this new technique it is possible to create current sources and current mirrors with very high output resistance together with maximum output voltage swing.

Acknowledgment: The authors would like to thank K. Martin at the University of Toronto for useful discussions about the high-swing cascode current mirror circuit. This work was supported by NSERC and by the Micronet.

13th December 1991

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BIT ERROR PROBABILITIES OF NCFSK AND DPSK SIGNALS IN MICROCELLULAR MOBILE RADIO SYSTEMS

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Indexing terms: Mobile radio systems, Digital communication systems

Microcellular mobile radio systems are studied where the desired signal has Rician statistics and the cochannel interferers experience Rayleigh fading. The probability density function of the signal-to-interference ratio is derived and is used to obtain the bit error probabilities of noncoherent frequency shift-keying and differential phase shift-keying signals.

Introduction: To extend the present medium/large cell mobile radio systems [1] towards ubiquitous communications, the microcellular approach looks promising [2]. A major problem in cellular radio systems, regardless of having micro or medium/large cell architecture, is the presence of cochannel interference because of frequency reuse. Extensive studies on the effects of cochannel interference in medium/large cell systems have been made [3–5]. Models from Rayleigh fading [3], lognormal shadowing [4] to superimposed Rayleigh fading and lognormal shadowing [5] have been used for signal environments. There is a common assumption in all these studies, however, that all signals received, desired and undesired, have the same statistical characteristics. For example, in Reference 3, both the desired signal and the undesired cochannel interferers are assumed to be subject to Rayleigh fading. To evaluate the performance of microcellular mobile radio systems, a microcell interference model, in which different fading statistics are used to characterise the desired and undesired signals, is introduced in Reference 6. The desired signal is assumed to have Rician statistics implying that a dominant multipath reflection exists in within-cell transmission [7]. The interference signals from cochannel cells are assumed to be subject to Rayleigh fading because of the absence of a line-of-sight propagation. This is the Rician/Rayleigh microcell interference model [6]. This model has been used to study the outage probability and spectral efficiency of microcellular mobile radio systems [6, 8].

This Letter further investigates the Rician/Rayleigh interference model by deriving the probability density function (PDF) of the power ratio between the desired and interfering signals. The PDF is then used to obtain the bit error probabilities of noncoherent frequency shift-keying (NCFSK) and differential phase shift-keying (DPSK) signals in microcellular systems with cochannel interference. The bit error probability issue in microcellular radio systems has been studied in Reference 9. However, in Reference 9, the cochannel interferers were approximated by independent Gaussian random variables. The microcell cochannel interference is thus not characterised using its fading statistics. In this Letter, we will derive the bit error probabilities in microcellular radio systems by considering both the Rician fading characteristics of the desired signal and the Rayleigh fading characteristics of the interferers. In situations where both the desired signal and the interferers have Rayleigh fading characteristics, the results apply to medium/large cell systems.

Microcell interference model: In the Rician/Rayleigh microcell interference model [6] the envelope of the desired signal has Rician statistics. The PDF of the corresponding signal power x is

$$p_x(x) = \frac{1}{X}\exp\left(-\frac{2x+s^2}{2X}\right)I_0\left[\frac{\sqrt{(2x)}s}{X}\right] \qquad (1)$$

where $I_0(\ )$ is the modified Bessel function of the first kind and order zero. The mean signal power is $X + s^2/2$, where X is contributed by the diffused signal component and $s^2/2$ by the direct line-of-sight signal component.

Each cochannel interferer is subject to Rayleigh fading and its power is exponentially distributed. Assuming that all interferers are independent and have the same mean power, Y_1 the PDF of the total interference power y of I interferers is obtained using an I-fold convolution of independent and identical exponential PDF. This results in a Gamma PDF of the form [6]

$$p_y(y) = \frac{y^{I-1}}{Y_1^I(I-1)!}\exp\left(-\frac{y}{Y_1}\right) \qquad (2)$$

Defining the signal-to-interference power ratio as $r = x/y$, we derive the PDF of r as follows:

$$\begin{aligned} p_r(r) &= \int_0^\infty y p_x(ry)p_y(y)\,dy \\ &= \frac{I}{b_1}\left(\frac{b_1}{r+b_1}\right)^{I+1}\exp\left(-\frac{ab_1}{r+b_1}\right) \\ &\quad \times \sum_{i=0}^{I} C_I^{I-i}\frac{1}{i!}\left(\frac{ar}{r+b_1}\right)^i \end{aligned} \qquad (3)$$

where $b_1 = X/Y_1$, $a = s^2/(2X)$ and C_I^{I-i} is a binomial coefficient, i.e. $I!/[i!(I-i)!]$.

The PDF given in eqn. 3 is valid for the case where all Rayleigh interferers have the same mean power. Where some interferers have different mean powers, different expressions for the PDF are needed. For the special case where all interferers have mutually different mean powers (i.e. the ith interferer ($1 \le i \le I$) has mean power Y_i and $Y_i \neq Y_j$ when $i \neq j$), the PDF of the total interference power is found to be [6]

$$p_y(y) = \sum_{i=1}^{I} Y_i^{I-2}\exp\left(-\frac{y}{Y_i}\right)\prod_{j=1,j\neq i}^{I}\frac{1}{Y_i - Y_j} \qquad (4)$$

and the PDF of the signal-to-interference power ratio is obtained as

$$\begin{aligned} p_r(r) &= \sum_{i=1}^{I}\frac{b_i}{(r+b_i)^2}\left(1+\frac{ar}{r+b_i}\right) \\ &\quad \times \exp\left(-\frac{ab_i}{r+b_i}\right)\prod_{j=1,j\neq i}^{I}\frac{b_j}{b_j-b_i} \end{aligned} \qquad (5)$$

where $b_i = X/Y_i$.

Bit error probabilities: There are classical results for the optimum detection of nonfading binary signals when the channel is perturbed by Gaussian noise. The 'static' probability of bit error (in a nonfading environment), P_e is given by

$$P_e(\mathrm{SNR}) = \tfrac{1}{2} \exp(-\beta \mathrm{SNR}) \tag{6}$$

where SNR is the predetection signal-to-noise power ratio and the coefficient β indicates the modulation type, $\beta = 1/2$ for NCFSK and $\beta = 1$ for DPSK. When the channel is subject to fading, SNR is a random variable and the 'dynamic' bit error probability is derived by averaging eqn. 6 over all possible values of SNR. This method can also be used to derive the bit error probability when the signal is subject to interfering in a fading environment [10]. Following this approach, the bit error probability in a microcellular system, assuming that all interferers have the same mean power, is obtained using eqns. 3 and 6,

$$\begin{aligned} P_e &= \frac{1}{2} \int_0^\infty \exp(-\beta r) p_r(r)\, dr \\ &= \frac{I}{2b_1} \int_0^\infty \left(\frac{b_1}{r+b_1}\right)^{I+1} \exp\left[-\left(\beta r + \frac{ab_1}{r+b_1}\right)\right] \\ &\quad \times \sum_{i=0}^{I} C_I^{I-i} \frac{1}{i!} \left(\frac{ar}{r+b_1}\right)^i dr \end{aligned} \tag{7}$$

Eqn. 7 can be numerically evaluated. For $a = 0$, i.e. the desired signal and cochannel interferers are subject to Rayleigh fading (a medium/large cell scenario), eqn. 7 is simplified as

$$\begin{aligned} P_e &= \frac{1}{2(I-1)!} \sum_{k=1}^{I} (k-1)!(-b_1\beta)^{I-k} \\ &\quad - \frac{(-b_1\beta)^I}{2(I-1)!} \exp(b_1\beta) Ei(-b_1\beta) \end{aligned} \tag{8}$$

where $Ei(\ )$ is an exponential integral function defined as

$$Ei(z) = -\int_{-z}^{\infty} \frac{\exp(-t)}{t\, dt} \tag{9}$$

for $z < 0$.

Eqns. 7 and 8 give the bit error probabilities of microcellular and medium/large cell radio systems when all interferers have the same mean power. If all interferers have mutually different mean powers, the bit error probabilities are found to be

$$\begin{aligned} P_e &= \frac{1}{2} \sum_{i=1}^{I} \int_0^\infty \frac{b_i}{(r+b_i)^2} \left(1 + \frac{ar}{r+b_i}\right) \\ &\quad \times \exp\left[-\left(\beta r + \frac{ab_i}{r+b_i}\right)\right] dr \prod_{j=1, j\neq i}^{I} \frac{b_j}{b_j - b_i} \end{aligned} \tag{10}$$

for microcellular radio systems and in a special case with $a = 0$,

$$\begin{aligned} P_e &= \frac{1}{2} \sum_{i=1}^{I} [b_i\beta \exp(b_i\beta) Ei(-b_i\beta) + 1] \\ &\quad \times \prod_{j=1, j\neq i}^{I} \frac{b_j}{b_j - b_i} \end{aligned} \tag{11}$$

Numerical results and conclusions: Fig. 1 shows the bit error probabilities of both NCFSK and DPSK signals in cellular radio systems with six cochannel interferers. As expected, the DPSK signal performs better than the NCFSK signal. In the same figure, performance comparison of microcellular and medium/large cell systems is also made. The case with $a = 5$ or 10 describes a microcellular environment and the one with $a = 0$ implies a medium/large cell system. It is interesting to note that lower error probabilities are achieved in microcellular radio systems.

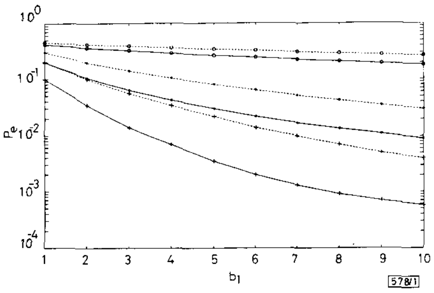

Fig. 1 *Bit error probability, comparison of microcellular and medium/large cell systems*

○ $a = 0$
* $a = 5$
\+ $a = 10$
- - - - NCFSK
——— DPSK
$I = 6$ (equal mean interference power)

A microcellular mobile radio system has been studied in this Letter. The PDF of signal-to-interference ratio is derived and is used to obtain the bit error probabilities of NCFSK and DPSK signals. The performance of medium/large cell systems is also evaluated by considering a special case where both the desired signal and interferers have the same fading characteristics (Rayleigh fading). It is observed that microcellular systems outperform medium/large cell systems.

12th December 1991

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