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Reverse link capacity of SIR-based power-controlled CDMA systems with antenna arrays

Jin Yu and Yu-Dong Yao*,†

Wireless Information Systems Engineering Laboratory (WISELAB), ECE Department, Stevens Institute of Technology, Hoboken, N.J. 07030, U.S.A.

SUMMARY

In this paper, reverse link capacity, in terms of user capacity, of a code-division multiple access (CDMA) system with both beamforming and antenna diversity under a multipath environment is analyzed and compared. Signal-to-interference ratio (SIR) based power control is assumed in the system. Antenna elements are divided into several groups to achieve antenna diversity gain. Antenna elements in a group are closely spaced to perform beamforming in order to suppress the own-cell and other-cell interference, while those groups are widely spaced to achieve the diversity gain. Based on the comparison, a most efficient use of the antenna array according to the distribution of antenna elements among the groups is obtained. Instead of using tedious iterative methods to evaluate user capacity, a simple closed-form capacity expression with respect to antenna diversity and beamforming gains, a target SIR and the CDMA processing gain is derived. Numerical results indicate significant capacity improvement with the antenna array. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: Code-division multiple access (CDMA); User capacity; Antenna diversity; Beamforming; Rake receiver

1. Introduction

With the advance of wireless communication technology, there is an explosive increase in the number of mobile users. Although second-generation (2G) wireless systems, such as the Global System for Mobile Communications (GSM) and IS-95, are successful in many countries [1], they still cannot meet the requirement of high-speed data and user capacity in high-user-density areas. Higher system capacity, better quality of service (QoS) and flexible accommodations of various wideband services (such as video and multimedia services) with different transmission rates are required in third-generation (3G) wireless communication systems [2]. Code-division multiple access (CDMA) has been chosen as the radio interface technology for 3G systems [3]. Unlike frequency-division multiple access (FDMA) and time-division multiple access (TDMA), which are primarily bandwidth or dimension limited in capacity, CDMA capacity is interference limited [4]. Thus any reduction of the interference will directly lead to capacity increases. The emerging technologies, such as antenna array and multiuser detection, could lead to a significant reduction in the interference and result in manyfold capacity increases [5–7]. Especially for antenna

*Correspondence to: Yu-Dong Yao, Wireless Information Systems Engineering Laboratory (WISELAB), ECE Department, Stevens Institute of Technology, Hoboken, N.J. 07030, U.S.A.
†E-mail: yyao@stevens-tech.edu

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array, three methods can be used to significantly improve the capacity of a CDMA system, which are beamforming, antenna diversity, and a combination of diversity and beamforming [8]. Capacity estimation is an important element in the design of CDMA systems and in the performance evaluation of these new technologies.

In CDMA performance evaluations, the user capacity is used to measure CDMA systems with continuously active users [9]. In this paper, user capacity is referred to as the number of users that a CDMA system could support at a desired signal-to-noise-plus-interference ratio (SNIR) without power constraints [5]. Gilhousen *et al.* [4] estimated CDMA reverse link user capacity, considering voice traffic only, where strength-based power control [10, 11] is assumed in the CDMA systems and total other-cell interference, I, is modeled as Gaussian noise [4, 12]. I increases with the number of active users per cell, N, which results in a decrease of signal-to-interference ratio (SIR). The maximum N can be found considering a target SIR, γ_0. Using similar methods as used in Reference [4], Kim and Sung estimated the reverse link user capacity of SIR-based power-controlled CDMA systems in References [13] and [14]. User capacity of multicode CDMA (MC-CDMA) systems, supporting voice and data traffic or heterogeneous constant-bit-rate traffic, was analyzed in Reference [15]. The effects of a Rake receiver and antenna diversity on reverse link user capacity are further investigated in Reference [2], where the received signals at different antenna groups are assumed to be independent of each other, and the user capacity increases linearly with the number of antenna groups. However, the limitation to reverse link user capacity, caused by the correlation between antenna groups, has not been investigated. The impact of beamforming, which is an important technique to suppress interference, on reverse link user capacity has not been thoroughly examined. The focus of this paper is to present the user capacity gain obtained through the use of beamforming and antenna diversity under a multipath environment.

In this paper, the use of beamforming in CDMA systems in a multipath fading environment is investigated and its impact on reverse link user capacity is analyzed first. Both transmit and receive beamformings are considered in the system. User capacity gain achieved through a combination of beamforming and receive antenna diversity is further investigated. The impact of correlation between antenna groups on reverse link user capacity is also examined. Traffic scenarios such as voice and multimedia services are considered in an MC-CDMA system. A simple closed-form equation, which is related to the array gain patterns, antenna diversity, a target SNIR and the CDMA processing gain is given to estimate the reverse link user capacity.

This paper is organized as follows. System models, including antenna array and other-cell interference, are given in Section 2. A closed-form equation to evaluate reverse link user capacity of a CDMA cellular system with antenna array under a multipath fading environment is derived in Section 3. Voice and multimedia traffic scenarios of an MC-CDMA system are considered in Section 4. Numerical results are given in Section 5. Finally, conclusions are drawn in Section 6.

2. System Model

2.1. Antenna Array

Beamforming has been widely used in wireless systems that employ a fixed set of antenna elements in an array. Considering receive beamforming in reverse link transmissions, the signals from these antenna elements are combined to form a movable beam pattern that can be steered to a desired direction that tracks mobile stations (MS) as they move. This allows the antenna system to focus radio frequency (RF) resources on a particular mobile station and minimize the impact of interference [16, 17]. This is achieved using a beamformer by placing nulls at the directions of interference, while the antenna array gain in the direction of the desired transmitter is maintained constant. While few antenna elements could be installed at a mobile station, large antenna arrays can be implemented at a base station (BS). When beamforming is used at the mobile station, the transmit beam pattern can be adjusted to minimize the interference to unintended receivers (such as base stations in other cells). At a base station, receive beamforming for each desired user could be implemented independently without affecting the performance of other links [17].

A linear equally spaced (LES) array for beamforming is considered here (Figure 1(a)). θ is the azimuthal angle and φ is the elevation angle of a plane wave incident on the array. In this paper, the LES array is used for both transmit and receive beamforming. Considering a two-dimensional multi-cell environment (in the horizontal plane) [16], we have $\varphi = 90°$. The distance d between the elements of the

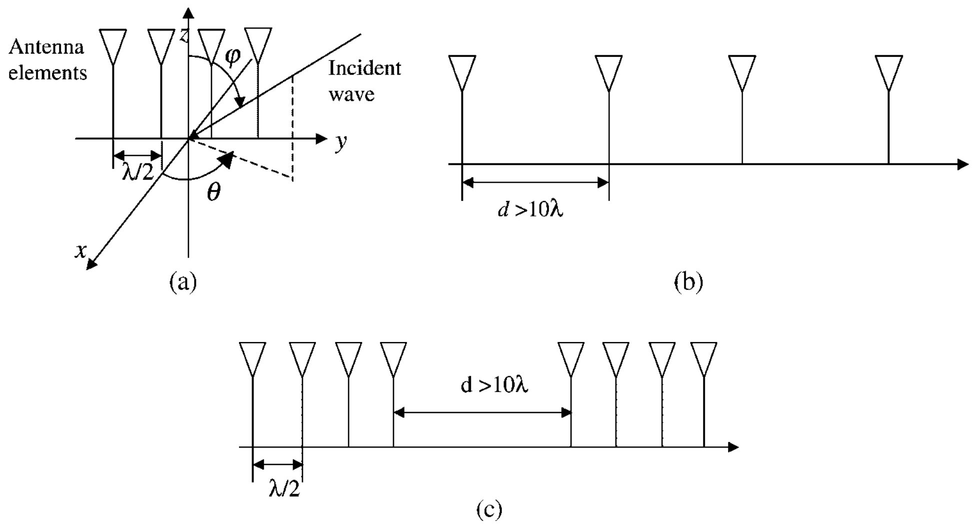


Fig. 1. (a) A linear equally spaced beamforming antenna array, (b) diversity antenna array, (c) antenna array combining beamforming and diversity.

LES array is assumed to be 0.5λ, where λ is the carrier wavelength. In the LES array system, a combining network connects an array of low gain antenna elements and could generate an antenna pattern [16,18],

$$G(\psi, \theta) = \left|\frac{f(\theta)}{M}\right|^2 = \left|\frac{\sin(0.5M\pi(\sin\psi - \sin\theta))}{M\sin(0.5\pi(\sin\psi - \sin\theta))}\right|^2, \quad (1)$$

where M is the number of antenna elements and ψ is a variable to adjust the beam direction. In the remaining of this paper, we will use the antenna pattern specified in Equation (1) to evaluate the impact of beamforming on the CDMA reverse link capacity.

When two or more widely spaced antennas are used at the base station, combining the received Rayleigh fading signals at different antennas can achieve the diversity gain to mitigate the effect of multipath propagation. And the spatial envelope autocorrelation coefficients between the antenna elements at the base stations, ρ_r, is given by [19]

$$\rho_r = \mathrm{E}^2[\cos\{kd\cos\phi_i\}] + \mathrm{E}^2[\sin\{kd\cos\phi_i\}], \quad (2)$$

where k is the wavenumber ($k = 2\pi/\lambda$, λ is the wavelength) and d is the distance between two antenna groups. ϕ_i is a random variable [20] and related to the incident angle θ and, in this paper, we assume that the 3 dB beamwidth is 3°. When the antenna array is used to perform beamforming, called beamforming antenna array, the spacing between array elements should be small enough so that there is no amplitude variation between the signals received at different elements; while in order to achieve antenna diversity gain, the spacing needs to be large enough to obtain independently received signals. A diversity antenna array is shown in Figure 1(b) and an example combining beamforming and antenna diversity is described in Figure 1(c) [8].

2.2. Rake Receiver

Multipath propagation in the radio channel leads to deep fading of the received signal. If the paths are independent and resolvable in time domain, i.e. the delay between different paths is greater than the chip duration T_c, a Rake receiver can be used to combine the paths to achieve diversity gains [20, 21]. The multipath fading can be characterized by a power delay profile (PDP), which is uniformly or exponentially distributed [2]. For a uniform PDP, we have

$$\mathrm{E}[a_l] = 1/L, \qquad 0 \le l \le L-1$$

and for an exponential PDP, we have

$$\mathrm{E}[a_l] = (1 - e^{-\varepsilon})e^{-\varepsilon l} \qquad l \ge 0,$$

where a_l is the square path gain of the lth path, L is the total number of paths in a uniform profile and ε is a decay factor for an exponential PDP, which can be set by considering the percentage of the signal power that is contained in the first L paths. Assuming that the Rake receiver at the base station has R fingers, the received power at the output is the combination of the R paths, $X = a_0 + a_1 + \cdots + a_{R-1}$. If P antenna groups are used to achieve antenna diversity gain,

$$X = \sum_{p=1}^{P} (a_{p,0} + a_{p,1} + \cdots + a_{p,R-1}) = \sum_{p=1}^{P} \sum_{r=0}^{R-1} a_{p,r}.$$

In this paper, we assume that the paths are temporally independent, but are spatially correlated and the correlation is specified through Equation (2).

2.3. Other-Cell Interference

A cellular structure is shown in Figure 2 with a reference cell (base station BS_0) and an interference cell (with base station BS_m). In a CDMA cellular system, an MS is power controlled by a BS in its home cell to ensure that the received SNIR at the BS is no less than a target value, assuming that SIR-based power control is in use. Considering a mobile station, $MS_{m,j}$, in the interference cell, let the received power at its base station (BS_m) be S. The received interference at the base station of the reference cell with a Rake receiver is [2]

$$I = S\left(\frac{r_m}{r_0}\right)^{\mu} 10^{(\xi_0 - \xi_m)/10} \ \frac{1}{X},$$

where r_0 and r_m are the distances from $MS_{m,j}$ to BS_0 and BS_m as shown in Figure 2. μ is a path loss

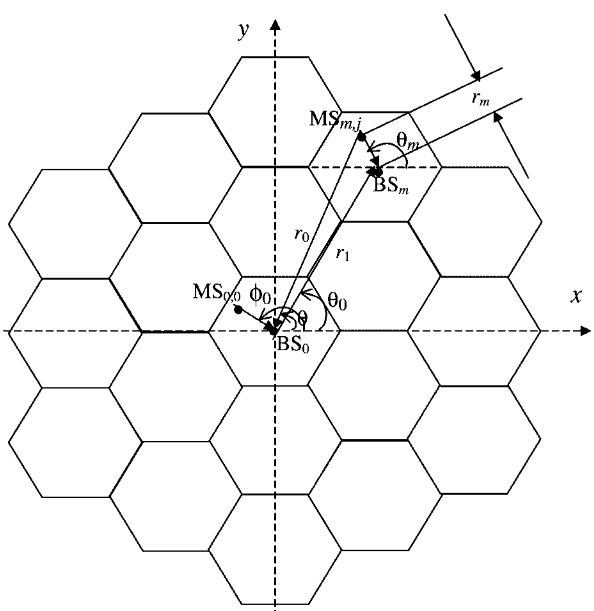


Fig. 2. Cellular structure and reverse link geometry.

exponent. ξ_0 and ξ_m describe the shadowing processes in the cells of BS_0 and BS_m, and the shadowing processes are assumed to be mutually independent and follow a lognormal distribution with standard deviation σ dB and zero mean. Considering all interfering mobile stations, the total other-cell interference at BS_0 is obtained by integrating the whole cellular coverage areas except the reference cell [14]

$$I = \iint S\left(\frac{r_m}{r_0}\right)^{\mu} 10^{(\xi_0 - \xi_m)/10} \ \phi\left(\xi_0 - \xi_m, \frac{r_m}{r_0}\right) \rho \frac{1}{X} \ dA,$$

where

$$\phi\left(\xi_0 - \xi_m, \frac{r_m}{r_0}\right) = \begin{cases} 1, & \text{if } (r_m/r_0)^{\mu} 10^{(\xi_0 - \xi_m)/10} \le 1 \\ 0, & \text{otherwise} \end{cases}$$

and $\rho = 2N/(3\sqrt{3})$ is the user density per unit area and there are N mobile stations in each cell. This assumes that the users are uniformly distributed in a cell and the radius of the hexagonal cell is normalized to unity. $\phi(\xi_0 - \xi_m,\ r_m/r_0)$ is an indicator function to show the cell areas that are excluded in the calculation of I since the mobile stations in these areas are not power controlled by BS_m but by BS_0. In computing the above integral, we simply consider the hexagonal areas of each cell rather than the actual coverage area of the base stations [4, 13, 14].

3. Reverse Link User Capacity with Antenna Array

When beamforming is applied at both transmit and receive sides with the antenna arrays along y axis, the total other-cell interference at BS_0 in a multipath fading environment can be expressed as

$$\begin{aligned} I = \iint & S\left(\frac{r_m}{r_0}\right)^{\mu} 10^{(\xi_0 - \xi_m)/10} \phi\left(\xi_0 - \xi_m, \frac{r_m}{r_0}\right) \\ & \times \rho G_t(\theta, \theta_m) G_r(\theta, \phi_0) \frac{1}{X} dA, \end{aligned} \tag{3}$$

where

$$\theta = \tan^{-1}\left(\frac{r_1 \sin\theta_0 + r_m \sin\theta_m}{r_1 \cos\theta_0 + r_m \cos\theta_m}\right)$$

and $G_t(\theta,\ \theta_m)$ and $G_r(\theta,\ \phi_0)$ are transmit and receive beamforming gain patterns. θ_m and θ are the azimuth angle of $MS_{m,j}$ to its home base station, BS_m, and that

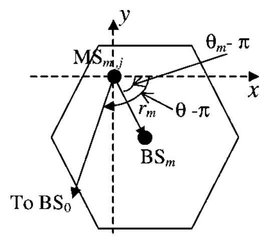


Fig. 3. Angle notations in transmit beamforming at $MS_{m,j}$.

to BS_0 respectively. r_1 is the distance between BS_m to BS_0. θ_0 is the azimuth angle of BS_m to BS_0. ϕ_0 is the azimuth angle of mobile station $MS_{0,0}$ to BS_0 as shown in Figure 2 and is uniformly distributed from 0 to 2π. Figure 3 indicates angle notations in transmit beamforming at $MS_{m,j}$. When M_t antenna elements are used with the beamforming pattern shown in (1), the transmit antenna gain in the direction from $MS_{m,j}$ to BS_0 is

$$G_t(\theta,\theta_m) = \left|\frac{\sin(0.5M_t\,\pi(\sin(\theta-\pi)-\sin(\theta_m-\pi)))}{M_t\,\sin(0.5\pi(\theta-\pi)-\sin(\theta_m-\pi)))}\right|^2 = \left|\frac{\sin(0.5M_t\,\pi(\sin\theta-\sin\theta_m))}{M_t\,\sin(0.5\pi(\sin\theta-\sin\theta_m))}\right|^2. \quad (4)$$

Similarly, when receive beamforming is used with M_r antenna elements at BS_0 for receiving signals from $MS_{0,0}$, the receive antenna gain in the direction from $MS_{m,j}$ to BS_0 is

$$G_r(\theta,\phi_0) = \left|\frac{\sin((0.5M_r\pi(\sin\theta-\sin\phi_0))}{M_r\sin(0.5\pi(\sin\theta-\sin\phi_0))}\right|^2. \quad (5)$$

The expected value of the total other-cell interference is

$$\mathrm{E}[I] = \mathrm{E}\left[\iint S\left(\frac{r_m}{r_0}\right)^{\mu} 10^{(\xi_0-\xi_m)/10}\phi\left(\xi_0-\xi_m,\frac{r_m}{r_0}\right) \times \rho G_t(\theta,\theta_m)G_r(\theta,\phi_0)\frac{1}{X}\,\mathrm{d}A\right],$$

which can be rewritten as

$$\mathrm{E}[I] = \mathrm{E}[S]F(\mu,\sigma)N\mathrm{E}\left[\frac{1}{X}\right] \quad (6)$$

with

$$F(\mu,\sigma) = \frac{2}{3\sqrt{3}}e^{\{\sigma\ln(10)/10\}^2} \times \iint\left(\frac{r_m}{r_0}\right)^{\mu}\Phi\left(\frac{10\mu}{\sqrt{2\sigma^2}}\log_{10}\left(\frac{r_0}{r_m}\right)-\sqrt{2\sigma^2}\frac{\ln(10)}{10}\right) \times G_t(\theta,\theta_m)E[G_r(\theta,\phi_0)]\,\mathrm{d}A,$$

where

$$\mathrm{E}[G_r(\theta,\phi_0)] = \frac{1}{2\pi}\int_0^{2\pi} G_r(\theta,\phi_0)\,\mathrm{d}\phi_0$$

and

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}\,\mathrm{d}t.$$

Similarly, the variance of I is expressed as

$$\mathrm{Var}[I] = \mathrm{Var}\left[\iint S\left(\frac{r_m}{r_0}\right)^{\mu}10^{(\xi_0-\xi_m)/10}\phi\left(\xi_0-\xi_m,\frac{r_m}{r_0}\right) \times \rho G_t(\theta,\theta_m)G_r(\theta,\phi_0)\frac{1}{X}\,\mathrm{d}A\right],$$

which, following Reference [4], is rewritten as

$$\mathrm{Var}[I] = \left\{U(\mu,\sigma)\mathrm{E}\left[S^2\right]-V(\mu,\sigma)\mathrm{E}^2[S]\right\}\mathrm{E}^2\left[\frac{1}{X}\right]N, \quad (7)$$

where

$$U(\mu,\sigma) = \frac{2}{3\sqrt{3}}e^{\{\sigma\ln(10)/5\}^2}\iint\left(\frac{r_m}{r_0}\right)^{2\mu}G_t^2(\theta,\theta_m) \times \Phi\left(\frac{10\mu}{\sqrt{2\sigma^2}}\log_{10}\left(\frac{r_0}{r_m}\right)-\sqrt{2\sigma^2}\frac{\ln(10)}{5}\right) \times \mathrm{E}\left[G_r^2(\theta,\phi_0)\right]\mathrm{d}A$$

$$V(\mu,\sigma) = \frac{2}{3\sqrt{3}}e^{2\{\sigma\ln(10)/10\}^2}\iint\left(\frac{r_m}{r_0}\right)^{2\mu} \times G_t^2(\theta,\theta_m)\Phi^2\left(\frac{10\mu}{\sqrt{2\sigma^2}}\log_{10}\left(\frac{r_0}{r_m}\right)-\sqrt{2\sigma^2}\frac{\ln(10)}{10}\right) \times \mathrm{E}^2\left[G_r(\theta,\phi_0)\right]\mathrm{d}A$$

and

$$\mathrm{E}[G_r^2(\theta,\phi_0)] = \frac{1}{2\pi}\int_0^{2\pi} G_r^2(\theta,\phi_0)\,\mathrm{d}\phi_0.$$

The value $F(\mu,\ \sigma)$, $U(\mu,\ \sigma)$ and $V(\mu,\ \sigma)$ can be obtained numerically. For example, when only the first and second-tier cells are considered, we find, for $\mu = 4$, $\sigma = 8$ dB, $M_r = 3$ and $M_t = 1$, $F(4,\ 8) = 0.2676$, $U(4,\ 8) = 0.1072$ and $V(4,\ 8) = 0.0197$.

In CDMA systems, the received SNIR, E_b/I_0, should be no less than a target value, γ_0, in order to maintain a required transmission quality. Following

Reference [2] and considering transmit and receive beamforming, we have

$$\frac{E_b}{I_0} = \sum_{p=1}^{P}\sum_{r=1}^{R}\frac{GSG_t(\phi_0,\phi_0)G_r(\phi_0,\phi_0)\frac{a_{p,r}}{X}}{\frac{2}{3}\left(S\sum_{m=0}^{N-1}\sum_{l=0}^{\infty}\mathrm{E}\left[G_r(\phi_m,\phi_0)G_t(\phi_m,\phi_m)\frac{a_{p,l}}{X} - SG_t(\phi_0,\phi_0)G_r(\phi_0,\phi_0)\frac{a_{p,r}}{X}\right] + I\right) + \eta_0 W}$$
$$\approx \frac{GS}{\frac{2}{3}\left(NS\mathrm{E}[G_r(\phi_m,\phi_0)]\mathrm{E}\left[\sum_{l=0}^{\infty}\frac{a_{p,l}}{X}\right] + I\right) + \eta_0 W} \tag{8}$$

and

$$\frac{E_b}{I_0} \geq \gamma_0, \tag{9}$$

where G is the CDMA processing gain, η_0 is the single-sided white noise power spectrum density and W is the spreading bandwidth. The factor 2/3 in the denominator is due to the assumption of a square chip pulse, if sinc waveform is used, the factor becomes 1. The denominator in Equation (8) includes other-cell interference as well as own-cell interference due to other mobile stations in the reference cell. Note that the transmit antenna gains of mobile stations at the reference cell in Equation (8) are all set to unity since their transmit beams are steered toward base station BS_0. ϕ_m is the azimuth angle of an interfering mobile station $MS_{0,m}$ to BS_0. Figure 4 illustrates angle notations of receive beamforming at BS_0. $G_r(\phi_m, \phi_0)$ is the receive beamforming gain of $MS_{0,0}$ to the direction of $MS_{0,m}$. ϕ_m and ϕ_0 are uniformly distributed in $[0, 2\pi)$. Let $\phi = \sin\phi_m - \sin\phi_0$, the probability density function of ϕ is found to be

$$f(\phi) = \begin{cases} \frac{1}{\pi^2}\int_{-1}^{\phi+1}\frac{1}{\sqrt{1-\tau^2}}\frac{1}{\sqrt{1-(\phi-\tau)^2}}\mathrm{d}\tau & -2 < \phi < 0 \\ \frac{1}{\pi^2}\int_{\phi-1}^{1}\frac{1}{\sqrt{1-\tau^2}}\frac{1}{\sqrt{1-(\phi-\tau)^2}}\mathrm{d}\tau & 0 < \phi < 2 \\ 0 & \text{otherwise} \end{cases}.$$

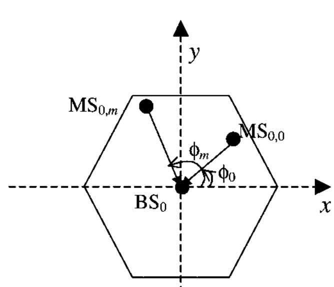


Fig. 4. Angle notations of receive beamforming at BS_0.

We further have

$$\mathsf{G}_r = \mathrm{E}[G_r(\phi_m,\phi_0)] = \int_{-2}^{2}\frac{\sin^2(0.5M_r\pi\phi)}{M_r^2\sin^2(0.5\pi\phi)}f(\phi)\,\mathrm{d}\phi.$$

Since the envelopes of received signals are considered as Rayleigh fading signals, their power profile $a_{p,r}$ is exponentially distributed. When antenna diversity is used in the system and the received signals of different antenna groups are assumed to be independent of each other, X is the sum of PR independent random variables, which are exponentially distributed. Thus the probability density function (PDF) of X is the convolution of all PR independent random variables. The terms including X, such as $\mathrm{E}[1/X]$, could be obtained theoretically or numerically. For uniform power delay profile

$$\mathrm{E}\left[\frac{1}{X}\right] = \frac{L}{PR-1}.$$

When the correlation between antenna groups is considered, the signals are not spatially independent. X can be rewritten as

$$X = \sum_{r=0}^{R-1}(a_{1,r} + a_{2,r} + \cdots + a_{P,r}) = \sum_{r=0}^{R-1}A_r,$$

where A_r, $r = 0, 1, \ldots, R-1$, is the sum of P spatially correlated random variables, while $A_0, A_1, \ldots, A_{R-1}$ are temporally independent. The PDF of A_r can be found by following the derivation in Reference [20]. For example, when $P=2$,

$$f(A_r) = \frac{1}{\lambda_1 - \lambda_2}\left[\exp(-A_r/\lambda_1) - \exp(-A_r/\lambda_2)\right] \tag{10}$$

where

$$\lambda_1 = P_r(1+\sqrt{\rho_r})$$
$$\lambda_2 = P_r(1-\sqrt{\rho_r}),$$

where P_r is the average received power at rth finger. Since the arrived signals follow the same slow fading and the distribution of power delay profile is assumed to be identical for different antenna groups, we assume that the average received power of the same Rake finger for different antenna groups are equal to each other. The PDF of X can also be obtained by convolving all A_r, $r=0, 1, \ldots, R-1$. Let us consider another term containing X in the denominator of Equation (8)

$$\mathrm{E}\left[\sum_{l=0}^{\infty}\frac{a_{p,l}}{X}\right] = \mathrm{E}\left[\sum_{l=0}^{R-1}\frac{a_{p,l}}{X}\right] + \mathrm{E}\left[\sum_{l=R}^{\infty}\frac{a_{p,l}}{X}\right].$$

Let $D = \mathrm{E}\left[\sum_{l=R}^{\infty} a_{p,l}/X\right]$, we could obtain [2]

$$D = \begin{cases} \mathrm{E}[1/X](L-R)/L & \text{for uniform PDP} \\ \mathrm{E}[1/X]\exp(-\varepsilon R) & \text{for exponential PDP} \end{cases} \tag{11}$$

and

$$\mathrm{E}\left[\sum_{l=0}^{R-1}\frac{a_{p,l}}{X}\right] \approx \frac{1}{P}. \tag{12}$$

Substituting Equations (11) and (12) into Equation (8), it can be simplified as

$$\frac{E_b}{I_0} = \frac{GS}{\frac{2}{3}(NS\mathsf{G}_r(D+1/P)+I)+\eta_0 W}.$$

Let S be the minimum power level satisfying Equation (9). The received power S could be expressed in terms of I as

$$S = \frac{I+1.5\eta_0 W}{C}, \tag{13}$$

where

$$C = 1.5G/\gamma_0 - N\mathsf{G}_r(D+1/P).$$

Now the user capacity N can be found via an iterative method [2, 13, 14], in which, there are two concatenated iteration loops. In the inner loop, for a given N value, determine E[I] and Var[I] using the following steps

1. Set E[I] and Var[I] as zeros.
2. Calculate E[S] and E[S^2] from Equation (13).
3. Calculate E[I] and Var[I] from Equations (6) and (7).
4. Repeat Steps 2 and 3 until the differences between old and new values of E[I] and Var[I] are less than 1% [2].

Using the E[I] and Var[I] obtained above and a specified maximum transmission power limit, calculate an outage probability (the transmission power exceeds the power constraint) [2]. If the outage probability does not exceed a required level, the outer loop increases N by 1 and enters the inter loop. The iteration loops stop when the calculated outage probability exceeds the required level. We then obtain the user capacity as $N-1$.

The iterative method to determine user capacity as described above is computationally complicated due to multiple loops of numerical integrations. In this paper, we derive a formula to directly estimate user capacity, which is related to all the relevant factors, such as γ_0, beamforming gains, diversity gains and CDMA processing gains. Solving Equations (6), (7) and (13), we get

$$\mathrm{E}[I] = \frac{1.5F(\mu,\sigma)N\mathrm{E}[1/X]}{C-\mathrm{E}[1/X]F(\mu,\sigma)N}\eta_0 W \tag{14}$$

$$\mathrm{Var}[I] = \frac{(U(\mu,\sigma)-V(\mu,\sigma))(\mathrm{E}[I]+1.5\eta_0 W)^2}{C^2/(N\mathrm{E}^2[1/X])-U(\mu,\sigma)}. \tag{15}$$

It is obvious that E[I] and Var[I] are greater than 0 because the noise or interference power is always positive. Thus a valid N has to ensure that Equations (14) and (15) greater than 0. Following the approach described in Reference [22], we are able to determine user capacity N as

$$N = \left\lfloor \frac{1.5G/\gamma_0}{\mathsf{G}_r(1/P+D)+\mathrm{E}[1/X]F(\mu,\sigma)} \right\rfloor, \tag{16}$$

where $\lfloor x \rfloor$ indicates a maximum integer no greater than x. In using the iterative method to determine the user capacity, we found that when the number of users is above the user capacity, N, power control will not work, since every mobile station tries to satisfy the target SNIR, γ_0, by increasing its transmit power until reaching its maximum transmit power allowed. This leads to an increase of interference to other users. The results from the closed-form expression, Equation (16), will be compared in Section 5 with those results obtained from the iterative method.

Note that in the computation of the other-cell interference in the correlated case of antenna diversity, X is a function of θ and $\mathrm{E}[I] = \mathrm{E}[S]H(\mu, \sigma)N$, where

$$H(\mu,\sigma) = \frac{2}{3\sqrt{3}} e^{\{\sigma \ln(10)/10\}^2} \iint \left(\frac{r_m}{r_0}\right)^{\mu} \times \Phi\left(\frac{10\mu}{\sqrt{2\sigma^2}}\log_{10}\left(\frac{r_0}{r_m}\right) - \sqrt{2\sigma^2}\frac{\ln(10)}{10}\right) \times G_t(\theta,\theta_m)\mathrm{E}[G_r(\theta,\phi_0)]\mathrm{E}\left[\frac{1}{X(\theta)}\right]\mathrm{d}A.$$

Notice that X is a function of $\phi_m (m = 0, 1, \ldots, N-1)$ (see Equation (8)), which is uniformly distributed in $[0, 2\pi)$, and we have $E[1/X] = \int_0^{2\pi} [1/X(\phi_m)] d\phi_m / 2\pi$. Using the above derivations, we are able to find the reverse link capacity of a CDMA system with antenna diversity (correlated antenna arrays)

$$N = \left\lfloor \frac{1.5G/\gamma_0}{\mathsf{G}_r(1/P + D) + H(\mu,\sigma)} \right\rfloor.$$

4. MC-CDMA

In Section 3, we derived the user capacity of a CDMA system with beamforming and diversity in a multipath fading environment. In this section, we extend the results to a CDMA system with multicode operation. MC-CDMA is a technique to support users requiring widely varying data rates for different applications [23]. In MC-CDMA, a user who wants to transmit different traffic at different rates is simply assigned different spreading codes and appears to the base station as multiple users. Thus different traffic from the same user will result in cochannel interference to each other. Heterogeneous constant bit rate (CBR) traffic, such as audio, data and video services, are considered in this paper, which require different spreading codes with processing gains g_a, g_d and g_v respectively. For example, we could assume that spreading gains for audio, data and video traffic are 128, 64, 32 respectively. If g_a is normalized to unity, $g_d = 1/2$ and $g_v = 1/4$. The total other-cell interference comes from all the traffic types is [13]

$$I = I_a + I_d + I_v.$$

If the traffic streams are assumed to be independent of each other, we could get

$$\mathrm{E}[I] = \mathrm{E}[I_a] + \mathrm{E}[I_d] + \mathrm{E}[I_v] \tag{17}$$

$$\mathrm{Var}[I] = \mathrm{Var}[I_a] + \mathrm{Var}[I_d] + \mathrm{Var}[I_v]. \tag{18}$$

And different traffic types with the use of beamforming and antenna diversity in a multipath fading environment have to satisfy different SNIR requirements,

$$\begin{aligned}
\left(\frac{E_b}{I_0}\right)_a &\approx \frac{GS_a}{\frac{2}{3}[\mathsf{G}_r(N_aS_a/g_a + N_dS_d/g_d + N_vS_v/g_v)(D+1/P) + I] + \eta_0 W} \geq \gamma_a \\
\left(\frac{E_b}{I_0}\right)_d &\approx \frac{GS_d}{\frac{2}{3}[\mathsf{G}_r(N_aS_a/g_a + N_dS_d/g_d + N_vS_v/g_v)(D+1/P) + I] + \eta_0 W} \geq \gamma_d \\
\left(\frac{E_b}{I_0}\right)_v &\approx \frac{GS_v}{\frac{2}{3}[\mathsf{G}_r(N_aS_a/g_a + N_dS_d/g_d + N_vS_v/g_v)(D+1/P) + I] + \eta_0 W} \geq \gamma_v
\end{aligned} \tag{19}$$

where subscript a, d and v represent audio, data and video traffic types respectively. N_i, γ_i, and S_i ($i = a$, d or v) represent the corresponding user capacity, target SNIR and received signal power, respectively. We could get the minimum received power S_a, S_d and S_v when we set the received E_b/I_0 of different traffic types equal to their target values. Following References [13, 24] and solving Equation (19), S_a, S_d and S_v could be expressed in terms of the total other-cell interference I as

$$S_a = \frac{I + 1.5\eta_0 W}{C}, \quad S_d = S_a\beta_d, \quad S_v = S_a\beta_v,$$

where

$$C = 1.5G/\gamma_a - \mathsf{G}_r(N_a/g_a + N_d\beta_d/g_d + N_v\beta_v/g_v)(D + 1/P)$$

and

$$\beta_d = \frac{\gamma_d}{\gamma_a}, \quad \beta_v = \frac{\gamma_v}{\gamma_a}.$$

Based on the derivations in Section 3, when a multipath fading environment with resolvable paths is considered and a combination of Rake receiver and antenna array is used in the MC-CDMA system, the user capacity can be derived as

$$N_a/g_a + N_d\beta_d/g_d + N_v\beta_v/g_v = \left\lfloor \frac{1.5G/\gamma_a}{\mathsf{G}_r(1/P+D) + \mathrm{E}[1/X]F(\mu,\sigma)} \right\rfloor, \quad (20)$$

where D and $\mathrm{E}[1/X]$ are the same as given in Section 3.

5. Numerical Results

Throughout this section, we assume propagation parameters, $\mu = 4$ and $\sigma = 8$ dB. Table I lists MC-CDMA system parameters. The basic data rate R, is assumed to be 32 kbps and the spreading chip rate is 4.096 Mbps. The first traffic type (voice) is of the basic rate and its required SNIR target γ_a is 5 dB. The second and third traffic types require higher data rates (64 kbps and 128 kbps) and their SNIR targets are 10 dB and 7 dB respectively.

Table II(a) considers a single transmit antenna element, and at the receive side, the number of antenna elements varies from 1, 3, 5, 7 to 9. As $F(\mu, \sigma)$ is proportional to other-cell interference, we see that the value of $F(\mu, \sigma)$ decreases with an increase of the number of receive antenna elements, M_r. The received own-cell interference is proportional to the expected received antenna gain, G_r, which decreases with increasing M_r as shown in the table. The values of $U(\mu, \sigma)$ and $V(\mu, \sigma)$ are also presented in the table, which will be used in evaluating user capacity via the iterative method as described in Section 3. The values of β_d and β_v are to be used in evaluating MC-CDMA systems. Table II(b) is similar to Table II(a) except that a different number of transmit antenna elements is considered.

Table I. System parameters.

R	W	g_a	g_d	g_v	γ_a	γ_d	γ_v
32 kbps	4.096 Mbps	1	0.5	0.25	5 dB	10 dB	7 dB

Table II. Computational parameters.

(a) $M_t = 1$					
(M_t, M_r)	(1, 1)	(1, 3)	(1, 5)	(1, 7)	(1, 9)
$F(\mu, \sigma)$	0.6611	0.2676	0.1724	0.1287	0.1035
$U(\mu, \sigma)$	0.2252	0.0800	0.0502	0.0372	0.0297
$V(\mu, \sigma)$	0.0451	0.0083	0.0037	0.0021	0.0014
G_r	1	0.3855	0.2487	0.1863	0.1501
β_d	2.9111	3.0595	3.0950	3.1116	3.1213
β_v	1.5311	1.5635	1.5710	1.5744	1.5765
(b) $M_t = 2$					
(M_t, M_r)	(2, 1)	(2, 3)	(2, 5)	(2, 7)	(2, 9)
$F(\mu, \sigma)$	0.3410	0.1458	0.0960	0.0726	0.0589
$U(\mu, \sigma)$	0.1085	0.0372	0.0242	0.0183	0.0150
$V(\mu, \sigma)$	0.0181	0.0038	0.0018	0.0011	0.0008

5.1. User Capacity of CDMA with Beamforming

Figure 5 presents the reverse link user capacity with only beamforming ($P = 1$) in a multipath fading environment and illustrates the effect of the number of receive antenna elements and the accuracy of the evaluation expression (16). In Figure 5, we assume that the power delay profile is exponentially distributed and 95% signal power exists in the first L paths, and L is set to be 4 and the number of Rake fingers R equals 2. Figure 5(a) clearly shows that the user capacity increases significantly when the number of antenna elements increases. However, as shown in Figure 5(b), the user capacity per antenna drops with the increase of antenna elements. In Figure 5(a), we also compare the capacity evaluations using a simple closed-form expression, Equation (16), and those based on the complex iterative method, where the outage probability, P_{outage}, as discussed in Section 3 is limited to be 0.05 [2] for $M_t = 1$ and $P_{\text{outage}} = 0.01$ for $M_t = 2$. The results obtained from the two evaluations differ from each other slightly when $M_t = 1$, while almost identical capacity results are obtained using the two evaluation methods when $M_t = 2$. The difference comes from the consideration of the outage probability. For example, when $M_r = 9$ and $M_t = 1$, from Equation (16), we obtain $N = 116$. When the iterative method is used, we get $N = 121$, which corresponds to an outage probability of 0.0368. However, a close examination reveals that the system with 121 users needs extremely high transmission power from the mobiles stations since the mean other-cell interference reaches $\mathrm{E}[I] \approx$ 2e+007 $\eta_0 W$. Note that, when $N = 116$, we have $\mathrm{E}[I] \approx 72.33\eta_0 W$ and when $N = 117$, $P_{\text{outage}} = 0$, $\mathrm{E}[I] \approx 520.13\eta_0 W$. Therefore when $N = 121$, the CDMA system is out of power control. Even with $N = 118$, although the outage probability is equal to 0.011, $\mathrm{E}[I] \approx$ 2e+007$\eta_0 W$. Based on numerical calculations, when $M_t = 2$ and P_{outage} is limited to 0.01, for some cases, they give identical results. In some other scenarios, they only differ by one user in capacity evaluations. Figure 5 (a) illustrates the effectiveness and accuracy using the simple closed-form expression (16) to evaluate user capacity.

The numerical results considering both uniform and exponential power delay profiles are shown in

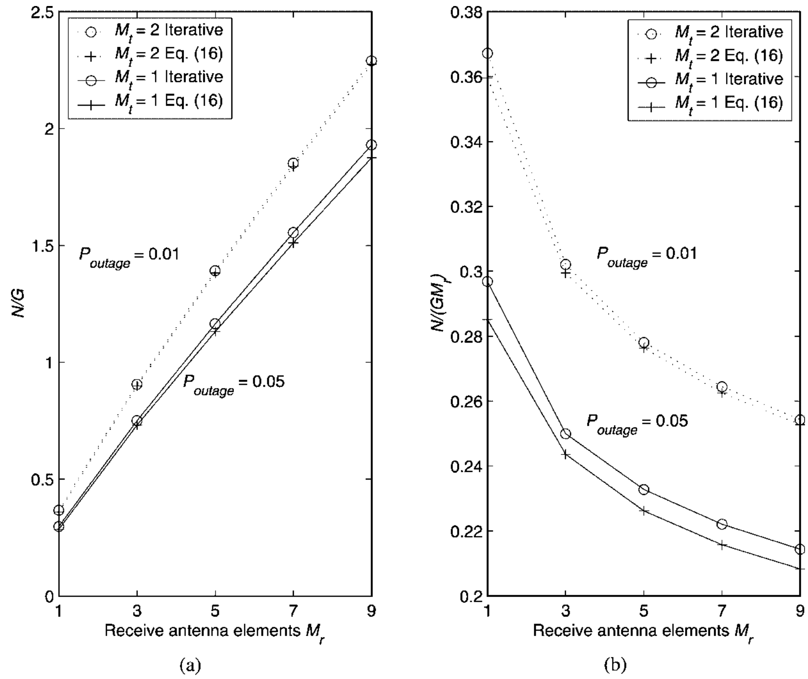


Fig. 5. Reverse link user capacity, (a) Effect of beamforming, (b) Per receive antenna element.

Figure 6. For comparison purpose, the number of transmit antenna elements, M_t, is set to be of 1 and 2, and the number of Rake receiver fingers, R, varies from 2, 3 to 4. Noticeable capacity improvements are seen when the number of Rake receiver fingers increases, which indicates the effectiveness of the Rake receiver in combination with the use of beamforming. Figures 5 and 6 present the capacity results of single-code CDMA systems, which are also applicable to MC-CDMA systems. When $N = N_a/g_a + N_d\beta_d/$

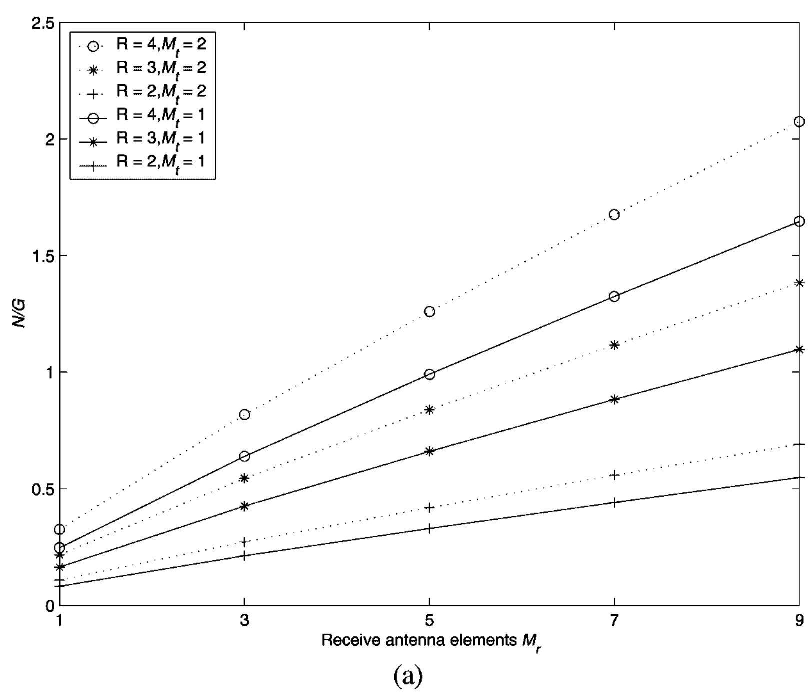
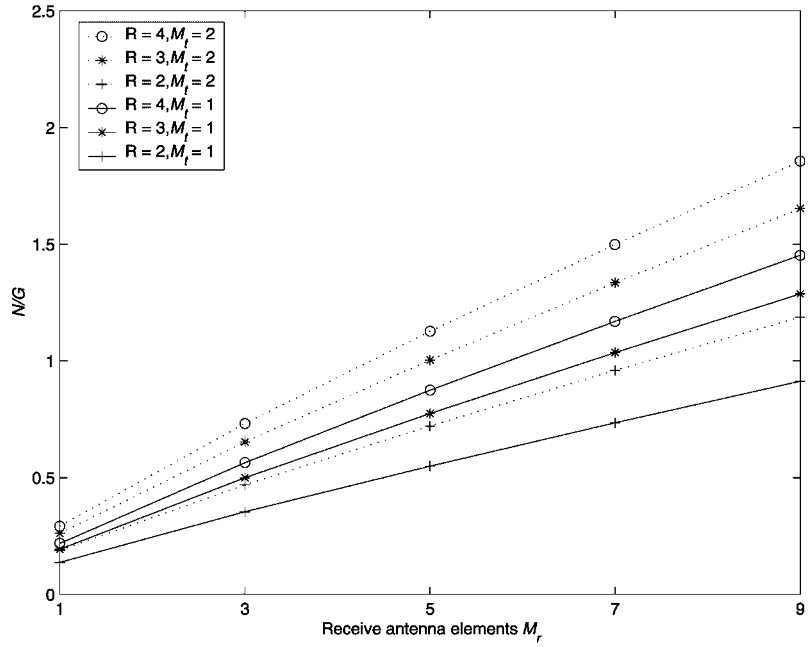


Fig. 6. User capacity, effect of beamforming and Rake receiver, (a) Uniform PDP, (b) Exponential PDP.



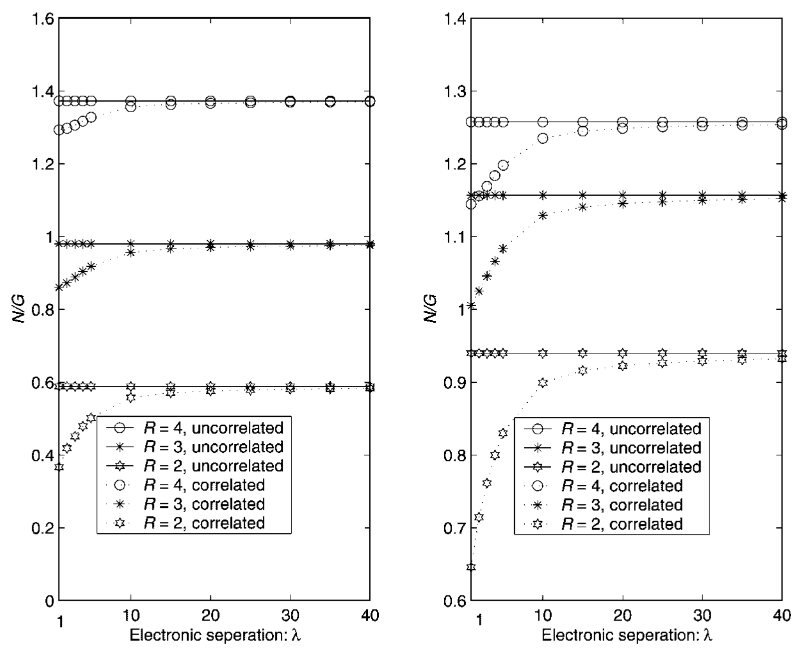
(b)

Fig. 6. Continued

$g_d + N_v\beta_v/g_v$, Equations (16) and (20) are identical. This suggests that once N is calculated, the values of (N_a, N_d, N_v) can be determined by a base station according to (g_a, g_d, g_v) and (β_d, β_v).

5.2. User Capacity of CDMA with Beamforming and Antenna Diversity

When both beamforming and antenna diversity are used, the reverse link user capacity can also be



(a) (b)

Fig. 7. Reverse link user capacity with correlated (dashed line) and independent antenna groups (solid line) with $P = 2$, $M_r = 3$ and $M_t = 1$ for (a) Uniform PDP, (b) Exponential PDP.

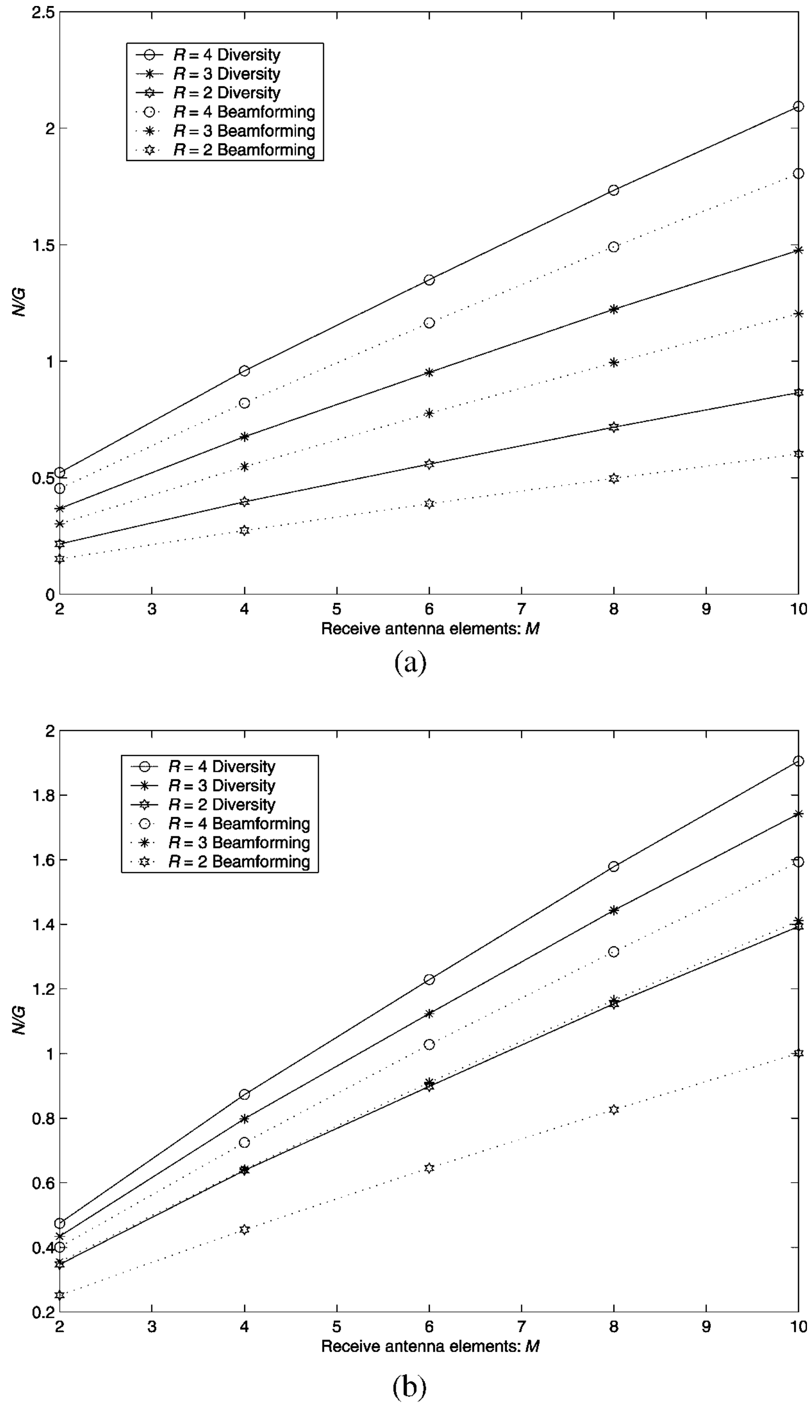


Fig. 8. Reverse link user capacity with beamforming and antenna diversity ($d = 10\lambda$) (a) Uniform PDP, (b) Exponential PDP.

evaluated using Equation (16). We still assume a mutlipath environment with $L = 4$ and R varies from 2, 3 to 4. Two antenna groups are used to achieve antenna diversity and each group has 3 antenna elements $M_r = 3$, and transmit antenna element $M_t = 1$. The numerical results of user capacity considering independent and correlated received signals are presented in Figure 7(a) and (b). Both uniform (Figure 7(a)) and exponential (Figure 7(b)) PDP's are considered. Dashed line represents the user capacity for correlated antenna groups. We found that when the distance between antenna groups is greater than 20λ, user capacity converges to that of independent antenna groups (solid line).

Figure 8 compares the user capacity of different combination of antenna elements. For example, if we have M antenna elements at the base station, one approach is to use all antenna elements to do beamforming and another one is to divide these M antenna elements into two groups to achieve antenna diversity and each group has $M/2$ antenna elements to steer the beam. And the number of transmit antenna element M_t is assumed to be 1. Both uniform (Figure 8(a)) and exponential (Figure 8(b)) PDP's are considered and the number of total paths, L, is assumed to be 4 and R varies from 2, 3 to 4. Noticeable capacity improvements are seen when antenna diversity is used with the same antenna elements in the base station, which indicates the effectiveness of the antenna diversity in combination with the use of beamforming.

6. Conclusions

A simple closed-form expression is derived to evaluate reverse link user capacity for CDMA and MC-CDMA systems with beamforming, antenna diversity and a Rake receiver in a multipath fading and multicell environment. Both transmit and receive beamforming are assumed in the system and significant capacity improvements are observed with an increase of the number of antenna elements. The capacity improvement due to the Rake receiver is also illustrated. The impact of the correlation between antenna groups on reverse link user capacity is analyzed. The relationships between the capacity and various system parameters, including the target SNIR, CDMA processing gain, antenna array gain patterns and the number of Rake receiver fingers, are reflected in the simple closed-form capacity equation.

In this paper, we considered an LES array for beamforming. The capacity evaluation method can be applied to CDMA systems with other types of arrays by considering different transmit and receive array gain patterns, G_t and G_r. Perfect fast transmit power control was assumed in this paper. In a practical system, the loop delay of power control and the accuracy of SNIR estimation will have an impact on the performance of power control, which will reduce the CDMA capacity. Furthermore, in a multipath environment, signals will arrive at the receiver through multiple paths from different directions, which will increase the complexity to steer the beam to each direction. All these issues affecting the system capacity will be studied in the future.

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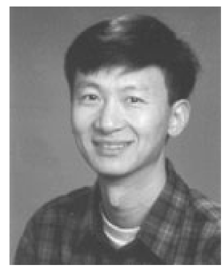
Authors' Biographies

Jin Yu received the B.S.E.E. from Wuhan University, China, in 1998, and M.S.E.E. from the University of Mississippi, Oxford, MS, in 2001. Currently he is studying toward the Ph.D. degree in electrical engineering at Stevens Institute of Technology. During his study at the University of Mississippi, his research work focused on the computational Electromagnetics and microwave circuits. At present he is doing research in the areas of spread-spectrum communication systems at WISELAB. His research interest areas include CDMA, DSP for wireless communications, MIMO systems, and microwave circuits.



Yu-Dong Yao received the B.Eng. and M.Eng. degrees from Nanjing University of Posts and Telecommunications, Nanjing, China, in 1982 and 1985, respectively, and the Ph.D. degree from Southeast University, Nanjing, in 1988, all in electrical engineering. Between 1989 and 1990, he was at Carleton University, Ottawa, as a research associate working on mobile radio communications. He was with Spar Aerospace Ltd., Montreal, between 1990 and 1994, where he was involved in research on satellite communications. He was with Qualcomm Inc., San Diego, from 1994 to 2000, where he participated in research and development in wireless CDMA systems. Dr Yao joined Stevens Institute of Technology, Hoboken, New Jersey, in 2000. He is an Associate Professor in the Department of Electrical and Computer Engineering and a director of Wireless Information Systems Engineering Laboratory (WISELAB). Dr Yao holds one Chinese patent and six U.S. patents. He is an associate editor of *IEEE Communications Letters* and *IEEE Transactions on Vehicular Technology*, and an editor of *IEEE Transactions on Wireless Communications*. His research interests include wireless communications and networks, spread spectrum and CDMA, and DSP for wireless systems.