Performance Evaluation of CDMA Reverse Links With Imperfect Beamforming in a Multicell Environment Using a Simplified Beamforming Model

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***Abstract*—Reverse link capacity of a direct-sequence code-division multiple-access (DS-CDMA) system in a multicell environment has been studied recently, and significant capacity improvements due to the use of beamforming have been observed. However, system performance with beamforming will be affected by several impairments, such as direction of arrival estimation errors, array perturbations, mutual coupling, and signal spatial spreads. In this paper, reverse link performance of CDMA systems with beamforming under these impairments (imperfect beamforming) is investigated. A simplified beamforming model is developed to evaluate the system performance in terms of user capacity, bit-error rates (BER), and outage probabilities. Both signal-to-interference-ratio-based power control and strength-based power control are considered in this paper. The capacity and BER degradations due to different impairments are shown, and outage probabilities under different power control schemes are examined.**

***Index Terms*—Array perturbation, beamforming, code-division multiple access (CDMA), mutual coupling, power control, spatial spread, user capacity.**

I. Introduction

IN THE PAST 15 years, there is an explosive increase in the number of mobile users. Although second-generation (2-G) wireless systems, such as the Global System for Mobile Communications (GSM) and IS-95, are successful in many countries [1], [2], they still cannot meet the requirements of high-speed data and user capacity in high-user-density areas. Code-division multiple access (CDMA) has been chosen as the radio interface technology for third-generation (3-G) systems [3]. Unlike frequency-division multiple access (FDMA) and time-division multiple access (TDMA), which are primarily bandwidth- or dimension-limited in capacity, CDMA capacity is interference-limited [4]. Thus, any reduction of the interference will directly lead to capacity increases. Emerging technologies, such as beamforming and multiuser detections, could lead to a significant reduction in the interference and result in considerable capacity increases [5]–[7]. Capacity estimation is an important element in the performance evaluation of these new technologies.

Gilhousen *et al.* [4] estimated CDMA reverse link user capacity, where strength-based power control [8], [9] is assumed and total other-cell interference I is modeled as Gaussian noise [4], [10]. The value of I increases with the number of active users per cell, N, which results in a decrease of signal-to-interference ratio (SIR). The maximum N can be found considering a target SIR, γ_0. Using similar methods employed in [4], Kim and Sung estimated the reverse link user capacity of SIR-based power-controlled CDMA systems in [11] and [12]. User capacity of multicode CDMA systems supporting voice and data traffic or heterogeneous constant-bit-rate traffic was analyzed in [13]. The effects of a Rake receiver and antenna diversity on reverse link user capacity were further investigated in [14]. Capacity improvements with base-station antenna arrays in strength-based power-controlled cellular CDMA systems have been analyzed [15]. CDMA reverse link capacity with the combined use of antenna arrays and Rake receivers under SIR-based power control scheme was investigated in [16]. Significant capacity improvement due to the use of beamforming has been observed in [15] and [16].

In the implementation of beamforming algorithms, the direction of arrival (DOA), which is a parameter that is used to steer the beams toward the desired signals, is usually obtained through estimation algorithms, such as the multiple signal classification (MUSIC) algorithm and the estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm [17], [18]. Any error in estimating arrival angles will cause the antenna array point away from the desired users and lead to a reduction of the received power of the desired signals. Even with perfect DOA estimations, array perturbations due to the position errors of antenna elements can also result in mismatches between exact DOAs and directions of main lobes [17].

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In addition to DOA estimation errors and array perturbations, the existence of mutual coupling between antenna elements also leads to a significant change of beam patterns [19], [20]. Imperfect power control in addition to mutual coupling and scattering is also considered in [19]. The obstacles around a transmitter (mobile station, MS), such as buildings, reflect the transmitted waves and result in multiple paths with different arrival angles at a base station (BS; angle spread). A propagation model to characterize the angle spread was reviewed in [21]. Recently, angle spreads have been measured and reported in [22] and [23]. For rural environments, angular spreads between 1° and 5° have been observed [22]; for urban and hilly terrain environments, considerably larger angular spreads as large as 20°, have been found [23]. Angle spreads not only reduce the received signal power, as the DOA estimation becomes random in the interval of arrival angles, but also cause DOA estimation uncertainty. Sensitivity of array signal processing/array antennas to errors and imperfections has also been described extensively in [17].

Notice that the CDMA system performance improvement has been investigated considering perfect beamforming (no impairments) in most studies [15], [16], [18], whereas imperfect beamforming considering mutual coupling and spatial spreads (angle spreads) has been investigated for a single-cell CDMA system [19]. This paper differs from previous studies in two aspects. First, we test a multicell CDMA system with both in-cell interference and other-cell interference. Second, we investigate all beamforming impairments discussed above, i.e., DOA estimation errors, array perturbations, mutual coupling, and spatial spreads. Additionally, a simplified beamforming model is developed in this paper to facilitate the evaluation of CDMA performance with imperfect beamforming. Signal strength-based power control and SIR-based power control are considered in the performance evaluation in terms of user capacity, bit-error rates (BER), and outage probabilities. In this paper, user capacity is referred to as the number of users that a CDMA system could support under a desired signal-to-noise-plus-interference ratio (SNIR) [6]. This paper is organized as follows. System models, including beamforming and other-cell interference, are given in Section II. A simplified beamforming model is presented in this section. The impacts of DOA estimation errors, array perturbations, mutual coupling, and spatial spreads on the interference statistics are analyzed in Section III. Performance of CDMA reverse link with imperfect beamforming is evaluated in Section IV, and numerical results are given in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

A. Beamforming

Beamforming has been widely used in wireless systems that employ a fixed set of antenna elements in an array. Considering receive beamforming in reverse link transmissions, the signals from these antenna elements are combined to form a movable beam pattern that can be steered to a desired direction that tracks MSs as they move. This allows the antenna system to focus radio frequency (RF) resources on a particular MS and minimize the impact of interference [18]–[24]. Although few antenna elements could be installed at an MS, large antenna arrays can be implemented at a BS. When beamforming is used at the MS, the transmit beam pattern can be adjusted to minimize the interference to unintended receivers (such as BSs in other cells). At a BS, receive beamforming for each desired user could be implemented independently without affecting the performance of other links [24]. A uniform linear array (ULA) considering a two-dimensional multicell environment $(\theta = 90^\circ)$ is considered and is shown in Fig. 1 [18]. The distance d between the antenna elements is assumed to be 0.5λ, where λ is the carrier wavelength. In the ULA system, a combining network connects an array of low-gain antenna elements and could generate an ideal antenna pattern [18], [25], i.e.,

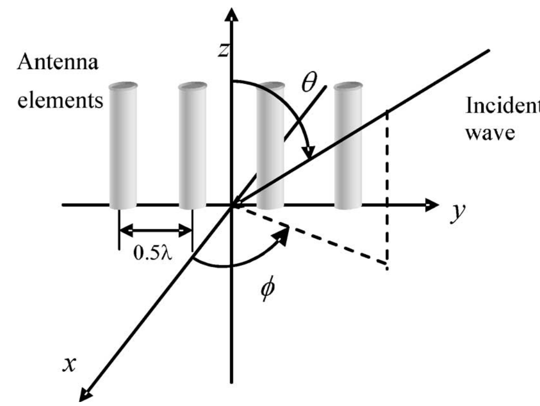

Fig. 1. ULA.

$$G(\phi, \varphi) = \left| \frac{1}{M} \sum_{m=0}^{M-1} \exp\left\{-j(\mathbf{k}\cdot\mathbf{p}_m - m\pi\sin\varphi)\right\} \right|^2 \quad (1)$$

where M is the number of antenna elements, $\exp(jm\pi\sin\varphi)$ is the weighting factor for the mth antenna element, $\mathbf{k} = 2\pi(\cos\phi, \sin\phi)/\lambda$ is the phase propagation vector, ϕ is the arrival angle of signals or interference, $\mathbf{p}_m = (x_m, y_m)$ is the vector for the position of the mth element, and φ is the scan angle. The beam could be steered to a desired direction by varying a single parameter φ, when φ is set to be equal to the arrival angle of the desired signal [18]. In the remaining of this paper, we will use the antenna pattern specified in (1) to evaluate the impact of beamforming on the CDMA reverse link performance.

B. Other-Cell Interference

A cellular structure is shown in Fig. 2 with a reference cell (BS_0) and an interference cell (with BS_m). In a CDMA cellular system, an MS is power-controlled by a BS in its home cell to ensure that the received SNIR (or power) at the BS is no less than a target value, assuming that SIR-based power control (or strength-based power control) is in use. Considering the jth MS in the mth cell, $\mathrm{MS}_{m,j}$, let the received power at its BS (BS_m) be S. Note that imperfect beamforming may result in the

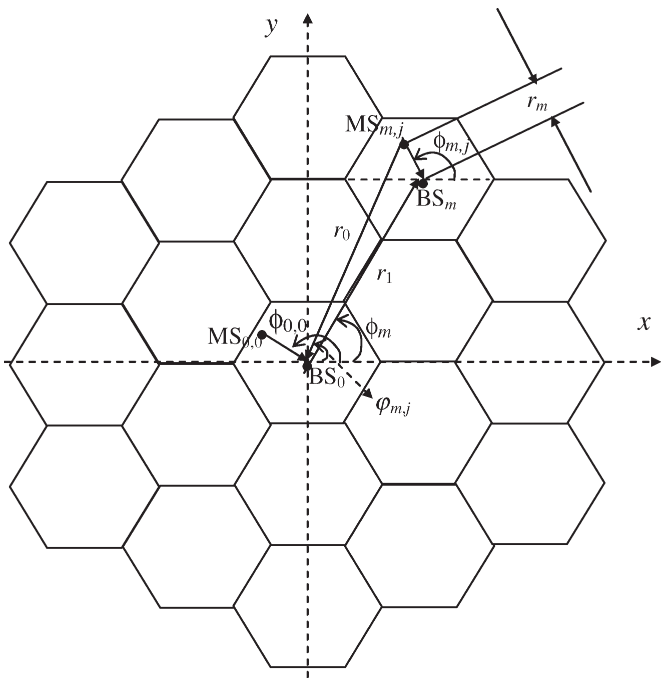

Fig. 2. Cellular structure and reverse link geometry.

change of the gain toward the desired user. The instantaneous received power is

$$S = P_T r_m^{-\mu} 10^{\xi_m/10} G_t(\phi_{m,j} - \pi, \hat{\phi}_{m,j} - \pi) G_r(\phi_{m,j}, \hat{\phi}_{m,j}) \tag{2}$$

where $\hat{\phi}_{m,j}$ is the estimated arrival angle that is used to steer the beam, $G_t(\phi_{m,j} - \pi, \hat{\phi}_{m,j} - \pi)$ and $G_r(\phi_{m,j}, \hat{\phi}_{m,j})$ are transmit and receive beamforming gains [16], and $\phi_{m,j} - \pi$ and $\phi_{m,j}$ are the transmit angle and arrival angle from the MS to its home BS, respectively.[1] The transmitted power P_T is obtained as

$$P_T = \frac{S}{r_m^{-\mu} 10^{\xi_m/10} G_t(\phi_{m,j} - \pi, \hat{\phi}_{m,j} - \pi) G_r(\phi_{m,j}, \hat{\phi}_{m,j})}. \tag{3}$$

The resulted interference at the BS of the reference cell BS_0 is

$$I = S(r_m/r_0)^{\mu} 10^{(\xi_0 - \xi_m)/10} \times \frac{G_t(\varphi_{m,j} - \pi, \hat{\phi}_{m,j} - \pi) G_r(\varphi_{m,j}, \hat{\phi}_{0,0})}{G_t(\phi_{m,j} - \pi, \hat{\phi}_{m,j} - \pi) G_r(\phi_{m,j}, \hat{\phi}_{m,j})} \tag{4}$$

where r_0 and r_m are the distances from $\mathrm{MS}_{m,j}$ to BS_0 and BS_m as shown in Fig. 2, $\phi_{0,0}$ is the arrival angle of the desired user $\mathrm{MS}_{0,0}$ to BS_0 as shown in Fig. 2 and is uniformly distributed from 0 to 2π, $\hat{\phi}_{0,0}$ is the estimated value of $\phi_{0,0}$, μ is a path loss exponent, and ξ_0 and ξ_m describe the shadowing processes in the cells of BS_0 and BS_m. The shadowing processes are assumed to be mutually independent and follow a lognormal distribution with standard deviation σ dB and zero mean. Considering all interfering MSs, the total other-cell interference at BS_0 is obtained by integrating the whole cellular coverage area except the reference cell [12], $\int\int(\cdot)dA$. The total other-cell interference is

$$I = \int\int S(r_m/r_0)^{\mu} 10^{(\xi_0 - \xi_m)/10} \varepsilon(\xi_0 - \xi_m, r_m/r_0) \times \rho \frac{G_t(\varphi_{m,j} - \pi, \hat{\phi}_{m,j} - \pi) G_r(\varphi_{m,j}, \hat{\phi}_{0,0})}{G_t(\phi_{m,j} - \pi, \hat{\phi}_{m,j} - \pi) G_r(\phi_{m,j}, \hat{\phi}_{m,j})} dA \tag{5}$$

where

$$\varepsilon(\xi_0 - \xi_m, r_m/r_0) = \begin{cases} 1, & \text{if } (r_m/r_0)^{\mu} 10^{(\xi_0 - \xi_m)/10} \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\varphi_{m,j} = \arctan\left(\frac{r_1 \sin\phi_m + r_m \sin\phi_{m,j}}{r_1 \cos\phi_m + r_m \cos\phi_{m,j}}\right)$$

ρ is the user density per unit area, N is the number of MSs in each cell ($\rho = 2N/(3\sqrt{3})$), assuming that the users are uniformly distributed in a cell and the radius of the hexagonal cell is normalized to unity, $\varepsilon(\xi_0 - \xi_m, r_m/r_0)$ is an indicator function to show the cell areas that are excluded in the calculation of I because the MSs in these areas are not power-controlled by BS_m but by BS_0, ϕ_m is the azimuth angle of BS_m, and r_1 is the distance between BS_m to BS_0. In computing the above integral, we simply consider the hexagonal areas of each cell rather than the actual coverage area of the BSs [4], [11], [12].

In the remainder of this paper, we consider receive beamforming only and set the number of transmit antenna elements to 1, so that $G_t(\varphi_{m,j} - \pi, \hat{\phi}_{m,j} - \pi) = G_t(\phi_{m,j} - \pi, \hat{\phi}_{m,j} - \pi) = 1$. The mean value of the total other-cell interference is thus

$$E[I] = E[S]F(\mu, \sigma)N \tag{6}$$

with

$$F(\mu, \sigma) = \frac{2}{3\sqrt{3}} \exp\left\{\left(\frac{\sigma \ln(10)}{10}\right)^2\right\} \int\int \left(\frac{r_m}{r_0}\right)^{\mu} \times \Phi\left(\frac{10\mu}{\sqrt{2\sigma^2}} \log_{10}\left(\frac{r_0}{r_m}\right) - \sqrt{2\sigma^2}\frac{\ln(10)}{10}\right) \times E\left[\frac{G_r(\varphi_{m,j}, \hat{\phi}_{0,0})}{G_r(\phi_{m,j}, \hat{\phi}_{m,j})}\right] dA \tag{7}$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left\{-\frac{t^2}{2}\right\} dt.$$

The variance of I is found to be

$$\mathrm{Var}[I] = \left\{U(\mu, \sigma)E[S^2] - V(\mu, \sigma)E^2[S]\right\} N \tag{8}$$

[1]Note that (2) describes the received power averaged over the small-scale fading. However, recent investigations [26] have shown that even the power distribution in the presence of both shadowing and small-scale fading can be well approximated by a lognormal distribution with a modified shadowing variance.

where

$$U(\mu,\sigma) = \frac{2}{3\sqrt{3}} \exp\left\{\left(\frac{\sigma \ln(10)}{5}\right)^2\right\} \int\int \left(\frac{r_m}{r_0}\right)^{2\mu} \times \Phi\left(\frac{10\mu}{\sqrt{2\sigma^2}} \log_{10}\left(\frac{r_0}{r_m}\right) - \sqrt{2\sigma^2}\frac{\ln(10)}{5}\right) \times E\left[\frac{G_r^2(\varphi_{m,j}, \hat{\phi}_{0,0})}{G_r^2(\phi_{m,j}, \hat{\phi}_{m,j})}\right] dA \tag{9}$$

and

$$V(\mu,\sigma) = \frac{2}{3\sqrt{3}} \exp\left\{2\left(\frac{\sigma \ln(10)}{10}\right)^2\right\} \int\int \left(\frac{r_m}{r_0}\right)^{2\mu} \times \Phi^2\left(\frac{10\mu}{\sqrt{2\sigma^2}} \log_{10}\left(\frac{r_0}{r_m}\right) - \sqrt{2\sigma^2}\frac{\ln(10)}{10}\right) \times E^2\left[\frac{G_r(\varphi_{m,j}, \hat{\phi}_{0,0})}{G_r(\phi_{m,j}, \hat{\phi}_{m,j})}\right] dA. \tag{10}$$

C. Simplified Beamforming Modeling: Signal and Interference

The value $F(\mu,\sigma)$, $U(\mu,\sigma)$, and $V(\mu,\sigma)$ can be obtained numerically [16]. However, the complexity considering the exact beam pattern is very high, especially for the evaluation under those impairments mentioned in Section I, due to multiple integrals. A simple Bernoulli model is introduced in [15] in which a signal is considered to be within a mainlobe ($G_r = 1$) or out of the mainlobe ($G_r = 0$), and the half-power beamwidth is defined as the beamwidth. This model is easy to use, but it neglects the impact of sidelobes and the effect of any specific beam patterns. The work of Spagnolini [27] provides a beamforming model with a triangular pattern to characterize the beam head. In the following, we develop an accurate, yet simple, beamforming model to account the impact of sidelobes and the real beam patterns. Assuming that the beamwidth is B (normalized by 2π), the gain of the mainlobe is normalized to unity, and the gain out of the mainlobe is α. This implies that the probability that one interferer is in the mainlobe is B. Considering the Bernoulli distribution, the first and second moments of the antenna gain with respect to all incident angles are $B + (1-B)\alpha$ and $B + (1-B)\alpha^2$, respectively. Considering the impact of the impairments of beamforming and normalizing the beam pattern by the gain at the desired direction, we set the first and second moments of the real beam pattern equal to that in the simplified model and obtain

$$B + (1-B)\alpha = E\left[\frac{G_r(\varphi, \hat{\phi})}{G_r(\psi, \hat{\psi})}\right] \tag{11}$$

and

$$B + (1-B)\alpha^2 = E\left[\frac{G_r^2(\varphi, \hat{\phi})}{G_r^2(\psi, \hat{\psi})}\right]. \tag{12}$$

Solving this equation set, we obtain the parameters for receive beamforming

$$\alpha = \frac{\Sigma_G - \Upsilon_G}{\Upsilon_G - 1} \tag{13}$$

and

$$B = \frac{\Sigma_G - \Upsilon_G^2}{\Sigma_G + 1 - 2\Upsilon_G} \tag{14}$$

where

$$\Upsilon_G = E\left[\frac{G_r(\varphi, \hat{\phi})}{G_r(\psi, \hat{\psi})}\right] \tag{15}$$

and

$$\Sigma_G = E\left[\frac{G_r^2(\varphi, \hat{\phi})}{G_r^2(\psi, \hat{\psi})}\right] \tag{16}$$

in which Υ_G and Σ_G are the antenna gain parameters, ϕ and φ are the arrival angles of the desired signal and the interference, $\hat{\phi}$ is an estimate of ϕ, ψ denotes the arrival angle of the interferer to its own BS, and $\hat{\psi}$ is an estimation of ψ. Note that the division in (15) and (16) is due to power control. When ψ is the arrival angle of an interferer in another cell (other-cell interference), the power control is performed at another cell. Therefore, ψ is independent of φ, and the denominator and numerator in (15) and (16) are independent of each other. However, when ψ is the arrival angle of an interferer from the same cell, power control is done at the reference cell and $\psi = \varphi$. We denote the array gain parameters for in-cell interference as Υ_G^I and Σ_G^I, and those for other-cell interference as Υ_G^O and Σ_G^O. The antenna gain ratios are averaged with respect to the arrival angles of the desired signal and the interference, ϕ, φ, and ψ, which are assumed to be uniformly distributed in the region from 0 to 2π. It is important to point out that a real beam pattern is used for evaluating the received power of a desired signal as shown in Fig. 3(a), and the simplified accurate model with parameters B and α is used for interference [Fig. 3(b)]. When the beamforming impairments are considered, φ, ϕ, and ψ will be related to those random variables (B, α), which are used to model the impairments. With the simplified/accurate model described above, we obtain the mean of the interference I

$$\begin{aligned} I &= NF(\mu,\sigma)E[S] \\ &= N\left(B + (1-B)\alpha\right) F_0(\mu,\sigma)E[S] \\ &= N\Upsilon_G^O F_0(\mu,\sigma)E[S] \end{aligned} \tag{17}$$

where

$$F_0(\mu,\sigma) = \frac{2}{3\sqrt{3}} \exp\left\{\left(\frac{\sigma \ln(10)}{10}\right)^2\right\} \int\int \left(\frac{r_m}{r_0}\right)^{\mu} \times \Phi\left(\frac{10\mu}{\sqrt{2\sigma^2}} \log_{10}\left(\frac{r_0}{r_m}\right) - \sqrt{2\sigma^2}\frac{\ln(10)}{10}\right) dA \tag{18}$$

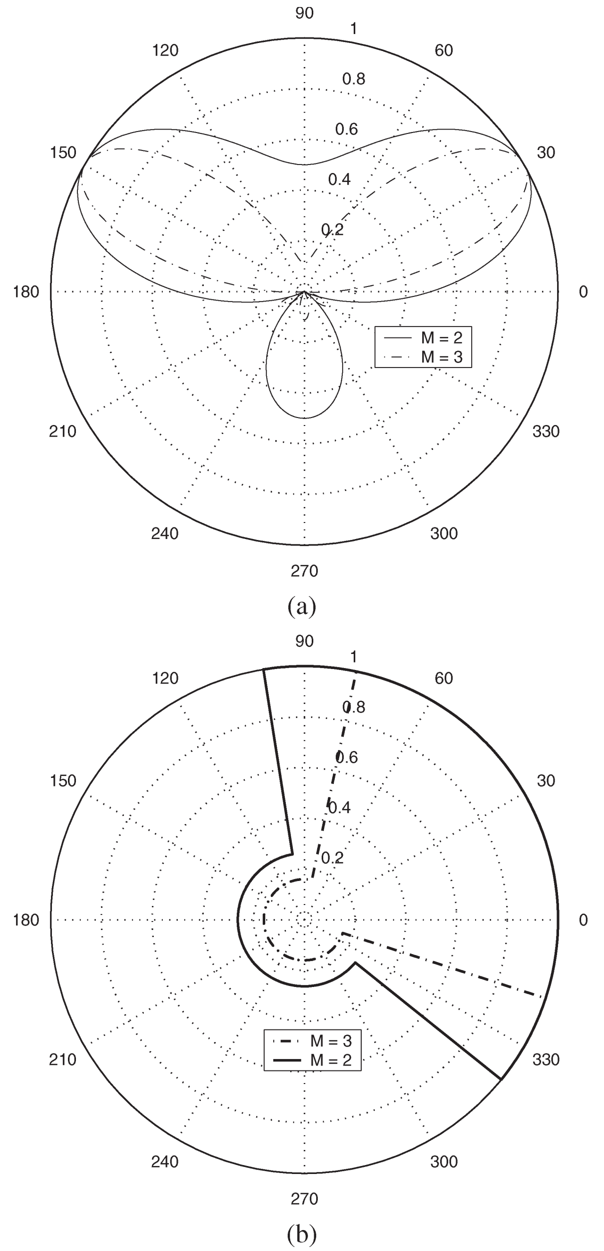

Fig. 3. Simplified model for beamforming with arrival angle $\phi = 30°$. (a) Signal model. (b) Interference model.

and similarly, the variance of the interference I can be simplified as

$$\mathrm{Var}[I] = \left\{U_0(\mu,\sigma)\Sigma_G^O E[S^2] - V_0(\mu,\sigma)E^2[S]\left(\Upsilon_G^O\right)^2\right\}N \tag{19}$$

where

$$U_0(\mu,\sigma) = \frac{2}{3\sqrt{3}}\exp\left\{\left(\frac{\sigma\ \ln(10)}{5}\right)^2\right\}\int\int\left(\frac{r_m}{r_0}\right)^{2\mu} \times \Phi\left(\frac{10\mu}{\sqrt{2\sigma^2}}\log_{10}\left(\frac{r_0}{r_m}\right) - \sqrt{2\sigma^2}\frac{\ln(10)}{5}\right)dA \tag{20}$$

and

$$V_0(\mu,\sigma) = \frac{2}{3\sqrt{3}}\exp\left\{2\left(\frac{\sigma\ \ln(10)}{10}\right)^2\right\}\int\int\left(\frac{r_m}{r_0}\right)^{2\mu} \times \Phi^2\left(\frac{10\mu}{\sqrt{2\sigma^2}}\log_{10}\left(\frac{r_0}{r_m}\right) - \sqrt{2\sigma^2}\frac{\ln(10)}{10}\right)dA \tag{21}$$

where $F_0(\mu,\sigma) = 0.6611$, $U_0(\mu,\sigma) = 0.2252$, and $V_0(\mu,\sigma) = 0.0451$ with $\mu = 4$ and $\sigma = 8$ [16]. In the evaluation of reverse link performance of a CDMA system with imperfect beamforming, the key issue is to find $\Sigma_G^{O,I}$ and $\Upsilon_G^{O,I}$. We will also show in Section IV that the numerical results with the use of the simplified and accurate model agree very well with the numerical results without the use of the model.

III. Interference Statistics

A. DOA Estimation Errors

In this section, the impact of DOA mismatch on the beamforming parameters (B and α, Υ_G and Σ_G) is analyzed. The estimated arrival angle $\hat{\phi}$ is characterized as a random variable with a uniform distribution or normal distribution [17], [21], [28] as (22), shown at the bottom of the page, where ϕ is the accurate arrival angle, Δ^2 represents the variance of the estimation errors for both uniform and normal distributions, B_w is the broadside null-to-null beamwidth, and

$$\kappa = \mathrm{erf}^{-1}\left(\frac{0.5B_w}{\sqrt{2}\Delta}\right)$$

in which

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)dt.$$

$$f(\hat{\phi}) = \begin{cases} \frac{1}{2\sqrt{3}\Delta},\ -\sqrt{3}\Delta \le (\hat{\phi}-\phi) \le \sqrt{3}\Delta, & \text{uniform distribution} \\ \frac{\kappa}{\sqrt{2\pi}\Delta}\exp\left\{-\frac{(\hat{\phi}-\phi)^2}{2\Delta^2}\right\},\ \hat{\phi}\in[-0.5B_w+\phi, 0.5B_w+\phi], & \text{normal distribution} \end{cases} \tag{22}$$

Therefore, the array gain parameters for the interference from the same cell can be found as

$$\Upsilon_G^I = E_{\hat{\phi},\hat{\varphi},\phi,\varphi}\left[\frac{G_r(\varphi,\hat{\phi})}{G_r(\varphi,\hat{\varphi})}\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\hat{\phi}}\int_{\hat{\varphi}} f(\hat{\phi})f(\hat{\varphi})\left(\frac{G_r(\varphi,\hat{\phi})}{G_r(\varphi,\hat{\varphi})}\right) d\hat{\varphi}\,d\hat{\phi}\,d\varphi\,d\phi \tag{23}$$

and

$$\Sigma_G^I = E_{\hat{\phi},\hat{\varphi},\phi,\varphi}\left[\frac{G_r^2(\varphi,\hat{\phi})}{G_r^2(\varphi,\hat{\varphi})}\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\hat{\phi}}\int_{\hat{\varphi}} f(\hat{\phi})f(\hat{\varphi})\left(\frac{G_r(\varphi,\hat{\phi})}{G_r(\varphi,\hat{\varphi})}\right)^2 d\hat{\varphi}\,d\hat{\phi}\,d\varphi\,d\phi. \tag{24}$$

For the interference from other cells, Υ_G^O and Σ_G^O can be found as

$$\Upsilon_G^O = E_{\hat{\phi},\phi,\varphi}\left[G_r(\varphi,\hat{\phi})\right]E_{\hat{\psi},\psi}\left[1/G_r(\psi,\hat{\psi})\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\hat{\phi}} f(\hat{\phi})\left(G_r(\varphi,\hat{\phi})\right)d\hat{\phi}\,d\varphi\,d\phi \times \frac{1}{2\pi}\int_0^{2\pi}\int_{\hat{\psi}} f(\hat{\psi})\left(\frac{1}{G_r(\psi,\hat{\psi})}\right)d\hat{\psi}\,d\psi \tag{25}$$

and

$$\Sigma_G^O = E_{\hat{\phi},\phi,\varphi}\left[G_r^2(\varphi,\hat{\phi})\right]E_{\hat{\psi},\psi}\left[1/G_r^2(\psi,\hat{\psi})\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\hat{\phi}} f(\hat{\phi})\left(G_r^2(\varphi,\hat{\phi})\right)d\hat{\phi}\,d\varphi\,d\phi \times \frac{1}{2\pi}\int_0^{2\pi}\int_{\hat{\psi}} f(\hat{\psi})\left(\frac{1}{G_r^2(\psi,\hat{\psi})}\right)d\hat{\psi}d\psi \tag{26}$$

where ϕ is the actual arrival angles of the desired signal to BS_0, $\hat{\phi}$ is an estimation of ϕ, φ and ψ represent the actual arrival angles of the users (interferers) to the reference BS and its own BS, $\hat{\varphi}$ and $\hat{\psi}$ are the estimates of φ and ψ, and ψ is also modeled as a uniform random variable in $[0, 2\pi)$.

B. Array Perturbations

Assuming there are odd number of antenna elements, $M = 2D+1$, which are symmetrically distributed along the y-axis with one at the center. The position of the bth element is $\mathbf{p}_b = (x_b, y_b) = (0, b\lambda/2)$ in the absence of a perturbation. The position errors due to array perturbation are modeled as random variables with a normal distribution. The weighting factor for the bth element is $w_b = \exp\{\mathbf{k}\cdot\mathbf{p}_b\}$, where $\mathbf{k}$ is the propagation vector of the signal. Let $p_b = x_b/\lambda$, $q_b = y_b/\lambda$, and we have the beam pattern (envelope) as [17]

$$A = \sum_{b=-D}^{D}\exp\left\{-j2\pi\left[(p_b\cos\varphi + q_b\sin\varphi) - 0.5b\sin\phi\right]\right\} \tag{27}$$

where p_b and q_b are independent with $E[p_b] = 0$ and $E[q_b] = 0.5d$ and $\mathrm{Var}[p_b] = \mathrm{Var}[q_b] = (\sigma_p/\lambda)^2$, where σ_p^2 is the variance of the position errors. Using $\mathbf{p}$ and $\mathbf{q}$ to represent $[p_{-D}\cdots p_0\cdots p_D]$ and $[q_{-D}\cdots q_0\cdots q_D]$, respectively, the beam pattern gain is expressed in (28), shown at the bottom of the page. Using $\mathbf{u}$ and $\mathbf{v}$ to represent $[u_1\cdots u_{2D}]$ and $[v_1\cdots v_{2D}]$, the beam pattern gain can be simplified as

$$G(\varphi,\phi,\mathbf{u},\mathbf{v}) = \frac{1}{M}\left\{1 + \sum_{l=1}^{M-1} G_l(\varphi,\phi,u_l,v_l)\right\} \tag{29}$$

where

$$G_l(\varphi,\phi,u_l,v_l) = 2(1-l/M) \times \cos\left\{2\pi(u_l\cos\varphi + v_l\sin\varphi) - l\pi\sin\phi\right\}$$

$l = a-b$, $u_l = p_a - p_b$, and $v_l = q_a - q_b$. Therefore, u_l and v_l are normally distributed with $E[u_l] = 0$, $E[v_l] = 0.5l$, and $\mathrm{Var}[u_l] = \mathrm{Var}[v_l] = 2(\sigma_p/\lambda)^2$. If the users are assumed to be uniformly distributed in the cell, φ and ϕ are uniform random variables in $[0, 2\pi)$. The array gain parameters for the interference from the same cell are obtained as

$$\Upsilon_G^I = E_{\varphi,\phi,\mathbf{u},\mathbf{v}}\left[\frac{G(\varphi,\phi,\mathbf{u},\mathbf{v})}{G(\varphi,\varphi,\mathbf{u},\mathbf{v})}\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\mathbf{u}}\int_{\mathbf{v}} f(\mathbf{u})f(\mathbf{v})\left[\frac{G(\varphi,\phi,\mathbf{u},\mathbf{v})}{G(\varphi,\varphi,\mathbf{u},\mathbf{v})}\right]d\mathbf{u}\,d\mathbf{v}\,d\varphi\,d\phi \tag{30}$$

$$G(\varphi,\phi,\mathbf{p},\mathbf{q}) = \frac{|AA^*|}{M^2} = \frac{1}{M^2}\left\{M + \sum_{a=-D}^{D}\sum_{a\neq b}\exp\left\{-j2\pi\left[((p_a-p_b)\cos\varphi + (q_a-q_b)\sin\varphi) - 0.5(a-b)\sin\phi\right]\right\}\right\} \tag{28}$$

and

$$\Sigma_G^I = E_{\varphi,\phi,\mathbf{u},\mathbf{v}}\left[\frac{G^2(\varphi,\phi,\mathbf{u},\mathbf{v})}{G^2(\varphi,\varphi,\mathbf{u},\mathbf{v})}\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\mathbf{u}}\int_{\mathbf{v}} f(\mathbf{u})f(\mathbf{v}) \times \left[\frac{G^2(\varphi,\phi,\mathbf{u},\mathbf{v})}{G^2(\varphi,\varphi,\mathbf{u},\mathbf{v})}\right] d\mathbf{u}\,d\mathbf{v}\,d\varphi\,d\phi \tag{31}$$

where $f(\mathbf{u})$ and $f(\mathbf{v})$ represent the probability density function of $\mathbf{u}$ and $\mathbf{v}$. The array gain parameters for the interference from other cells are obtained as

$$\Upsilon_G^O = E_{\varphi,\phi,\mathbf{u},\mathbf{v}}\left[G(\varphi,\phi,\mathbf{u},\mathbf{v})\right] E_{\psi,\mathbf{u},\mathbf{v}}\left[1/G(\psi,\psi,\mathbf{u},\mathbf{v})\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\mathbf{u}}\int_{\mathbf{v}} f(\mathbf{u})f(\mathbf{v})\left[G(\varphi,\phi,\mathbf{u},\mathbf{v})\right] d\mathbf{u}\,d\mathbf{v}\,d\varphi\,d\phi \times \frac{1}{2\pi}\int_0^{2\pi}\int_{\mathbf{u}}\int_{\mathbf{v}} f(\mathbf{u})f(\mathbf{v})\left[\frac{1}{G(\psi,\psi,\mathbf{u},\mathbf{v})}\right] d\mathbf{u}\,d\mathbf{v}\,d\psi \tag{32}$$

and

$$\Sigma_G^O = E_{\varphi,\phi,\mathbf{u},\mathbf{v}}\left[G^2(\varphi,\phi,\mathbf{u},\mathbf{v})\right] E_{\psi,\mathbf{u},\mathbf{v}}\left[1/G^2(\psi,\psi,\mathbf{u},\mathbf{v})\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\mathbf{u}}\int_{\mathbf{v}} f(\mathbf{u})f(\mathbf{v})\left[G^2(\varphi,\phi,\mathbf{u},\mathbf{v})\right] d\mathbf{u}\,d\mathbf{v}\,d\varphi\,d\phi \times \frac{1}{2\pi}\int_0^{2\pi}\int_{\mathbf{u}}\int_{\mathbf{v}} f(\mathbf{u})f(\mathbf{v})\left[\frac{1}{G^2(\psi,\psi,\mathbf{u},\mathbf{v})}\right] d\mathbf{u}\,d\mathbf{v}\,d\,d\psi. \tag{33}$$

C. Spatial Spreads

Spatial spread implies that the simultaneously arrived signals may come from different paths with different arrival angles, and the signal energy is spread in space. Assuming the spread energy follows the same distribution as (22) [21], with the normal distribution limited within $[-0.5\pi+\phi, 0.5\pi+\phi]$ and $\kappa = \mathrm{erf}^{-1}(0.5\pi/\sqrt{2}\Delta)$. Thus, the estimated arrival angle is assumed to follow the same distribution as that of the energy. The expected received energy should be averaged considering both the arrival angle estimations and the spatial spreads. Assuming the mean value of x as $\bar{x}$, the array gain parameters Υ_G^I and Σ_G^I for in-cell interference can be found as

$$\Upsilon_G^I = E_{\hat{\phi},\hat{\varphi},\varphi,\bar{\phi},\bar{\varphi}}\left[\frac{G_r(\varphi,\hat{\phi})}{G_r(\varphi,\hat{\varphi})}\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\hat{\phi}}\int_{\hat{\varphi}} f(\hat{\varphi})f(\hat{\phi}) \times \frac{\int_\varphi f(\varphi)G_r(\varphi,\hat{\phi})d\varphi}{\int_\varphi f(\varphi)G_r(\varphi,\hat{\varphi})d\varphi}\, d\hat{\varphi}\,d\hat{\phi}\,d\bar{\varphi}d\bar{\phi} \tag{34}$$

and

$$\Sigma_G^I = E_{\hat{\phi},\hat{\varphi},\varphi,\bar{\phi},\bar{\varphi}}\left[\frac{G_r^2(\varphi,\hat{\phi})}{G_r^2(\varphi,\hat{\varphi})}\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\hat{\phi}}\int_{\hat{\varphi}} f(\hat{\varphi})f(\hat{\phi}) \times \left(\frac{\int_\varphi f(\varphi)G_r(\varphi,\hat{\phi})d\varphi}{\int_\varphi f(\varphi)G_r(\varphi,\hat{\varphi})d\varphi}\right)^2 d\hat{\varphi}\,d\hat{\phi}\,d\bar{\varphi}\,d\bar{\phi}. \tag{35}$$

The integral with respect to φ indicates the accumulation of the spatially spread energy, and that with respect to $\hat{\phi}$ and $\hat{\varphi}$ considers the DOA estimations. The antenna gain parameters Υ_G^O and Σ_G^O for other-cell interference can be found as

$$\Upsilon_G^O = E_{\hat{\phi},\varphi,\bar{\phi},\bar{\varphi}}\left[G_r(\varphi,\hat{\phi})\right] E_{\hat{\psi},\psi,\bar{\psi}}\left[1/G_r(\psi,\hat{\psi})\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\hat{\phi}}\int_{\varphi} f(\hat{\phi})f(\varphi)G_r(\varphi,\hat{\phi})d\varphi\,d\hat{\phi}\,d\bar{\varphi}\,d\bar{\phi} \times \frac{1}{2\pi}\int_0^{2\pi}\int_{\hat{\psi}} f(\hat{\psi})\frac{1}{\int_\psi f(\psi)G_r(\psi,\hat{\psi})d\psi}\,d\hat{\psi}\,d\bar{\psi} \tag{36}$$

and

$$\Sigma_G^O = E_{\hat{\phi},\varphi,\bar{\phi},\bar{\varphi}}\left[G_r^2(\varphi,\hat{\phi})\right] E_{\hat{\psi},\psi,\bar{\psi}}\left[1/G_r^2(\psi,\hat{\psi})\right] = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\int_{\hat{\phi}}\int_{\varphi} f(\hat{\phi})f(\varphi)G_r^2(\varphi,\hat{\phi})d\varphi\,d\hat{\phi}\,d\bar{\varphi}\,d\bar{\phi} \times \frac{1}{2\pi}\int_0^{2\pi}\int_{\hat{\psi}} f(\hat{\psi})\left(\frac{1}{\int_\psi f(\psi)G_r(\psi,\hat{\psi})d\psi}\right)^2 d\hat{\psi}\,d\bar{\psi}. \tag{37}$$

D. Mutual Coupling

Antenna array mutual coupling has been found to have a significant impact on wireless communications. It can affect the estimation of the arrival angles, which results in the disturbance of the weighting vector in beamforming. Considering thin half-wavelength dipoles, mutual coupling can be characterized by a mutual coupling impedance matrix [20], [29], i.e.,

$$\mathbf{C} = (Z_T + Z_A)(\mathbf{Z} + Z_T\mathbf{I})^{-1} \tag{38}$$

where Z_A is the antenna impedance, Z_T is the terminating impedance, $\mathbf{I}$ denotes the identity matrix, and $\mathbf{Z}$ is the mutual impedance matrix. Assuming that the arrival angles are estimated correctly, the beam pattern is

$$A = \sum_{m=-B}^{B} \exp\{jm\pi\sin\phi\} \sum_{n=-B}^{B} \mathbf{C}_{m,n}\exp\{-j\pi n\sin\varphi\}. \tag{39}$$

Therefore, the normalized beamforming gain is

$$G(\varphi, \phi) = \frac{|AA^*|}{M^2}. \quad (40)$$

The antenna gain parameters Υ_G^I and Σ_G^I for the same cell interference can be found as

$$\Upsilon_G^I = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \frac{G(\varphi, \phi)}{G(\varphi, \varphi)} d\varphi \, d\phi \quad (41)$$

and

$$\Sigma_G^I = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \frac{G^2(\varphi, \phi)}{G^2(\varphi, \varphi)} d\varphi \, d\phi \quad (42)$$

where ϕ and φ are also modeled as uniform random variables in $[0, 2\pi)$. The array gain parameters for other-cell interference are

$$\Upsilon_G^O = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} G(\varphi, \phi) d\varphi \, d\phi \times \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{G(\psi, \psi)} d\psi \quad (43)$$

and

$$\Sigma_G^O = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} G^2(\varphi, \phi) d\varphi \, d\phi \times \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{G^2(\psi, \psi)} d\psi. \quad (44)$$

IV. Performance Evaluation

The performance of a CDMA system with imperfect beamforming is evaluated in terms of user capacity, error probabilities, and outage probabilities. User capacity is referred to as the maximum number of users that a CDMA system could support under a target SNIR. In this paper, a closed-form expression for the user capacity under the impairments is presented. Expressions for the error probability as a function of the number of users per cell, antenna gains $(\Upsilon_G^{I,O}, \Sigma_G^{I,O})$, and the CDMA processing gain are derived, and both the intracell and intercell interferences are taken into consideration in the evaluation. Expressions of the outage probability for a CDMA system with imperfect beamforming are derived considering both the strength-based power control and SIR-based power control.

A. User Capacity

Considering SIR-based power control in CDMA systems, the received SNIR, E_b/I_0, should be no less than a target value, γ_0, to maintain a required transmission quality. Considering receive beamforming, we have [16]

$$\begin{aligned} \frac{E_b}{I_0} &\approx \frac{GS}{\frac{2}{3}\left(\sum_{m=1}^{N-1} SE\left[\frac{G_r(\phi_{0,m}, \hat{\phi}_{0,0})}{G_r(\phi_{0,m}, \hat{\phi}_{0,m})}\right] + I\right) + \eta_0 W} \\ &= \frac{GS}{\frac{2}{3}\left((N-1)S\Upsilon_G^I + I\right) + \eta_0 W} \geq \gamma_0 \end{aligned} \quad (45)$$

and G is the CDMA processing gain, η_0 is the single-sided white noise power spectrum density, and W is the spreading bandwidth. The factor 2/3 in the denominator is due to the assumption of a square chip pulse. The denominator in (45) includes other-cell interference as well as own-cell interference due to other MSs in the reference cell. $\phi_{0,m}$ is the azimuth angle of an interfering $\mathrm{MS}_{0,m}$. $G_r(\phi_{0,m}, \hat{\phi}_{0,0})$ is the receive beamforming gain of $\mathrm{MS}_{0,0}$ to the direction of $\mathrm{MS}_{0,m}$. $\phi_{0,m}$ and $\phi_{0,0}$ are uniformly distributed in $[0, 2\pi)$. Considering $E[I] > 0$ and $\mathrm{Var}(I) > 0$, the user capacity can be derived as [16]

$$N = \left\lfloor \frac{\Upsilon_G^I + 1.5G/\gamma_0}{\Upsilon_G^I + \Upsilon_G^O F_0(\mu, \sigma)} \right\rfloor \quad (46)$$

where $\lfloor x \rfloor$ indicates maximum integer no greater than x.

B. Error Probability

Error probability performance of CDMA systems has been extensively investigated, considering standard Gaussian approximation (SGA), improved Gaussian approximation (IGA), and simplified improved Gaussian approximation (SIGA) [31], [32]. In this subsection, using the simplified beamforming model and interference statistics (Sections II and III), we test the impact of beamforming impairments on the error probabilities. The received signals at a BS from C cells are

$$\begin{aligned} r(t) = &\sum_{n=1}^{NC} A_n \alpha_n y_n d_n(t - \tau_n) \\ &\times c_n(t - \tau_n) \exp\{j(\omega_n + \varsigma_n)\} + n(t) \end{aligned} \quad (47)$$

where A_n is the nth user's signal amplitude, $d_n(t)$ and $c_n(t)$ represent the nth user's binary data stream and spreading sequence, respectively, τ_n indicates that each user has an independent delay due to asynchronous transmissions, ς_n is the carrier phase, $\alpha_n \exp\{j\omega_n\}$ is the complex path gain, and $n(t)$ is the additive white Gaussian noise (AWGN) with single-sided power spectral density η_0. In addition, y_n is also an indicator function given by

$$y_n = \begin{cases} 1, & n \leq N \\ (r_n/r_0)^{\mu/2} 10^{(\xi_0 - \xi_n)/20}, & n > N. \end{cases}$$

The first user is assumed to be the reference user, and when beamforming and power control is considered, the decision variable is obtained by correlating the spreading signal, i.e.,

$$\begin{aligned} r_T = &\sum_{n=1}^{NC} A_n \alpha_n y_n \sqrt{G_r(\varphi_n, \varphi_1)/G_r(\psi_n, \psi_n)} \\ &\times \cos(\omega_n - \omega_1 + \varsigma_n - \varsigma_1) I_n + N_w \end{aligned} \quad (48)$$

where $G_r(\varphi_n, \varphi_1)$ is the receive beam pattern, φ_n represents the nth user's azimuth angle, ψ_n is the angle from the nth user to its own BS, N_w is the noise component with a normal distribution $N(0, \eta_0 T/2)$, and I_n is given by [32]

$$I_n = \int_{\tau_1}^{T+\tau_1} d_n(t - \tau_n) c_n(t - \tau_n) c_1(t - \tau_1) dt$$

where T is the bit period, and T_c represents the chip period. Assume that the spreading sequences of different users are independent random binary sequences. Considering the strength-based power control ($A_n\alpha_n = \sqrt{S}$) and normalizing r_T by $A\alpha_n T_c$, we obtain $r'_T = r_T/(AT_c) = G + I_{\rm ic} + I_{\rm oc} + N_1$, in which $I_{\rm ic}$ is the normalized interference from other users in the same cell and $I_{\rm oc}$ is the cochannel interference from other cells, $N_1 \sim N(0, G^2/(2\gamma_b))$, where γ_b is the bit-energy-to-noise ratio. We have $\mathrm{Var}(I_{\rm ic}) \approx \Upsilon_G^I GN/3$, and

$$\mathrm{Var}(I_{\rm oc}) = \sum_{n=N+1}^{NC} E\left[\frac{y_n^2 G_r(\varphi_n, \varphi_1)}{G_r(\psi_n, \psi_n)}\right] G/3. \quad (49)$$

Similarly, assuming that all interfering MSs are uniformly distributed in a cell and the radius of the hexagonal cell is normalized to unity, $\mathrm{Var}(I_{\rm oc})$ can be obtained by integrating the whole cellular coverage area except the reference cell [14], [16], i.e.,

$$\mathrm{Var}(I_{\rm oc}) = F_0(\mu, \sigma)\Upsilon_G^O NG/3.$$

Inasmuch as $I_{\rm ic}$, $I_{\rm oc}$, and N_1 are independent of each other, the mean and variance of r'_T can be found as

$$E\left[r'_T\right] = G$$

and

$$\mathrm{Var}\left[r'_T\right] = \frac{\Upsilon_G^I (N-1)G}{3} + \frac{\Upsilon_G^O F_0(\mu, \sigma)NG}{3} + \frac{G^2}{2\gamma_b}.$$

We can obtain the error probabilities with Gaussian approximation, i.e.,

$$\begin{aligned} P_e &= Q\left(\frac{G}{\sqrt{\mathrm{Var}\left[r'_T\right]}}\right) \\ &= Q\left(\sqrt{\frac{G}{\frac{\Upsilon_G^I(N-1)}{3} + \frac{\Upsilon_G^O F_0(\mu,\sigma)N}{3} + \frac{G}{2\gamma_b}}}\right). \end{aligned} \quad (50)$$

When the number of users N is small, the method using simplified improved Gaussian approximation can improve the accuracy of the evaluation [31]. Following the derivation of SIGA [31], let $\nu = \sum_{n=2}^{NC} Z_n y_n^2 G_r(\varphi_n, \varphi_1)/G_r(\psi_n, \psi_n)$, where Z_n is a decision statistic as defined in [31], and the error probability can be derived as

$$\begin{aligned} P_e = &\frac{2}{3} Q\left(\frac{G}{\sqrt{\mathrm{Var}\left[r'_T\right]}}\right) + \frac{1}{6} \\ &\times \left[Q\left(\frac{G}{\sqrt{\mathrm{Var}[r'_T] + \sqrt{3}\sigma_\nu}}\right) + Q\left(\frac{G}{\sqrt{\mathrm{Var}\left[r'_T\right] - \sqrt{3}\sigma_\nu}}\right)\right] \end{aligned} \quad (51)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-t^2/2)dt$$

and the variance of ψ is

$$\begin{aligned} \sigma_\nu^2 = &(N-1)\left\{E\left[Z_n^2\right]\Sigma_G^I - E^2[Z_n]\left(\Upsilon_G^I\right)^2\right\} \\ &+ N\left\{E\left[Z_n^2\right]U_0(\mu, \sigma)\Sigma_G^O - E^2[Z_n]\left(\Upsilon_G^O\right)^2 V_0(\mu, \sigma)\right\} \\ &+ \Big\{(G-1)(N-1)(N-2)\left(\Upsilon_G^I\right)^2 \\ &\quad + 2(G-1)N(N-1)\Upsilon_G^I \Upsilon_G^O F_0(\mu, \sigma)\Big\}/36 \end{aligned}$$

where $E[Z_n] = G/3$ and $E[Z_n^2] = (7G^2 + 2G - 2)/40$. Notice that the error probability derived above is for a strength-based power-controlled CDMA system. The error probabilities for strength-based and SIR-based power-controlled CDMA systems are different due to different values of γ_b. In strength-based power-controlled CDMA systems, γ_b is constant, and typically, a very large value as the system is designed to be interference-limited, which is different in an SIR-based power-controlled system. Reverse link capacity of a CDMA system with SIR-based power control can be obtained from (46), which is denoted as N^*. To calculate the BER in an SIR-based power-controlled system, γ_b, when $N \le N^*$, can be found as $\gamma_b = 1.5/(1.5G/\gamma_0 - \Upsilon_G^I(N-1) - N\Upsilon_G^O F_0(\mu, \sigma))$, and the error probability is determined through (50) or (51); if the system loses power control, $N > N^*$, all users transmit with their maximum power under Rayleigh fading, and the error probability in this case can be found following [30].

C. Outage Probability

Let S be the minimum power level satisfying (45). We get the expression of S in terms of I, when SNIR satisfies the minimum requirement γ_0, i.e.,

$$S = \frac{I + 1.5\eta_0 W}{1.5G/\gamma_0 - \Upsilon_G^I(N-1)}. \quad (52)$$

Note that the user capacity N can be found via an iterative method [11], [14], in which there are two concatenated iteration loops. In the inner loop, for a given N value, determine $E[I]$ and $\mathrm{Var}[I]$ using the following steps.

1) Set $E[I]$ and $\mathrm{Var}[I]$ as zeros.
2) Calculate $E[S]$ and $\mathrm{Var}[S]$ from (52).
3) Calculate $E[I]$ and $\mathrm{Var}[I]$ from (17) and (19).
4) Repeat steps 2 and 3 until the differences between old and new values of $E[I]$ and $\mathrm{Var}[I]$ are less than 1% [14].

Using $E[I]$ and $\mathrm{Var}[I]$ obtained above and a specified maximum transmission power limit, an outage probability (the transmission power exceeds the power constraint) [14] can be calculated with a fixed N. If the outage probability does not exceeds a required level, the outer loop increases N by 1 and enters the inter loop. The iteration loops stop when the calculated outage probability exceeds the required level. We then obtain the user capacity as $N - 1$. The outage probability for an SIR-based power-controlled CDMA system considering power constraint S^* is

$$\begin{aligned} P_{\text{out}} &= 1 - \Pr\{0 < S \leq S^*\} \\ &= \Pr\left\{0 < \frac{I + 1.5\eta_0 W}{1.5G/\gamma_0 - \Upsilon_G^I(N-1)} \leq S^*, I \geq 0\right\}. \end{aligned} \tag{53}$$

It can be rewritten as

$$\begin{aligned} P_{\text{out}} &= 1 - \Pr\{0 < S \leq S^*\} \\ &= 1 - \Pr\left\{0 < I \leq S^*\left(1.5G/\gamma_0 - \Upsilon_G^I(N-1)\right) - 1.5\eta_0 W\right\}. \end{aligned} \tag{54}$$

Let $G_{\text{SIR}} = S^*(1.5G/\gamma_0 - \Upsilon_G^I(N-1)) - 1.5\eta_0 W$. Inasmuch as I is assumed to be a Gaussian distribution, we obtain

$$P_{\text{out}} = 1 + \mathrm{Q}\left(\frac{G_{\text{SIR}} - E[I]}{\sqrt{\mathrm{Var}[I]}}\right) - Q\left(-\frac{E[I]}{\sqrt{\mathrm{Var}[I]}}\right). \tag{55}$$

For a strength-based power-controlled CDMA system, S is fixed. The SIR expression (45) is rewritten as

$$I/S \leq \frac{1.5G}{\gamma_0} - (N-1)\Upsilon_G^I - 1.5\eta_0 W/S.$$

Let $G_S = 1.5G/\gamma_0 - (N-1)\Upsilon_G^I - 1.5/\eta$, where η is signal-to-noise ratio (SNR) and $\eta = S/\eta_0 W$. Inasmuch as $I \geq 0$, we obtain the outage probability

$$\begin{aligned} P_{\text{out}} &= 1 - \Pr\{0 \leq (I/S) \leq G_S\} \\ &= 1 + Q\left(\frac{G_S - E[I/S]}{\sqrt{\mathrm{Var}[I/S]}}\right) - Q\left(-\frac{E[I/S]}{\sqrt{\mathrm{Var}[I/S]}}\right). \end{aligned} \tag{56}$$

V. Numeric Results

Throughout this section, we assume the following propagation parameters, path loss exponent $\mu = 4$ and shadowing process standard deviation $\sigma = 8$ dB. The CDMA processing gain G is assumed to be 128, and the required SNIR target γ_0 is 5 dB.

A. User Capacity

Fig. 4 presents the impact of the DOA estimation errors on the reverse link user capacity and illustrates the effect of the number of receive antenna elements. Both uniformly and normally distributed DOA errors are considered. In this figure, we focus on the comparison of the capacity evaluation results using the simple/accurate model (Section II-C) and those based on direct integral computations (7). Good matches of the capacity evaluation results are observed. As shown in the figure, for some cases, the two evaluation methods give identical results. In some other scenarios, they only differ by one or two users in the capacity evaluations. In the evaluations presented in Fig. 4, the DOA errors follow the distributions specified in (22) with a mean ϕ and a standard deviation $0.15B_w$. The normally distributed estimation errors are limited to be within $[-0.45B_w + \phi, 0.45B_w + \phi]$.

Fig. 5 presents the impact of various impairments on the reverse link user capacity, in which δ, Δ, and σ_p represent

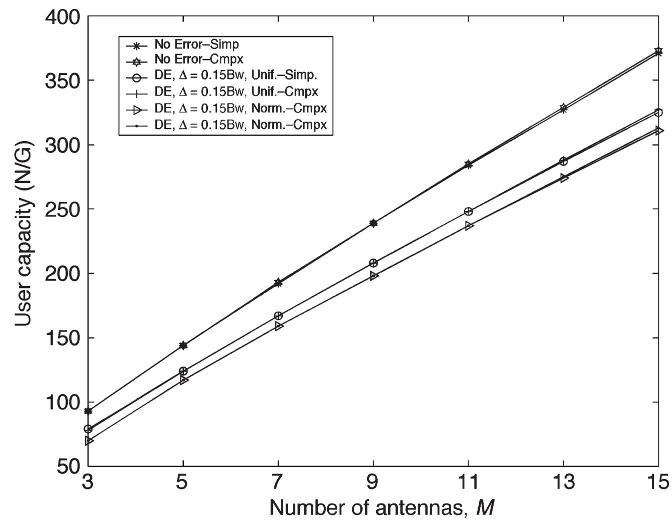

Fig. 4. Accuracy of using simple model to evaluate user capacity, considering DOA estimation errors (DE) with $\Delta = 0.15B_w$ (Cmpx: (7), Simp: using the simplified model).

the standard deviations of the DOA estimation errors, spatial spreads, and perturbation errors, respectively. In considering the DOA estimation errors [Fig. 5(a)], the errors are assumed to follow a normal or uniform distribution with a standard deviation of $0.20B_w$. It is seen that user capacity drops approximately 30% due to the DOA estimation errors. In Fig. 5(b), the impact of array perturbations is considered. The perturbation follows a normal distribution with zero mean and a standard deviation of 0.05λ, 0.1λ, and 0.2λ. The user capacity decrease ranges from approximately 4% ($\sigma_p = 0.05\lambda$), 9% ($\sigma_p = 0.1\lambda$), to 29% ($\sigma_p = 0.2\lambda$). The impact of spatial spreads is examined in Fig. 5(c). The spreads follow a uniform or normal distribution with a standard deviation of 5° and 10°. User capacity decreases are significant. When the number of antenna elements is equal to 9, the capacity drop is as high as 28% when $\Delta = 5^\circ$ and 55% when $\Delta = 10^\circ$ (normally distributed spatial spread). The performance drop of user capacity increases with an increase of the number of antenna elements because of the fact that, whereas the spatial spread is fixed (5° or 10°), the antenna beamwidth reduces when the number of antenna elements becomes larger. This leads to reduced received signal energy and a more significant impact due to DOA estimation errors. Fig. 5(d) exams the impact of mutual coupling, considering a scenario in which the terminating impedance matches the antenna impedance $Z_T = Z_A^*$ ($*$ represents conjugate). It is observed that the user capacity increases slightly with the presence of mutual coupling (3% with $M = 7$ or 9). A similar observation (performance improvement due to mutual coupling) has also been reported for a diversity system [20]. Notice that the impact of DOA estimation errors and spatial spreads is fully taken into account because the real beam pattern is used to characterize the desired signal.

B. Error Probabilities

The error probabilities of a CDMA system with imperfect beamforming due to the impairments are evaluated using (50), and the corresponding numerical results are shown in Fig. 6

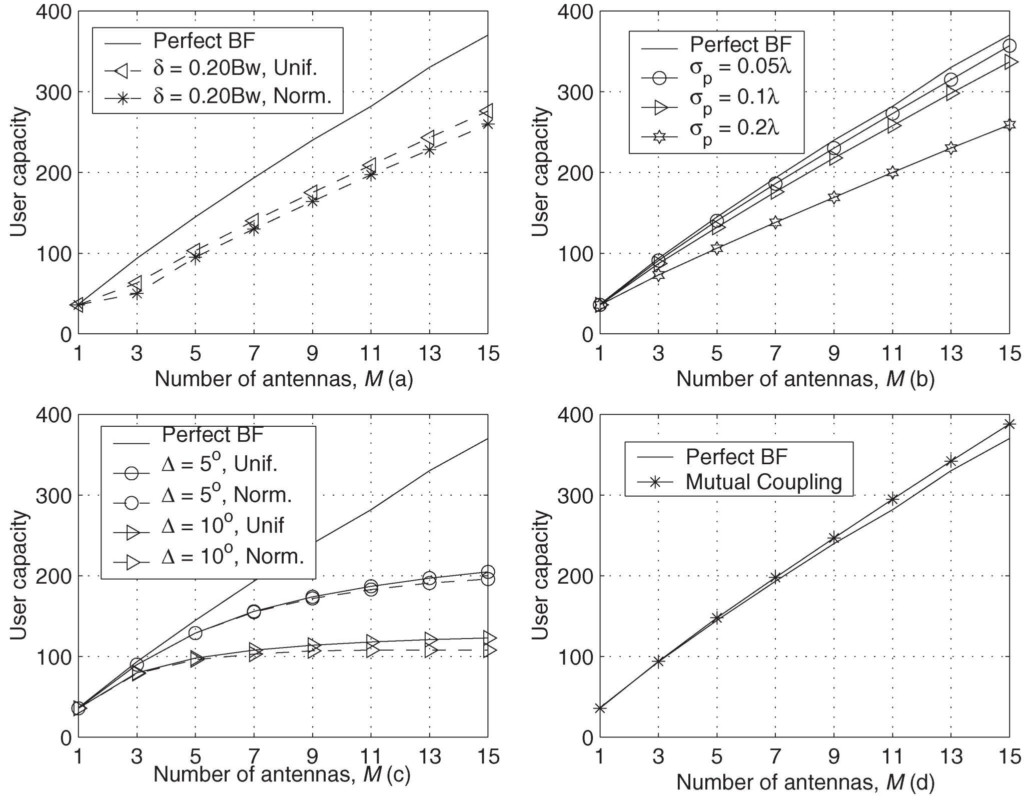

Fig. 5. User capacity of CDMA systems with beamforming under (a) DOA estimation errors, (b) array perturbations, (c) spatial spreads, and (d) mutual coupling.

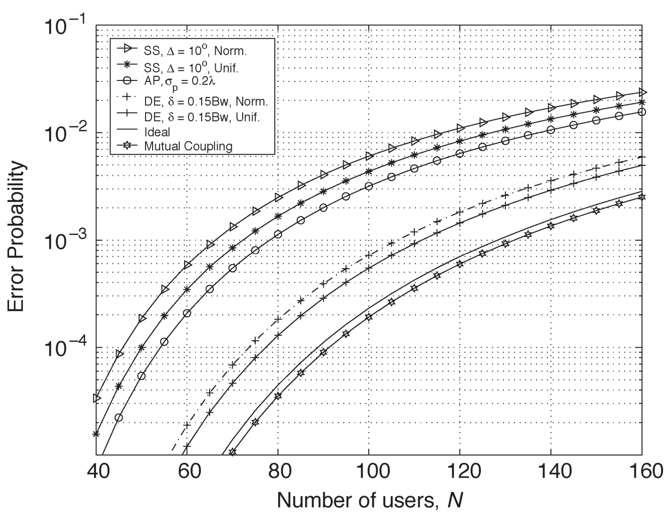

Fig. 6. Error probabilities of CDMA systems with $M = 7$, imperfect beamforming, and strength-based power control (DE: DOA estimation errors, AP: array perturbations, SS: spatial spreads).

with $M = 7$ and $\gamma_b = 30$ dB (an interference-limited environment). Although noticeable error probability increases are seen due to DOA estimation errors, array perturbations, and spatial spreads, the error performance improves slightly in the presence of mutual coupling.

C. Outage Probabilities

Strength-based power control expects a constant received power level regardless of the interference. The total other-cell interference, which is modeled as a random variable, results in a nonzero outage probability even when the CDMA system has a small number of users. However, SIR-based power control tries to maintain SIR above a target value. Thus, if SIR-based power control operates normally in a system, the required power levels are within a reasonable range, and the outage probability is close to 0. When the system loses power control, the system becomes unstable and every user tries to meet the requirement of SIR by transmitting its maximum allowed power, and the outage probability will suddenly jump to a large value. From Fig. 7, we observe that, for a given outage probability, e.g., 0.01, CDMA systems with SIR-based power control have larger capacity than those with strength-based power control, where $S^*/\eta_0 W$ is assumed to be 60 dB in setting a signal power constraint.

VI. Conclusion

The impact of DOA estimation errors, spatial spreads, array perturbations, and mutual coupling on reverse link performance of a multicell CDMA system is investigated in this paper. A simple beamforming model is developed, which is shown to be accurate in performance evaluations. User capacity, error probabilities, and outage probabilities are derived to characterize the CDMA performance. Noticeable performance degradations are observed due to the impairments such as DOA estimation errors, array perturbations, and spatial spreads. However, a minimal performance improvement is seen in the presence of mutual coupling. The performance degradations due to the beamforming impairments are illustrated. Notice that errors

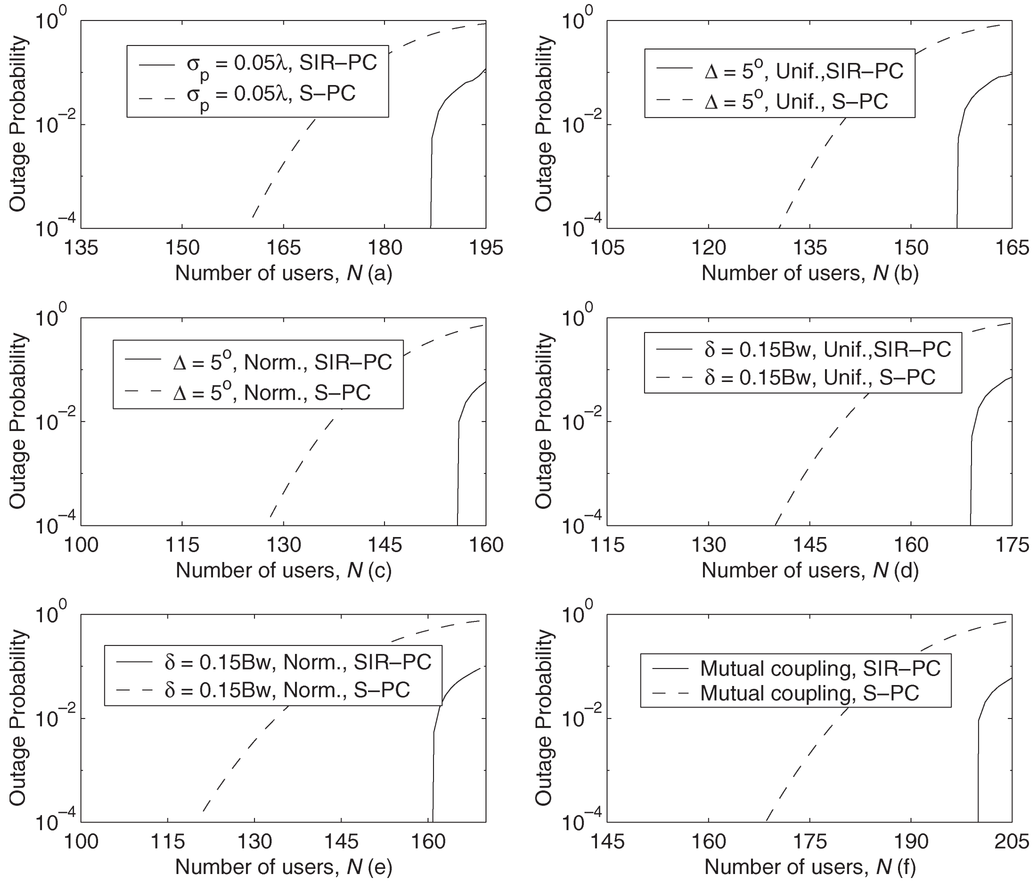

Fig. 7. Outage probabilities of CDMA systems with $M = 7$ and imperfect beamforming under different power control schemes and impairments. (a) Array perturbations. (b) Spatial spreads with uniform distribution. (c) Spatial spreads with normal distribution. (d) DOA errors with uniform distributions. (e) DOA errors with normal distributions. (f) Mutual coupling (SIR-PC: SIR-based power control, S-PC: strength-based power control).

due to array perturbations (antenna mispositioning) can be eliminated or reduced by appropriate calibration of the antenna array. However, the errors due to misestimation of the DOA and due to spatial spreads cannot be reduced through antenna array calibration.

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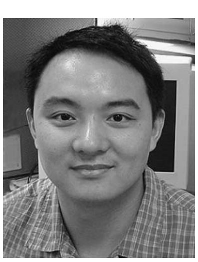

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