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Nonlinear decentralized control of large-scale power systems[☆]

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Abstract

This paper describes an application of nonlinear decentralized robust control (Guo, Jiang & Hill, 1998) to large-scale power systems. Decentralized power controllers are designed explicitly to maintain transient stable closed-loop systems. For the first time, nonlinear bounds of generator interconnections are used which achieves less-conservative control gains. The proposed controllers are robust with regard to uncertain network parameters and attenuate the persistent disturbances in the sense that the L_2-gain from the disturbance to the power frequency is reduced to a certain level. Simulations on a two-generator infinite bus power system exhibit enhancement of system transient stability at different conditions of operation points, fault locations and network parameters.

Keywords: Large-scale systems; Decentralized control; Multimachine power systems; Transient stability

1. Introduction

Power systems are modelled as large nonlinear highly structured systems. Conventional linear control is limited since it can only deal with small disturbances about an operating point. Since differential geometric tools were introduced to nonlinear control systems design (Isidori, 1995), various stabilizing control results based on nonlinear power system models have been obtained (Mark, 1992; Wang, Xie, Hill & Middleton, 1992) for single-machine systems and (Chapman, Ilic, King, Eng & Kaufman, 1993; King, Chapman & Ilic, 1994; Wang, Guo & Hill, 1997; Jain & Khorrami, 1997b) for multimachine systems. Two important issues for power systems control are robustness and a decentralized structure. The robustness issue arises to deal with sources of uncertainties which mainly come from the varying network topology and the dynamic variation of the load. Since physical limitation on the system structure makes information transfer among subsystems unfeasible, decentralized controllers for multimachine systems must be used.

There have been numerous results on decentralized robust control of power systems. Among the decentralized excitation control works (Wang et al., 1997; Chapman et al., 1993; King et al., 1994; Lu, Sun, Xu & Mochizuki, 1996; Sun, Zhao, Sun & Lu, 1996; Jain, Khorrami & Fardanesh, 1994), we consider the approach in Wang et al. (1997) which applies the direct feedback linearization to transfer a nonlinear multimachine power system model to a linear one; then robust decentralized control is applied. Solving a set of algebraic Riccati equations gives controllers which guarantee the overall stability of the excitation system. Among the decentralized turbine-governor control works (Wang, Hill & Guo, 1998; Lu & Sun, 1989; Jiang, Cai, Dorsey & Qu, 1997; Jain & Khorrami, 1997b), we consider Jain & Khorrami (1997b) where use of adaptive backstepping is made to design output feedback controllers which maintain the closed-loop stability and reject bounded disturbances. Although the essential interconnections in the large-scale power systems are nonlinear (sinuous functions of machine angles), all the above results manage the interconnections with linear bounds which may cause conservatism of the control gains for nonlocal behaviour.

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Nomenclature

δ_i	the power angle of the ith generator, in rad
ω_i	the relative speed of the ith generator, in rad/s
P_{mi}	the mechanical input power, in p.u.
P_{ei}	the electrical power, in p.u.
ω_0	the synchronous machine speed, in rad/s
D_i	the per unit damping constant
H_i	the inertia constant, in s
E'_{qi}	the transient EMF in the quadrature axis, in p.u.
E_{qi}	the EMF in the quadrature axis, in p.u.
E_{fi}	the equivalent EMF in the excitation coil, in p.u.
T'_{doi}	the direct axis transient short-circuit time constant, in s
x_{di}	the direct axis reactance, in p.u.
x'_{di}	the direct axis transient reactance, in p.u.
B_{ij}	the ith row and jth column element of nodal susceptance matrix at the internal nodes after eliminating all physical buses, in p.u.
Q_{ei}	the reactive power, in p.u.
I_{fi}	the excitation current, in p.u.
I_{di}	the direct axis current, in p.u.
I_{qi}	the quadrature axis current, in p.u.
k_{ci}	the gain of the excitation amplifier, in p.u.
u_{fi}	the input of the SCR amplifier, in p.u.
x_{adi}	the mutual reactance between the excitation coil and the stator coil, in p.u.
x_{Ti}	the transformer reactance, in p.u.
x_{ij}	the transmission line reactance between the ith generator and the jth generator, in p.u.
V_{ti}	the terminal voltage of the ith generator, in p.u.
X_{ei}	the steam valve openning of the ith generator, in p.u.
P_{ci}	the power control input of the ith generator, in p.u.
T_{mi}	the time constant of the ith machine's turbine, in s
K_{mi}	the gain of the ith machine's turbine
T_{ei}	the time constant of the ith machine's speed governor, in s
K_{ei}	the gain of the ith machine's speed governor
R_i	the regulation constant of the ith machine, in p.u.

Recently, in a general nonlinear decentralized control result, Guo et al. (1998) extend the result of Jain & Khorrami (1997a) by allowing general nonlinear bounds of interconnections. Robust backstepping and the centralized H_∞ almost disturbance decoupling method have been combined with nonlinear decentralized design. Since a significant application of decentralized robust control is in large-scale power systems, in this paper we apply the nonlinear decentralized control scheme developed in Guo et al. (1998) to this problem. We design both excitation control and steam valve control to enhance the transient stability. Lower-triangular structured models are used, interconnections among subsystems are nonlinear, and persistent disturbances (arising from permanent faults, load change, etc.) enter the system without matching conditions. The decentralized controllers are designed explicitly and the resulting closed-loop systems are transiently stable and attenuate the effect of persistent disturbances. The bounds of disturbances are not known a priori in the design. The proposed controllers are simulated on a two-generator infinite bus power system where the saturation effects of magnetizing inductances are also considered. The simulation results exhibit the effectiveness of the designed controllers, both of excitors and steam valve controllers, in the sense that the transient stability of the system is enhanced in the presence of variation of operation points, fault location and network parameters.

The layout of the paper is as follows. In Section 2 we give the dynamic model of power systems, in which excitation control loop and steam valve control loop models are represented, respectively. In Section 3, the theory background of the nonlinear decentralized control scheme is stated briefly. Then in Section 4, excitation control and steam valve control are designed using the above-mentioned scheme. In Section 5, simulation results for both excitation and steam valve control performance are given to support the theoretical claims. And finally, the paper is concluded by brief remarks in Section 6.

The notation used in this paper is standard. $|\cdot|$ denotes the usual Euclidean norm for vectors. We say that $z:(0,T)\to\Re^k$ is in $L_2(0,T)$ if $\int_0^T|z(t)|^2\,dt<\infty$. A continuous function $\phi:\Re_+\to\Re_+$ is said to be of class K_∞ if it is strictly increasing and satisfies $\phi(0)=0$, and $\phi(s)\to\infty$ as $s\to\infty$. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and the minimum eigenvalue of any square matrix P.

2. Power system dynamic model

For a large-scale power system consisting of n generators interconnected through a transmission network, we apply the classic dynamic model (Bergen, 1986; Kundur, 1994). A model for each generator with both excitation and power control can be written as follows:

Mechanical equations:

$$\dot{\delta}_i=\omega_i, \tag{1}$$

$$\dot{\omega}_i = -\frac{D_i}{2H_i}\omega_i + \frac{\omega_0}{2H_i}(P_{mi} - P_{ei}) + d_i. \tag{2}$$

Generator electrical dynamics:

$$\dot{E}'_{qi} = \frac{1}{T'_{doi}}(E_{fi} - E_{qi}). \tag{3}$$

Turbine dynamics:

$$\dot{P}_{mi} = -\frac{1}{T_{mi}}P_{mi} + \frac{K_{mi}}{T_{mi}}X_{ei}. \tag{4}$$

Turbine valve control:

$$\dot{X}_{ei} = -\frac{K_{ei}}{T_{ei}R_i\omega_0}\omega_i - \frac{1}{T_{ei}}X_{ei} + \frac{1}{T_{ei}}P_{ci}. \tag{5}$$

Electrical equations:

$$E_{qi} = E'_{qi} + (x_{di} - x'_{di})I_{di}, \tag{6}$$

$$E_{fi} = k_{ci}u_{fi}, \tag{7}$$

$$P_{ei} = \sum_{j=1}^{n} E'_{qi}E'_{qj}B_{ij}\sin(\delta_i - \delta_j), \tag{8}$$

$$Q_{ei} = -\sum_{j=1}^{n} E'_{qi}E'_{qj}B_{ij}\cos(\delta_i - \delta_j), \tag{9}$$

$$I_{di} = -\sum_{j=1}^{n} E'_{qj}B_{ij}\cos(\delta_i - \delta_j), \tag{10}$$

$$I_{qi} = \sum_{j=1}^{n} E'_{qj}B_{ij}\sin(\delta_i - \delta_j), \tag{11}$$

$$E_{qi} = x_{adi}I_{fi}, \tag{12}$$

$$V_{ti} = \sqrt{(E'_{qi} - x'_{di}I_{di})^2 + (x'_{di}I_{qi})^2}. \tag{13}$$

The notation for the multimachine power system model is given in the Nomenclature. Disturbance d_i is the persistent disturbance which could be, for example, a consistent load change, or increase of the mechanical input power. We concern the disturbance effect on the power system frequency, i.e. $f_i = (1/2\pi)\omega_i$.

2.1. Excitation control loop

Since we only consider the excitation loop, $P_{mi} = P_{mi0}$ is a constant, and the plant can be modelled by (1)–(3). By applying direct feedback linearization compensation to (1)–(3) — see Wang et al. (1997) for details, we obtain

$$\dot{\delta}_i = \omega_i,$$

$$\dot{\omega}_i = -\frac{D_i}{2H_i}\omega_i - \frac{\omega_0}{2H_i}\triangle P_{ei} + d_i,$$

$$\triangle\dot{P}_{ei} = -\frac{1}{T'_{doi}}\triangle P_{ei} + \frac{1}{T'_{doi}}v_{fi} + \gamma_i(\delta,\omega), \tag{14}$$

where

$$\triangle P_{ei} = P_{ei} - P_{mi0}, \tag{15}$$

$$\gamma_i(\delta,\omega) = E'_{qi}\sum_{j=1}^{n} \dot{E}'_{qj}B_{ij}\sin(\delta_i - \delta_j) - E'_{qi}\sum_{j=1}^{n} E'_{qj}B_{ij}\cos(\delta_i - \delta_j)\omega_j, \tag{16}$$

$$v_{fi} = I_{qi}k_{ci}u_{fi} - (x_{di} - x'_{di})I_{qi}I_{di} - P_{mi0} - T'_{d0i}Q_{ei}\omega_i. \tag{17}$$

Now we seek the bound of the interconnection term $\gamma_i(\delta,\omega)$. Since the electrical power P_{ei} and the reactive power Q_{ei} of each generator and the electrical power flow through each transmission line are all bounded, and the excitation voltage E_{fi} may raise by up to 5 times of the E_{qi} when there is no load in the system, we have

$$|E'_{qi}E'_{qj}B_{ij}| \le |P_{ei}|_{\max}, \tag{18}$$

$$|\dot{E}'_{qj}| \le \left|\frac{1}{T'_{doj}}[E_{fj} - E_{qj}]\right|_{\max} \le 4|E_{qj}|_{\max}\frac{1}{|T'_{doj}|_{min}} \tag{19}$$

which is followed by

$$\begin{aligned}|\gamma_i(\delta,\omega)| &\le \sum_{j=1,j\ne i}^{n} \frac{4}{|T'_{doj}|_{\min}}|P_{ei}|_{\max}|\sin(\delta_i - \delta_j)| + \sum_{j=1}^{n} |Q_{ei}|_{\max}|\omega_j| \\ &\le \sum_{j=1,j\ne i}^{n} \frac{4p_{1ij}}{|T'_{doj}|_{\min}}|P_{ei}|_{\max}(|\sin\delta_i| + |\sin\delta_j|) + \sum_{j=1}^{n} p_{2ij}|Q_{ei}|_{\max}|\omega_j| \\ &= \sum_{j=1}^{n} (\gamma_{i1j}|\sin\delta_j| + \gamma_{i2}|\omega_j|),\end{aligned} \tag{20}$$

where

$$\gamma_{i1j} \triangleq \begin{cases} \sum_{j=1,j\ne i}^{n}\frac{4p_{1ij}}{|T'_{doj}|_{\min}}|P_{ei}|_{\max} & \text{when } j = i, \\ \frac{4p_{1ij}}{|T'_{doj}|_{\min}}|P_{ei}|_{\max} & \text{when } j \ne i, \end{cases}$$

$$\gamma_{i2} \triangleq p_{2ij}|Q_{ei}|_{\max}. \tag{21}$$

and p_{1ij}, p_{2ij} are constants with values either 1 or 0. (If they are 0, it means that jth subsystem has no connection with the ith subsystem.)

2.2. Steam valve control loop

The steam valve control loop can be modeled by (1), (2), (4), (5), i.e.

$$\dot{\delta}_i = \omega_i,$$

$$\dot{\omega}_i = -\frac{D_i}{2H_i}\omega_i + \frac{\omega_0}{2H_i}[P_{mi} - g_i(\delta)] + d_i,$$

$$\dot{P}_{mi} = -\frac{1}{T_{mi}}P_{mi} + \frac{K_{mi}}{T_{mi}}X_{ei}$$

$$\dot{X}_{ei} = -\frac{K_{ei}}{T_{ei}R_i\omega_0}\omega_i - \frac{1}{T_{ei}}X_{ei} + \frac{1}{T_{ei}}u_i, \tag{22}$$

where

$$g_i(\delta) = \sum_{j=1}^{n} E'_{qi}E'_{qj}B_{ij}\sin(\delta_i - \delta_j), \tag{23}$$

$$u_i = P_{ci}. \tag{24}$$

Because of (18), we can bound the interconnection term $g_i(\delta)$ as the following nonlinear function regardless of uncertain E'_{qi}, E'_{qj} and network parameters:

$$\begin{aligned} |g_i(\delta)| &\le \sum_{j=1,\, j\ne i}^{n} |P_{ei}|_{\max}|\sin(\delta_i - \delta_j)| \\ &\le \sum_{j=1,\, j\ne i}^{n} p_{1ij}|P_{ei}|_{\max}(|\sin\delta_i| + |\sin\delta_{j1}|) \\ &\le \sum_{j=1}^{n} g_{ij}|\sin x_{j1}|, \end{aligned} \tag{25}$$

where

$$g_{ij} \triangleq \begin{cases} \sum_{j=1,j\ne i}^{n} p_{1ij}|P_{ei}|_{\max} & \text{when } j = i, \\ p_{1ij}|P_{ei}|_{\max} & \text{when } j \ne i, \end{cases} \tag{26}$$

and p_{1ij} is as defined in (21).

2.3. Control task

We consider the transient stability of the power control system, i.e. the ability of the system to preserve synchronism after sudden severe disturbances. The fault we consider in this paper is a symmetrical three-phase short circuit on one of the transmission lines. We test the transient stability of the power control system under the following temporary fault sequence:

Stage 1: The system is in a prefault steady state.

Stage 2: A fault occurs at $t = t_0$.

Stage 3: The fault is removed by opening the breakers of the faulted line at $t = t_1$.

Stage 4: The transmission lines are restored at $t = t_2$.

Stage 5: The system is in a postfault state.

The control task we address in this paper is as follows: Design decentralized nonlinear feedback control law u_{fi} and u_i $(i = 1, 2, \dots, n)$ for the excitation control loop and steam valve control loop respectively, such that the resulting closed-loop systems are transiently stable when a major fault occurs; furthermore, the effect of the persistent disturbance on the system frequency is reduced to a certain level.

3. Nonlinear decentralized control scheme

In this section, we briefly state the results of Guo et al. (1998) as the background for the decentralized controller design in the next section. The class of large-scale nonlinear systems S considered are composed of the single-input single-output (SISO) subsystems S_i $(1 \le i \le N)$:

$$\begin{bmatrix} \dot{z}_{i1} \\ \vdots \\ \dot{z}_{i,\kappa-1} \\ \dot{z}_{i\kappa} \end{bmatrix} = \begin{bmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ 0 & 0 & & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} z_{i1} \\ \vdots \\ z_{i,\kappa-1} \\ z_{i\kappa} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \{\xi_{i1} + \gamma_{i0}(t,z) + p_{i0}(t,z)\omega_i\}$$

$$\triangleq A_i z_i + B_i[\xi_{i1} + \gamma_{i0}(t,z) + p_{i0}(t,z)\omega_i],$$

$$\dot{\xi}_{i1} = \xi_{i2} + \gamma_{i1}(t,z,\xi_{i1}) + p_{i1}(t,z,\xi_{i1})\omega_i,$$

$$\dot{\xi}_{i2} = \xi_{i3} + \gamma_{i2}(t,z,\xi_{i1},\xi_{i2}) + p_{i2}(t,z,\xi_{i1},\xi_{i2})\omega_i,$$

$$\vdots$$

$$\dot{\xi}_{i,n-\kappa} = u_i + \gamma_{i,n-\kappa}(t,z,\xi_i) + p_{i,n-\kappa}(t,z,\xi_i)\omega_i,$$

$$y_i = z_{i1}, \tag{27}$$

where

$$z_i = (z_{i1}, \dots, z_{i\kappa}) \in \Re^{\kappa}, \quad z = (z_1^{\mathrm{T}}, \dots, z_N^{\mathrm{T}})^{\mathrm{T}},$$

$$\xi_i = (\xi_{i1}, \dots, \xi_{i,n-\kappa}) \in \Re^{n-\kappa}, \quad \text{and} \quad \xi = (\xi_1^{\mathrm{T}}, \dots, \xi_N^{\mathrm{T}})^{\mathrm{T}},$$

z and ξ are the state vectors, $u_i \in \Re$ is the control input, $\omega_i \in \Re^{m_i}$ is the disturbance input, $y_i \in \Re$ is the to-be-controlled output; the unknown functions γ_{il}, p_{il}, $0 \le l \le n - \kappa$ are locally Lipschitz in states and piecewise continuous in t, and γ_{il} $(t,0,0,\dots,0) = 0$.

Both parametric and dynamic uncertainties are considered in the terms γ_{ij}, p_{ij}. We allow general nonlinear interconnections among subsystems whose bounds are given by:

$$|\gamma_{il}(t,z,\xi_{i1},\dots,\xi_{il})-\gamma_{il}(t,0,\xi_{i1},\dots,\xi_{il})| \le \sum_{j=1}^{N} a_{ilj}(\xi_{i1},\dots,\xi_{il})\varphi_{ilj}(|z_j|), \tag{28}$$

$$|p_{il}(t,z,\xi_{i1},\dots,\xi_{il})-p_{il}(t,0,\xi_{i1},\dots,\xi_{il})| \le \sum_{j=1}^{N} b_{ilj}(\xi_{i1},\dots,\xi_{il})\Phi_{ilj}(|z_j|); \tag{29}$$

and the time-varying local terms satisfy

$$|\gamma_{il}(t,0,\xi_{i1},\dots,\xi_{il})| \le \psi_{il}(|(\xi_{i1},\dots,\xi_{il})|), \tag{30}$$

$$|p_{il}(t,0,\xi_{i1},\dots,\xi_{il})| \le \Psi_{il}(|(\xi_{i1},\dots,\xi_{il})|), \tag{31}$$

where $a_{ilj}(\cdot), b_{ilj}(\cdot), \varphi_{ilj}(\cdot), \Phi_{ilj}(\cdot), \psi_{il}(\cdot), \Psi_{il}(\cdot)$ are smooth known functions, with $\varphi_{ilj}(0)=\Phi_{ilj}(0)=0$, $\psi_{il}(0)=0$, $0\le l\le n-k$.

The following theorem states the main result for disturbance attenuation:

Theorem 1. *Decentralized smooth state feedback controllers $u_i=u_i(z_i,\xi_i)$ can be found for system* (27), *such that, for any given positive constant μ, the closed-loop interconnected system satisfies the following dissipation inequality*:

$$\int_0^T |y|^2\,dt \le \mu\int_0^T |w|^2\,dt + v(z(0),\xi(0)) \qquad \forall \omega\in L_2(0,T),\ \forall T\ge 0 \tag{32}$$

where v is a positive-semidefinite function and $(z(0),\xi(0))$ *is the initial condition. Furthermore*, *the origin is globally uniformly asymptotically stable* (*GUAS*) *if* $w=0$.

The proof of Theorem 1 uses a combination of centralized H_∞ almost disturbance decoupling method (Marino, Respondek, van der Schaft & Tomei, 1994), its robust version (Jiang & Jiang, 1997) and decentralized designs (Han & Chen, 1995; Jain & Khorrami, 1997a). A stepwise procedure is presented by application of robust backstepping to the large-scale system (27) in Guo et al. (1998), where a detailed proof of Theorem 1 can be found.

Remark 1. The necessary and sufficient geometric conditions to characterize a subclass of system (27) can be found in Jain and Khorrami (1997a). Nonlinear gain bounds on the interconnections are motivated by ideas used in Mareels and Hill (1992), Jiang, Teel and Praly (1994). The idea of dominance which appeared in early decentralized work (M-matrix design) (Moylan & Hill, 1978) is implemented in this nonlinear systems design.

In the next section the nonlinear decentralized control scheme will be applied to the multimachine power systems, where design procedures are stated in detail with decentralized controllers constructed explicitly.

4. Decentralized robust controller design

4.1. Excitation control

Defining the states as $[z_{i1},z_{i2},\xi_i]=[\delta_i,\omega_i,\triangle P_{ei}]$, we represent the excitation system model after coordinate transformation in state space:

$$\dot z_{i1}=z_{i2},$$

$$\dot z_{i2}=-\frac{D_i}{2H_i}z_{i2}-\frac{\omega_0}{2H_i}\xi_i+d_i,$$

$$\dot\xi_i=-\frac{1}{T'_{doi}}\xi_i+\frac{1}{T'_{doi}}v_{fi}+\gamma_i(z), \tag{33}$$

where d_i is the disturbance, v_{fi} is the to-be-designed input, and

$$|\gamma_i(z)| \le \sum_{j=1}^{n}(\gamma_{i1j}|\sin z_{j1}|+\gamma_{i2}|z_{j2}|). \tag{34}$$

γ_{i1j}, γ_{i2} are as defined in (21).

The to-be-controlled output (power system frequency) is defined as

$$y_i=\frac{1}{2\pi}z_{i2}. \tag{35}$$

It is clear that (33) exhibits a lower-triangular dependence on the local coordinates, the disturbance enters the system without matching conditions, and the interconnection among subsystems is bounded by a nonlinear function. The nonlinear decentralized control scheme of Section 3 is applicable.

Step 1: Considering the $z_i=(z_{i1},z_{i2})$-subsystem, and seeing ξ_i as the virtual control, we have

$$\dot z_i=A_iz_i+B_i(\xi_i+k_id_i), \tag{36}$$

where

$$A_i=\begin{bmatrix}0 & 1\\ 0 & -\frac{D_i}{2H_i}\end{bmatrix},\quad B_i=\begin{bmatrix}0\\ -\frac{\omega_0}{2H_i}\end{bmatrix},\quad k_i=-\frac{2H_i}{\omega_0}. \tag{37}$$

Choose the Lyapunov function candidate:

$$V_i(z_i)=\phi_i(V_{i0}(z_i)), \tag{38}$$

where ϕ_i is a smooth K_∞ function which will be defined in the last step of the design, and $V_{i0}=z_i^{\mathrm T}P_iz_i$ with $P_i>0$ solving the algebraic Riccati equation:

$$A_i^{\mathrm T}P_i+P_iA_i-2\varepsilon_iP_iB_iB_i^{\mathrm T}P_i+Q_i=0 \tag{39}$$

with $\varepsilon_i>0$, and $Q_i>0$.

Differentiating (38) along the solution of z_i-subsystem, we have

$$\dot{V}_i = \frac{\partial \phi_i}{\partial V_{i0}} 2 z_i^{\mathrm{T}} P_i [A_i z_i + B_i(\xi_i + k_i d_i)]. \tag{40}$$

Notice that

$$\frac{\partial \phi_i}{\partial V_{i0}} 2 z_i^{\mathrm{T}} P_i B_i k_i d_i \le \frac{1}{\tau_i}\left(\frac{\partial \phi_i}{\partial V_{i0}}\right)^2 (z_i^{\mathrm{T}} P_i B_i k_i)^2 + \tau_i |d_i|^2. \tag{41}$$

Choosing the virtual control $\xi_i = \xi_i^*$ as

$$\xi_i^* = -\left\{\varepsilon_i B_i^{\mathrm{T}} P_i z_i + 0.5\,\frac{1}{\tau_i}\,\frac{\partial \phi_i}{\partial V_{i0}} (z_i^{\mathrm{T}} P_i B_i) k_i^2\right\}, \tag{42}$$

we have the dissipation inequality

$$\dot{V}_i \le -\frac{\partial \phi_i}{\partial V_{i0}} (z_i^{\mathrm{T}} Q_i z_i) + \tau_i |d_i|^2. \tag{43}$$

Step 2: Augment the z_i-subsystem with the ξ_i-subsystem, and choose a Lyapunov function as

$$W_i(z_i,\xi_i) = V_i(z_i) + (\xi_i - \xi_i^*)^2. \tag{44}$$

Note that

$$\dot{\xi}_i^* = \frac{\partial \xi_{i1}^*}{\partial z_i}(A_i z_i + B_i \xi_i + B_i k_i d_i)$$

$$\triangleq \vartheta_i(z_i, \xi_i) + \sigma_i(z_i) d_i. \tag{45}$$

Differentiating W_{i1} along the solutions of the (z_i, ξ_i)-subsystem yields

$$\dot{W}_i \le \left\{-\frac{\partial \phi_i}{\partial V_{i0}} (z_i^{\mathrm{T}} Q_i z_i) + \tau_i |d_i|^2\right\} + 2\tilde{\xi}_i \left\{\frac{\partial \phi_i}{\partial V_{i0}} z_i^{\mathrm{T}} P_i B_i - \frac{1}{T'_{doi}} \xi_i + \frac{1}{T'_{doi}} v_{fi} + \gamma_i(z) - \vartheta_i - \sigma_i(z_i) d_i\right\}, \tag{46}$$

where $\tilde{\xi}_i = \xi_i - \xi_i^*$.

For the interconnected term in (46), we have

$$2\tilde{\xi}_i \gamma_i(z) \le |\tilde{\xi}_i|^2 \sum_{j=1}^{n} \varrho_j + \sum_{j=1}^{n} \varrho_j^{-1} (\gamma_{i1j} |\sin z_{j1}| + \gamma_{i2} |z_{j2}|)^2. \tag{47}$$

For the disturbance term in (46), we have

$$2\tilde{\xi}_i [-\sigma_i(z_i) d_i] \le \frac{1}{\tau_i} \tilde{\xi}_i^2 \sigma_i^2(z_i) + \tau_i |d_i|^2. \tag{48}$$

Choosing the true control as

$$v_{fi} = -T'_{doi}\left\{c_i \tilde{\xi}_i + \frac{\partial \phi_i}{\partial V_{i0}} z_i^{\mathrm{T}} P_i B_i - \frac{1}{T'_{doi}} \xi_i - \vartheta_i + 0.5 |\tilde{\xi}_i| \sum_{j=1}^{n} \varrho_j + 0.5 \frac{1}{\tau_i} \tilde{\xi}_i \sigma_i^2(z_i)\right\} \tag{49}$$

we have the dissipation inequality

$$\dot{W}_i \le -\frac{\partial \phi_i}{\partial V_{i0}} (z_i^{\mathrm{T}} Q_i z_i) - c_i \tilde{\xi}_i^2 + 2\tau_i |d_i|^2 + \sum_{j=1}^{n} \varrho_j^{-1} (\gamma_{i1j} |\sin z_{j1}| + \gamma_{i2} |z_{j2}|)^2. \tag{50}$$

Step 3: We construct the K_∞ function ϕ_i in this step.

Define the Lyapunov function for the whole interconnected system as

$$W(z,\xi) = \sum_{i=1}^{n} W_i = \sum_{i=1}^{n} \{\phi_i(V_{i0}(z_i)) + (\xi_i - \xi_i^*)^2\}. \tag{51}$$

From (50), we obtain

$$\dot{W}(z,\xi) \le \sum_{i=1}^{n} \left\{-\frac{\partial \phi_i}{\partial V_{i0}} (z_i^{\mathrm{T}} Q_i z_i) - c_i \tilde{\xi}_i^2 + 2\tau_i |d_i|^2 + \sum_{j=1}^{n} \varrho_j^{-1} (\gamma_{i1j} |\sin z_{j1}| + \gamma_{i2} |z_{j2}|)^2\right\}. \tag{52}$$

Notice that

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \varrho_j^{-1} (\gamma_{i1j} |\sin z_{j1}| + \gamma_{i2} |z_{j2}|)^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \varrho_i^{-1} (\gamma_{j1i} |\sin z_{i1}| + \gamma_{j2} |z_{i2}|)^2. \tag{53}$$

Construct the derivative of ϕ_i as

$$\frac{\partial \phi_i}{\partial (V_{i0})} = l_i + \frac{1}{\lambda_{\min}(Q_i)} \sum_{j=1}^{N} \varrho_i^{-1} \max\{\gamma_{j1i}^2, \gamma_{j2}^2\} \tag{54}$$

which ensures that ϕ_i is a smooth K_∞ function, and

$$\sum_{i=1}^{n} \frac{\partial \phi_i}{\partial V_{i0}} (z_i^{\mathrm{T}} Q_i z_i) \ge \sum_{i=1}^{n} l_i \lambda_{\min}(Q_i) |z_i|^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \varrho_j^{-1} (\gamma_{i1j} |\sin z_{j1}| + \gamma_{i2} |z_{j2}|)^2. \tag{55}$$

So we have

$$\dot{W}(z,\xi) \le \sum_{i=1}^{n} \{-l_i \lambda_{\min}(Q_i) |z_i|^2 - c_i \tilde{\xi}_i^2 + 2\tau_i |d_i|^2\}. \tag{56}$$

Remark 2. The design procedure of excitation control involves an application of robust backstepping. A storage function W_i is built up and associated control v_{fi} is constructed iteratively. The dissipation inequality (50) is obtained for the ith isolated subsystem. Then we choose storage function W for the interconnected system (33) to be the sum of all subsystem storage functions W_i, $i=1,\dots,n$ (see (51)). To get net energy dissipation for the system, we dominate other subsystem interactions by the subsystem stability margins. This is done by carefully choosing the storage function ϕ_i of (38). For this case, the function ϕ_i turns out to be linear. This can be explained from (34) that the least conservative bound of $|\gamma_i(z)|$ in terms of $|z_j|$ is linear.

The properties of the above designed control law can be summarized in the following theorem:

Theorem 2. *The decentralized control* (49) *globally stabilizes the excitation control system* (33), *with all admissible uncertainties in the interconnections which satisfy* (34); *and the L_2 gain from the persistent disturbance to the power system frequency can be reduced to a given level.*

Proof. With the decentralized control law (49), in the absence of the disturbance (i.e. $d_i=0$), from (56) we obtain

$$\dot{W}(z,\xi) \le \sum_{i=1}^{n} \{-l_i\lambda_{\min}(Q_i)|z_i|^2 - c_i\tilde{\xi}_i^2\}$$

$$\triangleq -W_a(z,\xi) \le 0. \tag{57}$$

From (51), $W(z,\xi)$ is a continuously differentiable, positive definite and radially unbounded function and its derivative is negative definite. Hence we obtain the GUAS of the closed-loop system when $d_i=0$.

When d_i is not zero, from (56), we have

$$\dot{W}(z,\xi) \le \sum_{i=1}^{n} \{-l_i\lambda_{\min}(Q_i)|z_{i2}|^2 + 2\tau_i|d_i|^2\}. \tag{58}$$

Taking the integral of (58) along time t, and by (35), the L_2-gain from d to y of the closed-loop system is obtained as

$$\int_0^T |y|^2\,dt \le \mu\int_0^T |d|^2\,dt + v(z(0),\xi(0)), \tag{59}$$

where

$$\mu = \min_{1\le i\le n}\{2\tau_i\}\bigg/\left(2\pi\max_{1\le i\le n}\{l_i\lambda_{\min}(Q_i)\}\right), \tag{60}$$

$$v(z(0),\xi(0)) = W(z(0),\xi(0))\bigg/\left(2\pi\max_{1\le i\le n}\{l_i\lambda_{\min}(Q_i)\}\right). \tag{61}$$

It is easy to see that the L_2 gain μ can be designed arbitrarily by appropriately choosing parameters, which also provides the tradeoff with control effort.

Remark 3. Since the coordinate transformation between (1)–(3) and (33) is a diffeomorphism, the control task defined in Section 2 is achieved for the excitation control system (1)–(3). The excitation control input u_{fi} can be obtained by an inverse transform of (17) which gives

$$u_{fi} = \frac{1}{k_{ci}I_{qi}}\{v_{fi} + P_{\mathrm{m}i0} + (x_{di}-x'_{di})I_{qi}I_{di} + T'_{d0i}Q_{\mathrm{e}i}\omega_i\}. \tag{62}$$

Note that u_{fi} is well-defined since $I_{qi}=0$ is not in the normal working region for a generator.

Remark 4. In power systems, $P_{\mathrm{e}i}, Q_{\mathrm{e}i}$ and I_{fi} are readily measurable variables. From (8)–(11), we can obtain

$$P_{\mathrm{e}i} = E'_{qi}I_{qi}, \quad Q_{\mathrm{e}i} = -E'_{qi}I_{di}.$$

And from (6) and (12), we know that I_{di} and I_{qi} can be calculated using these available variables. Since ω_i is also measurable and the method for measuring the power angle δ_i can be found in de Mello (1994), the compensating law (62) is practically realizable using only local measurements.

4.2. Steam valve control

Defining the states as $[z_{i1}, z_{i2}, \xi_{i1}, \xi_{i2}] = [\delta_i, \omega_i, P_{\mathrm{m}i}, X_{\mathrm{e}i}]$, the state-space representation of the steam valve control is as follows:

$$\begin{aligned}
\dot{z}_{i1} &= z_{i2},\\
\dot{z}_{i2} &= -\frac{D_i}{2H_i}z_{i2} + \frac{\omega_0}{2H_i}[\xi_{i1} - g_i(z)] + d_i,\\
\dot{\xi}_{i1} &= -\frac{1}{T_{\mathrm{m}i}}\xi_{i1} + \frac{K_{\mathrm{m}i}}{T_{\mathrm{m}i}}\xi_{i2},\\
\dot{\xi}_{i2} &= -\frac{K_{\mathrm{e}i}}{T_{\mathrm{e}i}R_i\omega_0}z_{i2} - \frac{1}{T_{\mathrm{e}i}}\xi_{i2} + \frac{1}{T_{\mathrm{e}i}}u_i,
\end{aligned} \tag{63}$$

where d_i is the disturbance, u_i is the input, and

$$|g_i(z)| \le \sum_{j=1}^{n} g_{ij}|\sin z_{j1}|. \tag{64}$$

g_{ij} is as defined as in (26). The to-be-controlled output (power system frequency) is

$$y_i = \frac{1}{2\pi}z_{i2}. \tag{65}$$

The model (63) also exhibits a lower-triangular structure, the disturbance enters the system without a matching condition, and interconnections are bounded by nonlinear functions. The stepwise controller design is as follows.

Step 1: Considering the z_i-subsystem, seeing ξ_{i1} as the virtual control, we have

$$\dot{z}_i = A_i z_i + B_i[\xi_{i1} - g_i(z) + k_i d_i], \tag{66}$$

where

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{D_i}{2H_i} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ \dfrac{\omega_0}{2H_i} \end{bmatrix}, \quad k_i = \frac{2H_i}{\omega_0}. \tag{67}$$

Choose the Lyapunov function candidate:

$$V_i(z_i) = \phi_i(V_{i0}(z_i)), \tag{68}$$

where ϕ_i is a K_∞ function which will be defined in the last step of the design, and $V_{i0} = z_i^{\mathrm{T}} P_i z_i$ were $P_i > 0$ solves algebraic Riccati equation (39) with system matrix defined in (67).

Differentiating (68) along the solution of z_i-subsystem, we have

$$\dot{V}_i = \frac{\partial \phi_i}{\partial V_{i0}} 2 z_i^{\mathrm{T}} P_i \{A_i z_i + B_i[\xi_{i1} - g_i(z) + k_i d_i]\}. \tag{69}$$

Notice that

$$\frac{\partial \phi_i}{\partial V_{i0}} 2 z_i^{\mathrm{T}} P_i B_i k_i d_i \le \frac{1}{\tau_i}\left(\frac{\partial \phi_i}{\partial V_{i0}}\right)^2 (z_i^{\mathrm{T}} P_i B_i k_i)^2 + \tau_i |d_i|^2, \tag{70}$$

$$\frac{\partial \phi_i}{\partial V_{i0}} 2 z_i^{\mathrm{T}} P_i B_i(-g_i(z)) \le \left(\frac{\partial \phi_i}{\partial V_{i0}}\right)^2 (z_i^{\mathrm{T}} P_i B_i)^2 \sum_{j=1}^{n} \varrho_j + \sum_{j=1}^{n} \varrho_j^{-1} g_{ij}^2 (\sin z_{j1})^2. \tag{71}$$

Choose the virtual control $\xi_{i1} = \xi_{i1}^*$ as

$$\xi_{i1}^* = -\left\{ \varepsilon_i B_i^{\mathrm{T}} P_i z_i + 0.5 \frac{1}{\tau_i} \frac{\partial \phi_i}{\partial V_{i0}} (z_i^{\mathrm{T}} P_i B_i) k_i^2 + 0.5\left(\frac{\partial \phi_i}{\partial V_{i0}}\right)(z_i^{\mathrm{T}} P_i B_i) \sum_{j=1}^{n} \varrho_j \right\}, \tag{72}$$

we have the dissipation inequality

$$\dot{V}_i \le -\frac{\partial \phi_i}{\partial V_{i0}} (z_i^{\mathrm{T}} Q_i z_i) + \tau_i |d_i|^2 + \sum_{j=1}^{n} \varrho_j^{-1} g_{ij}^2 \sin^2 z_{j1}. \tag{73}$$

Step 2: Augment the z_i-subsystem with the ξ_{i1}-subsystem, and choose a Lyapunov function as

$$W_{i1}(z_i, \xi_{i1}) = V_i(z_i) + (\xi_{i1} - \xi_{i1}^*)^2. \tag{74}$$

Note that

$$\dot{\xi}_{i1}^* = \frac{\partial \xi_{i1}^*}{\partial z_i}[A_i z_i + B_i \xi_{i1} - B_i g_i(z) + B_i k_i d_i] \triangleq \vartheta_{i1}(z_i, \xi_{i1}) - \sigma_i(z_i) g_i(z) + \sigma_i(z_i) k_i d_i. \tag{75}$$

Differentiating along the solutions of the (z_i, ξ_{i1})-subsystem yields

$$\dot{W}_{i1} \le \left\{ -\frac{\partial \phi_i}{\partial V_{i0}} (z_i^{\mathrm{T}} Q_i z_i) + \tau_i |d_i|^2 + \sum_{j=1}^{n} \varrho_j^{-1} g_{ij}^2 (\sin z_{j1})^2 \right\} + 2\tilde{\xi}_{i1} \left\{ \frac{\partial \phi_i}{\partial V_{i0}} z_i^{\mathrm{T}} P_i B_i - \frac{1}{T_{\mathrm{m}i}} \xi_{i1} + \frac{K_{\mathrm{m}i}}{T_{\mathrm{m}i}} \xi_{i2} - \vartheta_{i1}(z_i, \xi_{i1}) + \sigma_i(z_i) g_i(z) - \sigma_{i1}(z_i) k_i d_i \right\}, \tag{76}$$

where $\tilde{\xi}_{i1} = \xi_{i1} - \xi_{i1}^*$.

Notice that

$$2\tilde{\xi}_{i1} \sigma_{i1}(z_i) g_i(z) \le (\tilde{\xi}_{i1})^2 \sigma_{i1}^2(z_i) \sum_{j=1}^{n} \varrho_j + \sum_{j=1}^{n} \varrho_j^{-1} g_{ij}^2 (\sin z_{j1})^2, \tag{77}$$

$$2\tilde{\xi}_{i1}[-\sigma_{i1}(z_i) k_i d_i] \le \frac{1}{\tau_i} \tilde{\xi}_{i1}^2 \sigma_{i1}^2(z_i) k_i^2 + \tau_i |d_i|^2. \tag{78}$$

Choose the virtual control $\xi_{i2} = \xi_{i2}^*$ as

$$\xi_{i2}^* = -\frac{T_{\mathrm{m}i}}{K_{\mathrm{m}i}} \left\{ c_{i1} \tilde{\xi}_{i1} + \frac{\partial \phi_i}{\partial V_{i0}} z_i^{\mathrm{T}} P_i B_i - \frac{1}{T_{\mathrm{m}i}} \xi_{i1} - \vartheta_{i1}(z_i, \xi_{i1}) + 0.5(\tilde{\xi}_{i1}) \sigma_{i1}^2(z_i) \sum_{j=1}^{n} \varrho_j + 0.5 \frac{1}{\tau_i} \tilde{\xi}_{i1} \sigma_{i1}^2(z_i) k_i^2 \right\}, \tag{79}$$

then we obtain the dissipation inequality

$$\dot{W}_{i1} \le -\frac{\partial \phi_i}{\partial V_{i0}} (z_i^{\mathrm{T}} Q_i z_i) - c_{i1} \tilde{\xi}_{i1}^2 + 2\tau_i |d_i|^2 + \sum_{j=1}^{n} 2\varrho_j^{-1} g_{ij}^2 \sin^2 z_{j1}. \tag{80}$$

Step 3: Augment the (z_i, ξ_{i1})-subsystem with ξ_{i2}-subsystem, and choose a Lyapunov function as

$$W_{i2}(z_i, \xi_{i1}) = W_{i1}(z_i, \xi_{i1}) + (\xi_{i2} - \xi_{i2}^*)^2. \tag{81}$$

Denote that

$$\dot{\xi}_{i2}^* = \frac{\partial \xi_{i2}^*}{\partial z_i}[A_i z_i + B_i \xi_{i1} - B_i g_i(z) + B_i k_i d_i] + \frac{\partial \xi_{i2}^*}{\partial \xi_{i1}}\left(-\frac{1}{T_{\mathrm{m}i}} \xi_{i1} + \frac{K_{\mathrm{m}i}}{T_{\mathrm{m}i}} \xi_{i2}\right) \triangleq \vartheta_{i2}(z_i, \xi_{i1}, \xi_{i2}) - \sigma_{i2}(z_i, \xi_{i1}) g_i(z) + \sigma_{i2}(z_i, \xi_{i1}) k_i d_i. \tag{82}$$

Following the similar procedure as stated in Step 2 we can obtain

$$\dot{W}_{i2} \leq -\frac{\partial \phi_i}{\partial V_{i0}}(z_i^{\mathrm{T}} Q_i z_i) - c_{i1}\tilde{\xi}_{i1}^2 - c_{i2}\tilde{\xi}_{i2}^2 + 3\tau_i |d_i|^2 + \sum_{j=1}^{n} 3\varrho_j^{-1} g_{ij}^2 \sin^2 z_{j1}, \tag{83}$$

and the true control is chosen as

$$u_i = -T_{ei}\Bigg\{c_{i2}\tilde{\xi}_{i2} + 0.5\frac{\partial W_{i1}}{\partial \xi_{i1}}\frac{K_{mi}}{T_{mi}} - \frac{K_{ei}}{T_{ei}R_i\omega_0}z_{i2} - \frac{1}{T_{ei}}\xi_{i2} - \vartheta_{i2}(z_i, \xi_{i1}, \xi_{i2}) + 0.5(\tilde{\xi}_{i2})\sigma_{i2}^2 \sum_{j=1}^{n} \varrho_j + 0.5\frac{1}{\tau_i}\tilde{\xi}_{i2}\sigma_{i2}^2 k_i^2\Bigg\}. \tag{84}$$

Step 4: We construct the K_∞ function ϕ_i in this step. Define a Lyapunov function for the whole interconnected system as

$$W(z,\xi) = \sum_{i=1}^{n} W_{i2} = \sum_{i=1}^{n} \{\phi_i(V_{i0}(z_i)) + (\xi_{i1} - \xi_{i1}^*)^2 + (\xi_{i2} - \xi_{i2}^*)^2\}. \tag{85}$$

From (83) we obtain

$$\dot{W}(z,\xi) \leq \sum_{i=1}^{n}\Bigg\{-\frac{\partial \phi_i}{\partial V_{i0}}(z_i^{\mathrm{T}} Q_i z_i) - c_{i1}\tilde{\xi}_{i1}^2 - c_{i2}\tilde{\xi}_{i2}^2 + 3\tau_i|d_i|^2 + \sum_{j=1}^{n} 3\varrho_j^{-1} g_{ij}^2 \sin^2 z_{j1}\Bigg\}. \tag{86}$$

We bound the $\sin(\cdot)$ function as

$$\sin^2|z_i| \leq |z_i|^2 \varphi_i(|z_i|),$$

where φ_i is a decreasing C function:

$$\varphi_i(|z_i|) = \begin{cases} 1 & \text{when } |z_i| = 0, \\ \frac{\sin^2|z_i|}{|z_i|^2} & \text{when } |z_i| \leq \frac{\pi}{2}, \\ \frac{1}{|z_i|^2} & \text{when } |z_i| > \frac{\pi}{2}, \end{cases} \tag{87}$$

Since

$$|z_i| \geq \sqrt{\frac{V_{i0}(z_i)}{\lambda_{\max}(P_i)}} \triangleq \eta_i(V_{i0}), \tag{88}$$

we obtain

$$\varphi_i(|z_i|) \leq \varphi_i \circ \eta_i(V_{i0}). \tag{89}$$

Construct the derivative of ϕ_i as

$$\frac{\partial \phi_i}{\partial (V_{i0})} = l_i + \frac{1}{\lambda_{\min}(Q_i)} \sum_{j=1}^{N} 3\varrho_i^{-1} g_{ji}^2 \varphi_i \circ \eta_i(V_{i0}), \tag{90}$$

then we have

$$\sum_{i=1}^{n}\left\{\frac{\partial \phi_i}{\partial V_{i0}}(z_i^{\mathrm{T}} Q_i z_i)\right\} \geq \sum_{i=1}^{n}\left\{l_i \lambda_{\min}(Q_i)|z_i|^2 + \sum_{j=1}^{n} 3\varrho_j^{-1} g_{ij}^2 \sin^2 z_{j1}\right\}, \tag{91}$$

which is followed by

$$\dot{W}(z,\xi) \leq \sum_{i=1}^{n}\{-l_i\lambda_{\min}(Q_i)|z_i|^2 - c_{i1}\tilde{\xi}_{i1}^2 - c_{i2}\tilde{\xi}_{i2}^2 + 3\tau_i|d_i|^2\}. \tag{92}$$

Remark 5. The function ϕ_i in this case is nonlinear since the interconnection (64) is bounded by a $\sin(\cdot)$ function of $|z_j|$.

Remark 6. To maintain less conservatism of controller gains, we choose $\varphi_i(\cdot)$ as a decreasing function instead of nondecreasing one as proposed in Guo et al. (1998); and correspondingly we use the upper bound of V_{i0} as in (88). Note that $V_{i0} = 0$ is not in the normal working region of power systems which ensures that the derivatives of $\eta_i(V_{i0})$ are well-defined. Since φ_i is a C function, u_i has a non-continuous point at $\eta_i(V_{i0}) = \pi/2$.

The following theorem summarizes the property of the above-designed controller:

Theorem 3. *The decentralized control* (84) *globally stabilizes the steam valve control system* (63), *with all admissible uncertainties in the interconnection which satisfies* (64); *and the L_2 gain from the persistent disturbance to the power system frequency can be reduced to a given level.*

The proof of Theorem 3 can be obtained similarly as the proof of Theorem 2.

Remark 7. The decentralized control law (84) employs feedback of state variables $\delta_i, \omega_i, P_{mi}$ and X_{ei}. Since ω_i, P_{mi}, X_{ei} are directly measurable variables, and the power angle δ_i can be found using the method described in de Mello (1994), (84) is practically realizable.

Remark 8. In this section we designed decentralized controllers for the power excitation and steam valve control loops respectively. Instead of bounding the interconnections with first-order polynomials (linear), we bound them by nonlinear functions, which provides less conservatism of the controller gains. The stepwise design procedure involves choosing nonlinear storage functions which achieve net energy dissipation along the closed-loop system trajectories. There are always different kinds of persistent disturbances entering power systems which cause unexpected effects on the power frequency. Besides giving stability, the designed controller also attenuates the persistent disturbance, in the sense that the L_2-gain from the disturbance to the power frequency is reduced to a certain level.

5. Simulation results

The decentralized controller designed above was simulated on a two-generator infinite bus power system which is shown in Fig. 1. The generator and the transmission line parameters are listed in Table 1. The fault sequence is as stated in Section 2, where it is chosen that $t_0 = 0.1$ s, $t_1 = 0.25$ s, $t_2 = 1$ s.

5.1. Excitation control performance

According to Table 1, we have the system data used in Section 4.1 as

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.625 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -39.27 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 0 & -0.2941 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -30.8 \end{bmatrix},$$

$$\gamma_{111} = \gamma_{112} = 0.7817, \quad \gamma_{12} = 1.4,$$

$$\gamma_{211} = \gamma_{212} = 0.9662, \quad \gamma_{22} = 1.5. \tag{93}$$

Table 1
System parameters

	Generator #1	Generator #2
x_d (p.u.)	1.863	2.36
x'_d (p.u.)	0.257	0.319
x_T (p.u.)	0.129	0.11
x_{ad} (p.u.)	1.712	1.712
T'_{d0} (p.u.)	6.9	7.96
H (s)	4	5.1
D (p.u.)	5	3
T_m (s)	0.35	0.35
T_e (s)	0.1	0.1
R	0.05	0.05
K_m	1.0	1.0
K_e	1.0	1.0
k_c	1	1
x_{12} (p.u.)	0.55	
x_{13} (p.u.)	0.53	
x_{23} (p.u.)	0.6	
ω_0 (rad/s)	314.159	

In the controller design, we choose $Q_1 = Q_2 = 0.8I$, $\varepsilon_1 = \varepsilon_2 = 14$ to get

$$P_1 = \begin{bmatrix} 0.8043 & 0.0043 \\ 0.0043 & 0.0043 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.8055 & 0.0055 \\ 0.0055 & 0.0055 \end{bmatrix},$$

and other design parameters are chosen as

$$\varrho_1 = \varrho_2 = 0.1, \quad \tau_1 = \tau_2 = 0.5,$$

$$l_1 = l_2 = 2, \quad c_1 = c_2 = 0.1.$$

Using the above parameters, the decentralized controllers we derived are

$$v_{f1} = 19.68(\delta_1 - \delta_{10}) + 20.60\omega_1 - 93.81(P_{e1} - P_{m10}),$$
$$v_{f2} = 19.69(\delta_2 - \delta_{20}) + 21.45\omega_2 - 73.95(P_{e2} - P_{m20}), \tag{94}$$

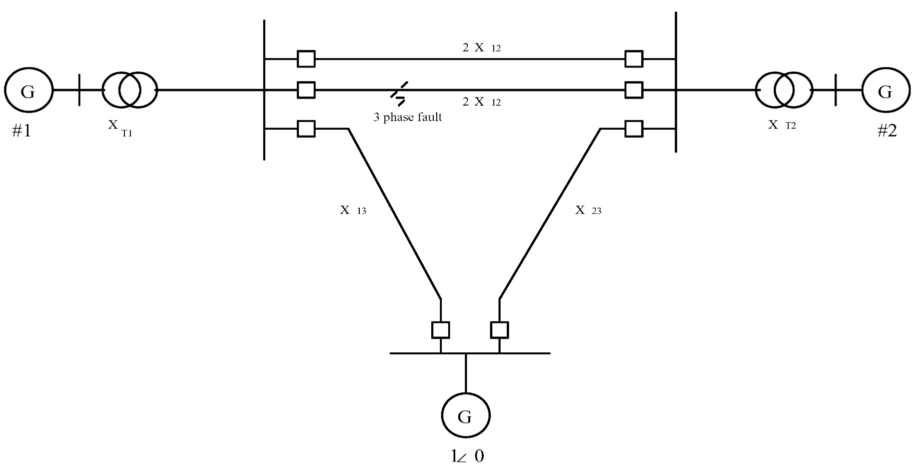

Fig. 1. A two-machine infinite bus power system.

and the original excitation control for the two-generator infinite bus power system are

$$u_{f1} = \frac{1}{k_{c1} I_{q1}} \{ v_{f1} + P_{m10} - (x_{d1} - x'_{d1}) I_{q1} I_{d1} + T'_{d01} Q_{e1} \omega_1 \},$$

$$u_{f2} = \frac{1}{k_{c2} I_{q2}} \{ v_{f2} + P_{m20} - (x_{d2} - x'_{d2}) I_{q2} I_{d2} + T'_{d02} Q_{e2} \omega_2 \}. \quad (95)$$

The deduced L_2 gain from the persistent disturbance to the system frequency is obtained by (60) as $\mu_1 = \mu_2 = 0.63/(2\pi) = 0.1$.

In the simulation, saturation of synchronous machines is also considered, so (3) becomes

$$\dot{E}'_{qi} = \frac{1}{T'_{doi}} [E_{fi} - E_{qi} - (1 - k_{fi}) E'_{qi}], \quad (96)$$

where

$$k_{fi} = 1 + \frac{b_i}{a_i} (E'_{qi})^{(n_i - 1)},$$

with

$$a_1 = 0.95, \quad b_1 = 0.051, \quad n_1 = 8.727,$$

$$a_2 = 0.935, \quad b_2 = 0.064, \quad n_2 = 10.878. \quad (97)$$

The excitation control input limitations are

$$-3 \le E_{fi} = k_{ci} u_{fi} \le 6, \quad i = 1,2.$$

We demonstrate the performance of the proposed excitation controller in the following three cases of different sets of operation points, fault locations and network parameters. The symmetrical three-phase short circuit fault occurs on one of the transmission lines between the generator #1 and the generator #2. The fault location is indexed by a constant λ which is the fraction of the line to the left of the fault. The persistent disturbance of 30% is used in the simulation, i.e. $d_1 = d_2 = 0.3$ p.u.

Case 1: The operating points are:

$$\delta_{10} = 60.78^\circ, \quad P_{m10} = 1.10 \text{ p.u.}, \quad V_{t10} = 1.0 \text{ p.u.},$$

$$\delta_{20} = 60.64^\circ, \quad P_{m20} = 1.01 \text{ p.u.}, \quad V_{t20} = 1.0 \text{ p.u.} \quad (98)$$

The fault location is $\lambda = 0.2$. The corresponding closed-loop system responses are shown in Fig. 2.

Case 2: The operating points are:

$$\delta_{10} = 30.5^\circ, \quad P_{m10} = 0.57 \text{ p.u.}, \quad V_{t10} = 1.01 \text{ p.u.},$$

$$\delta_{20} = 32.5^\circ, \quad P_{m20} = 0.56 \text{ p.u.}, \quad V_{t20} = 1.00 \text{ p.u.} \quad (99)$$

The fault location is $\lambda = 0.05$. The corresponding closed-loop system responses are shown in Fig. 3.

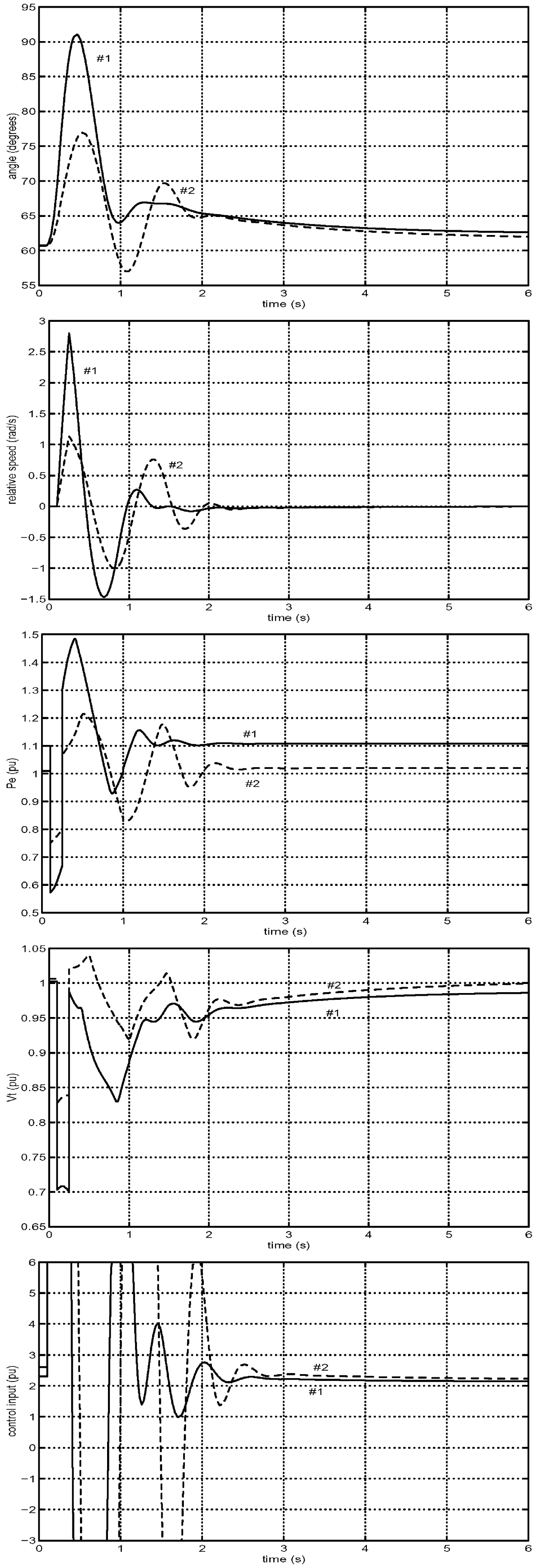

Fig. 2. Responses of excitation control: Case 1.

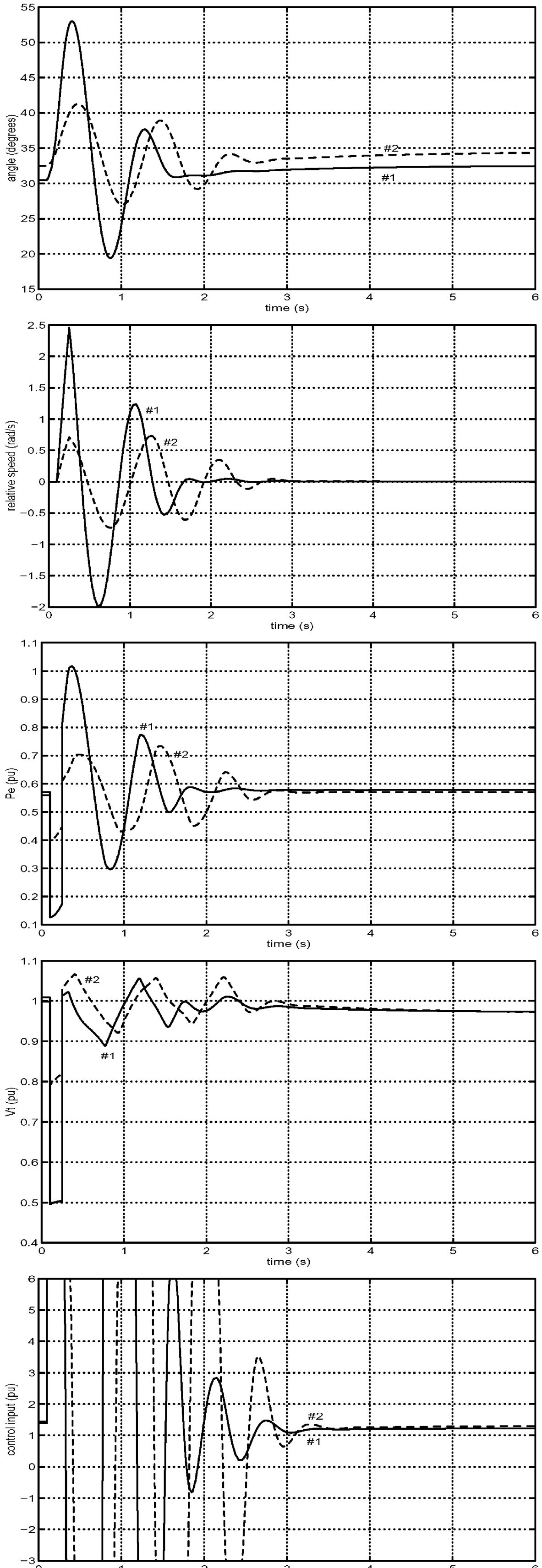

Fig. 3. Responses of excitation control: Case 2.

Case 3: The transmission line parameters are changed to

$$x_{12} = x_{13} = x_{23} = 0.7.$$

The operating points are

$$\delta_{10} = 64.08^\circ, \quad P_{m10} = 0.95 \text{ p.u.}, \quad V_{t10} = 1.0 \text{ p.u.},$$

$$\delta_{20} = 65.33^\circ, \quad P_{m20} = 0.95 \text{ p.u.}, \quad V_{t20} = 1.0 \text{ p.u.} \quad (100)$$

The fault location is $\lambda = 0.5$. The corresponding closed-loop system responses are shown in Fig. 4.

From the simulation results we can see that the proposed excitation control enhances the system transient stability and dampens out the power angle oscillations. Also the effect of a persistent disturbance on the system frequency is reduced to be quite small.

5.2. Steam valve control performance

According to Table 1, we have the system data used in Section 4.2 as

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.625 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 39.27 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 0 & -0.2941 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 30.8 \end{bmatrix},$$

$$\gamma_{11} = \gamma_{12} = 1.4, \quad \gamma_{21} = \gamma_{22} = 1.5. \quad (101)$$

In the controller design, we choose $Q_1 = Q_2 = 0.8I$, $\varepsilon_1 = \varepsilon_2 = 20$ to get

$$P_1 = \begin{bmatrix} 0.8036 & 0.0036 \\ 0.0036 & 0.0036 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.8046 & 0.0046 \\ 0.0046 & 0.0046 \end{bmatrix},$$

and other design parameters are chosen as

$$\varrho_1 = \varrho_2 = 0.005, \quad \tau_1 = \tau_2 = 0.5,$$

$$l_1 = l_2 = 2, \quad c_1 = c_2 = 0.01.$$

The deduced L_2 gain from the persistent disturbance to the system frequency is $\mu_1 = \mu_2 = 0.94/(2\pi) = 0.15$. The persistent disturbance $d_1 = d_2 = 0.3$ p.u. is used in the simulation.

Case 4: The operating points are same as Case 2, while the fault location is $\lambda = 0.05$. The corresponding closed-loop system responses are shown in Fig. 5.

Case 5: The transmission line parameters and the operating points are the same as in Case 3, while the fault location is $\lambda = 0.01$. The corresponding closed-loop system responses are shown in Fig. 6.

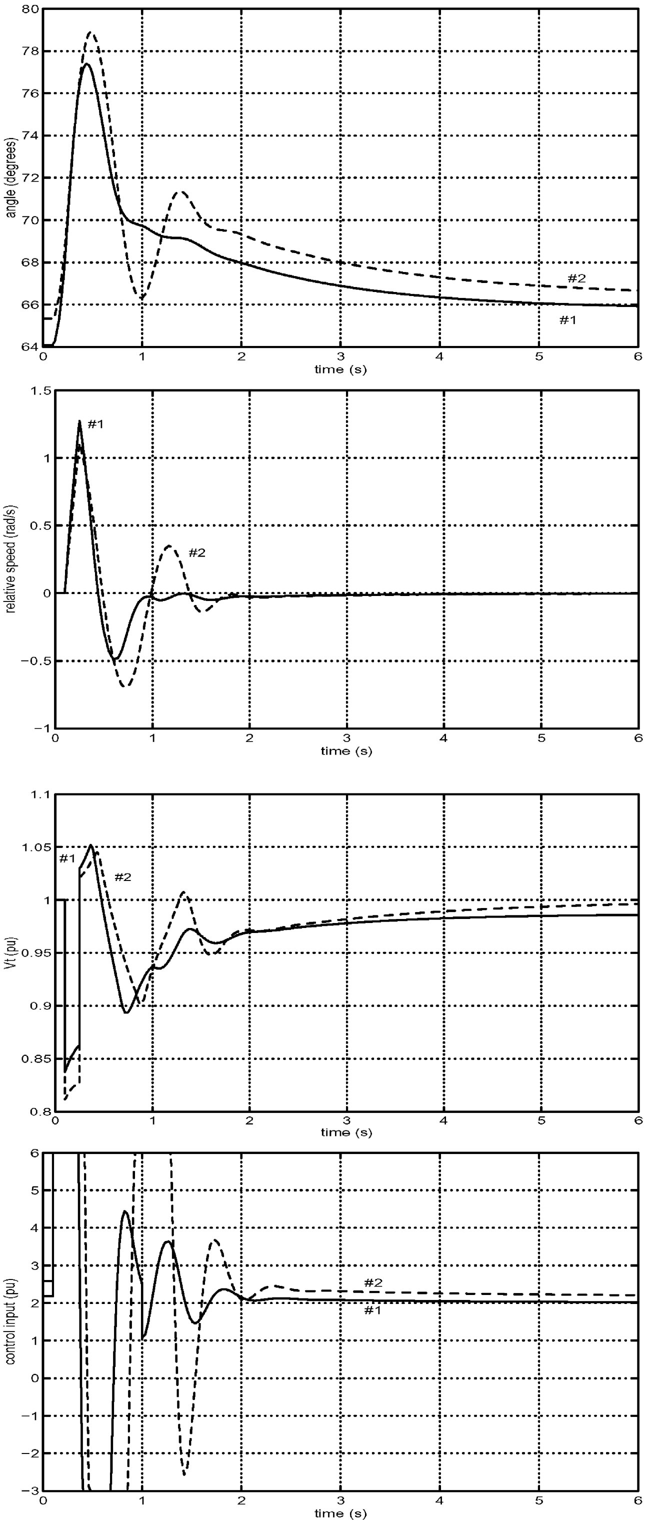

Fig. 4. Responses of excitation control: Case 3.

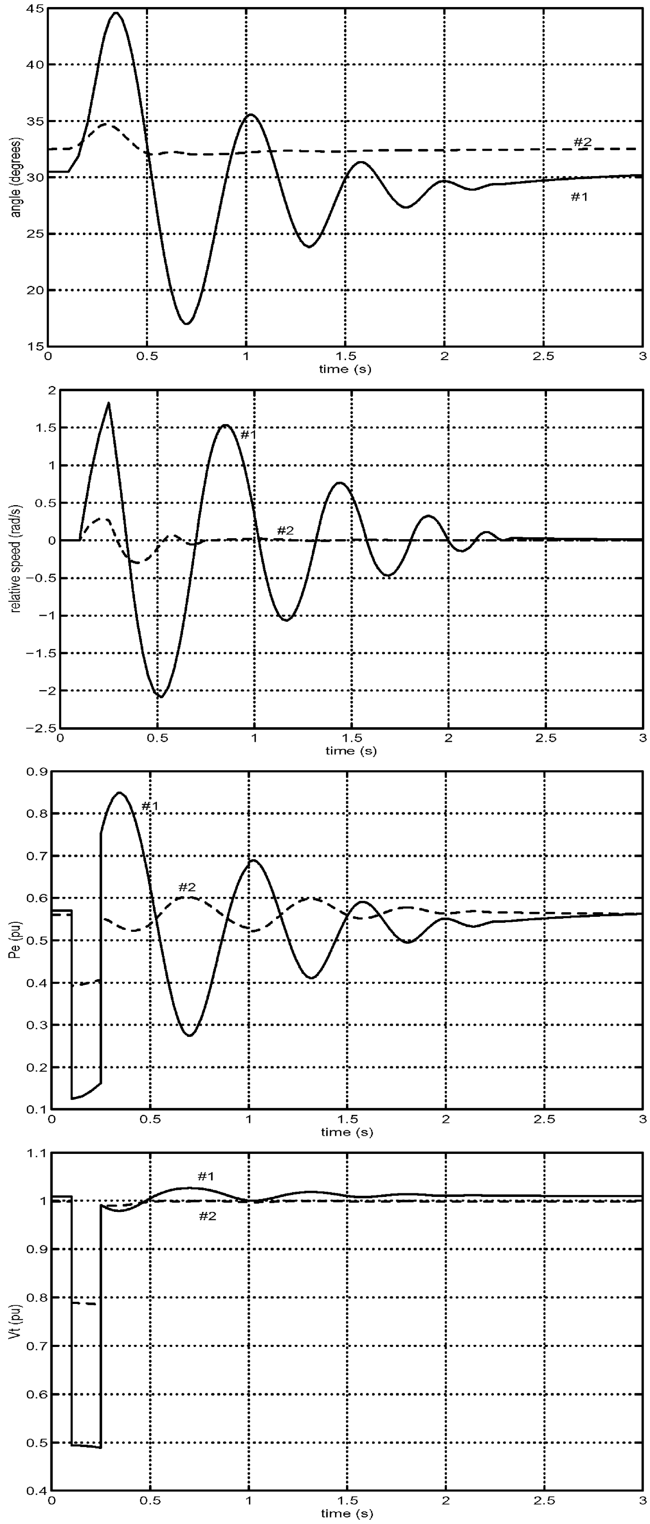

Fig. 5. Responses of steam valve control: Case 4.

The simulations testify the enhancement of transient stability of the proposed steam valve controller in face of different conditions of operation points, fault locations and network parameters.

6. Conclusions

In this paper we apply the recently developed nonlinear decentralized control scheme (Guo et al., 1998) to

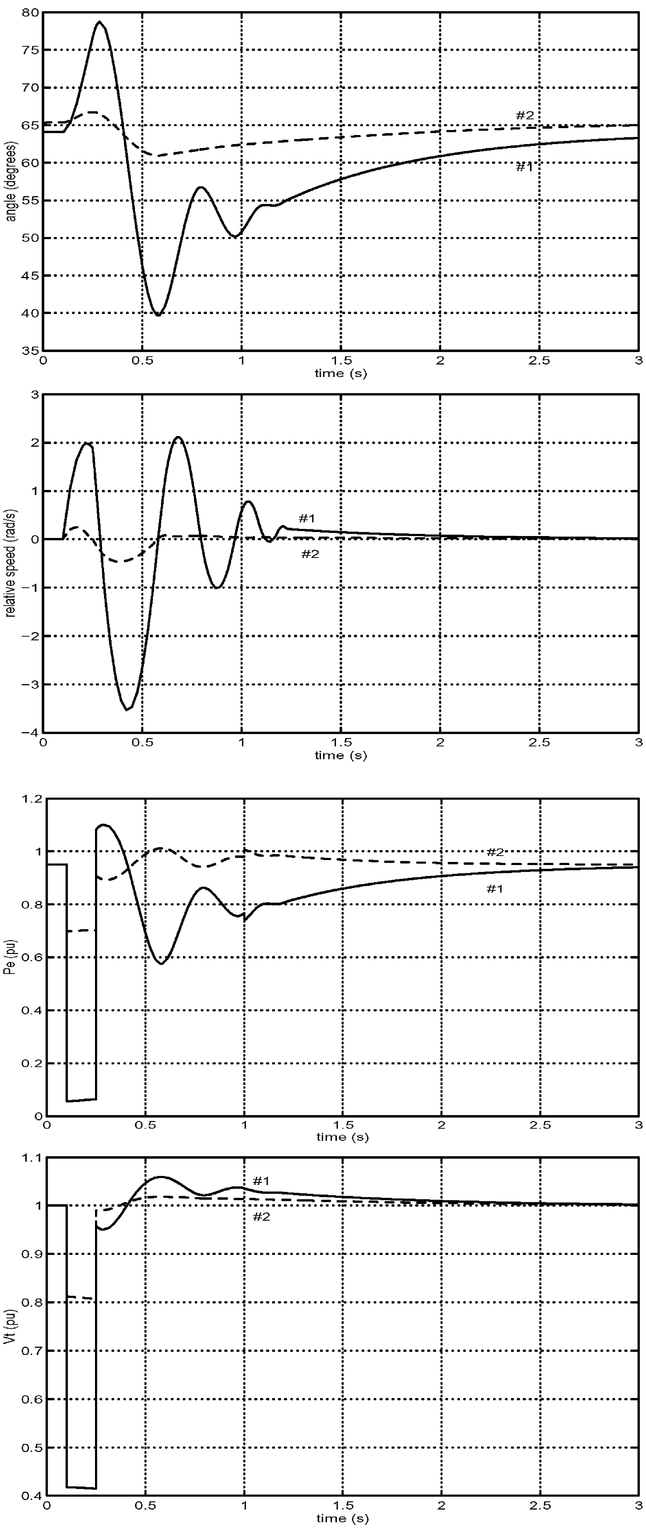

Fig. 6 . Responses of steam valve control: Case 5.

large-scale power systems. Nonlinear decentralized robust controllers are designed explicitly for the excitation model and turbine-governor model, where the interconnections among generators are bounded by nonlinear functions. Nonlinear storage functions are chosen to maintain the net energy dissipation of the closed-loop systems. The proposed controllers guarantee the overall stability of the large-scale power systems and are robust with regard to uncertain network parameters. The effectiveness of the controller is demonstrated on a two-generator infinite bus example system. The simulations show that the transient stability is greatly enhanced regardless of different condition of operation points, fault locations and network parameters, and the effect of persistent disturbance on system frequency is effectively attenuated.

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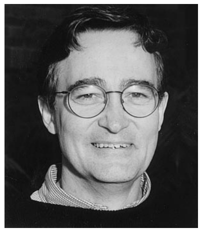

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