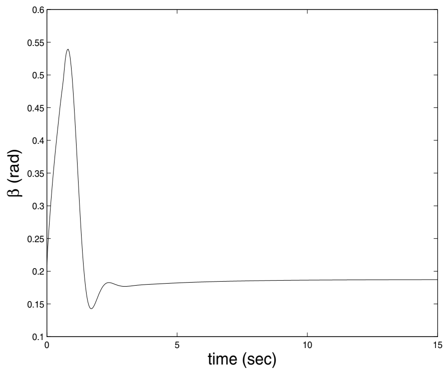

Fig. 4. Response of β.

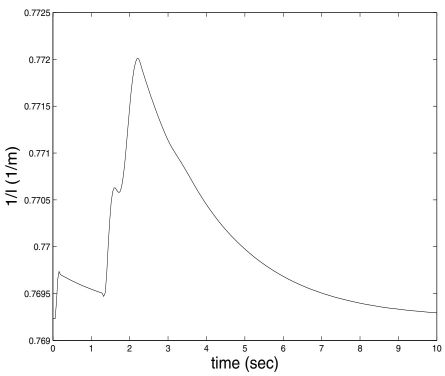

Fig. 5. Response of $\widehat{b}$.

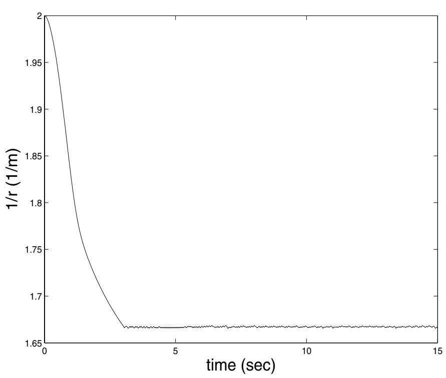

Fig. 6. Response of $\widehat{c}$.

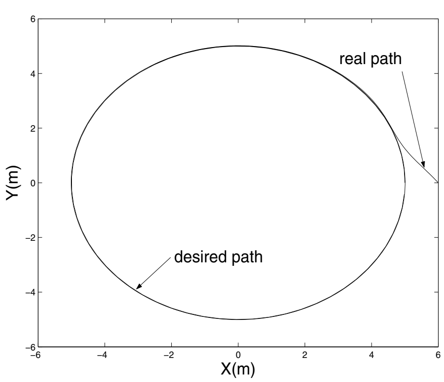

Fig. 7. Desired path and real path in X-Y plane.

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