Opportunistic Spectrum Access in Cognitive Radio Networks: Global Optimization Using Local Interaction Games

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***Abstract*—We investigate the problem of achieving global optimization for distributed channel selections in cognitive radio networks (CRNs), using game theoretic solutions. To cope with the lack of centralized control and local influences, we propose two special cases of local interaction game to study this problem. The first is local altruistic game, in which each user considers the payoffs of itself as well as its neighbors rather than considering itself only. The second is local congestion game, in which each user minimizes the number of competing neighbors. It is shown that with the proposed games, global optimization is achieved with local information. Specifically, the local altruistic game maximizes the network throughput and the local congestion game minimizes the network collision level. Also, the concurrent spatial adaptive play (C-SAP), which is an extension of the existing spatial adaptive play (SAP), is proposed to achieve the global optimum both autonomously as well as rapidly.**

***Index Terms*—Cognitive radio networks (CRNs), local interaction game, local altruistic game, local congestion game, spatial adaptive play (SAP).**

I. Introduction

OPPORTUNISTIC spectrum access (OSA), which is mainly built on the cognitive radio (CR) technology [1], has been regarded as a promising solution to address the spectrum shortage problem and has drawn great attention recently [2]–[4]. In OSA systems, there are two types of users. One is the primary user which is the licensed owner of the spectrum, and the other is the cognitive user which is allowed to transmit in the licensed spectrum at a particular time and location when and where the primary users are not active [5]. There are two fundamental tasks in OSA systems: spectrum discovery, i.e., detecting the spectrum holes [6], and spectrum utilization, i.e., selecting the best unoccupied channel for transmissions [7]. In this paper, we consider the problem of distributed channel selections in cognitive radio networks (CRNs), where mutual interfere occurs only between neighboring users [8]–[10] instead of all users in the network.

The users in CRNs are generally distributed in different locations; moreover, normally multiple primary users exist, with their interference regions partially but seldom completely overlap. Although the topic of distributed channel selections for CR systems has been widely studied, using, e.g., game theory [11], partially observable Markov decision process [12], optimal stopping rule [13], etc., the following distinctive features of CRNs were rarely considered: 1) only local information of neighbors rather than global information of all other users is readily available, 2) the transmission of a user only interferes with its neighbors rather than with all other users, and 3) the spectrum opportunities are generally heterogeneous, i.e., vary from user to user. Therefore, we need to reinvestigate the problem of distributed channel selections in CRNs with the above considerations.

Notably, the considered CRNs are characterized by a lack of centralized control and the restriction that global information is not available, which requires that the channel selection algorithms should be completely distributed relying on local information. However, as was pointed in [14], it is a challenging task to achieve global optimization for distributed systems where only local information is available. Thus, for a better understanding of the distributed channel selection problem in CRNs, the task of how to achieve global optimization with local information should be first addressed.

The lack of a centralized control and restricted access to global information motivate us to employ local interaction games [15], which have been recently introduced in CRN research known as graphical game in [9], to study this problem. The reason of using game model rather than other decentralized optimization approaches is that game model is a powerful tool to analyze the interactions among autonomous decisions. In a local interaction game, the utility function of a player is only dependent on itself and its neighboring players. This aligns with the nature of local interactions among users in CRNs. Although some progress has been achieved in [9], the problem is not yet

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solved. Specifically, in local interaction games, or in general game models, players are assumed to be selfish, which leads to inefficiency and dilemma. This is referred to as the *tragedy of commons* [16] and is the inherent limitation of game models.

The main contribution in this paper is that we propose two special cases of local interaction games, the *local altruistic game* and the *local congestion game*, to achieve global optimization for CRNs in terms of network throughput maximization and network collision minimization. In the local altruistic game, we do not adhere to the assumption of selfishness of users as in traditional game models; instead, we consider local altruistic behaviors between neighboring users. In order to reduce the communication overhead between neighbors, we then propose a local congestion game, in which each user minimizes the number of competing neighbors. It is analytically shown that with the two games, the global optimization is achieved via just local information exchanges. In comparing the two local games:

- the local altruistic game maximizes network throughput while requiring relatively more information exchange between neighbors;
- the local congestion game minimizes network collision level while requiring less exchange of information.

In addition, we investigate two learning algorithms to achieve global optimization for the proposed games. First, a spatial adaptive play (SAP) [17] based algorithm is developed for the CRN channel selections. Second, a concurrent spatial adaptive play (C-SAP) is proposed, which is implemented in an autonomous manner, to overcome the drawbacks of SAP (e.g., requiring a global coordination mechanism and slow convergence).

The rest of this paper is organized as follows. In Section II, we present related work and highlight the differences between this work with existing work. In Section III, we present the system model and establish the local interaction game framework. In Section IV, we present the local altruistic game model and the local congestion game, and investigate the properties of their Nash equilibrium. In Section V, we propose two learning algorithms that converge to the global optimum with arbitrary high probability. In Section VI, simulation results are presented. Finally, we present discussion in Section VII and make conclusion in Section VIII.

II. Related Work

Game theory [18], as a powerful tool for distributed decision problems where the individual decision mutually influences each other, has been widely applied to CR networks and various versions of game models can be found in the literature [11], [19]–[25]. Most existing work do not consider the spatial aspect of CR networks. That is, they explicitly or implicitly assume that the CR users are located closely and hence the transmission of a user interferes with all other users. Notably, existing solutions can not be applied to CRNs where limited interference ranges and scenarios are considered.

There are approaches to improve the Nash equilibrium (NE) efficiency in the literature, and the current methods mainly include using coordination games [26], pricing [27] and bargaining [28], [29]. However, a main drawback of these methods is that they need global information, which leads to unsustainable communication overhead, especially when the network scale is large. Thus, as stated before, the desired solutions for CRNs should be completely distributed relying on local information.

It should be mentioned that our proposed local altruistic game belongs to noncooperative games. In other words, NE is still central to local altruistic game. Moreover, a game similar to the local congestion game, called spatial congestion game [30], [31], has been proposed recently. Both the local congestion game and spatial congestion game capture the local interactions in CRNs. In [30] and [31], the authors focus on analyzing the existence of NE and investigating the finite improvement properties of best response dynamic, whereas we not only pay attention to the existence of NE but also develop algorithms achieving global optimization with local information.

The problem of resource sharing in distributed networks in which mutual influence occurs only between neighbors (called as spatial resource sharing networks [33]) begins to draw attention recently [8], [9], [30]–[32], and our work differs from existing work in the following key aspects: 1) the spectrum opportunities are heterogeneous (vary for different users), 2) the proposed games are potential games with network throughput or network collision level serving as the potential functions, and 3) global optimization is achieved via just local information exchange between neighbors.

III. System Model and Problem Formulation

A. System Model

We consider a CRN involving N cognitive transmitter–receiver pairs and M licensed channels, $N > M$. For simplicity, we call a cognitive transmitter–receiver pair as a CR link or CR user. The licensed channels are owned by the primary users and can be opportunistically used by the CR users when not occupied by the primary users. That is, the CR users use the licensed channels in an *overlay* manner. Denote the set of the CR users as $\mathcal{N}$, i.e., $\mathcal{N} = \{1, 2, \ldots, N\}$, and the set of the licensed channels as $\mathcal{M}$, i.e., $\mathcal{M} = \{1, \ldots, M\}$. It is assumed that all the channels support the same transmission rate for all users. This represents the case that all channels yield the same bandwidth and same transmission rate to each user, although different users may experience different channel conditions [31]. Note that such an assumption holds in some practical systems, e.g., IEEE 802.16d/e standard [34]. An example of the considered CRN is shown in Fig. 1, which involves four CR links or CR users, two primary users and four licensed channels (1,2,3,4). It is shown in the figure that different primary users occupy different channels and their interference range partially overlap, which leads to heterogeneous spectrum opportunities for the CR users.

First, we characterize the heterogeneous spectrum opportunities by the channel availability vector $\mathbf{C}_n$, for each $n \in \mathcal{N}$. Specifically, $\mathbf{C}_n = \{C_{n1}, C_{n2}, \ldots, C_{nM}\}$, where $C_{nm} = 1$, $m \in \mathcal{M}$, indicates that channel m is available for user n, while $C_{nm} = 0$ means that it is not available. For the CRN example shown in Fig. 1, we have $\mathbf{C}_1 = \{0\ 1\ 0\ 1\}$, $\mathbf{C}_2 = \{0\ 1\ 0\ 0\}$ and $\mathbf{C}_3 = \mathbf{C}_4 = \{1\ 1\ 1\ 0\}$. Moreover, it is assumed that the spectrum opportunities vary slowly in time.

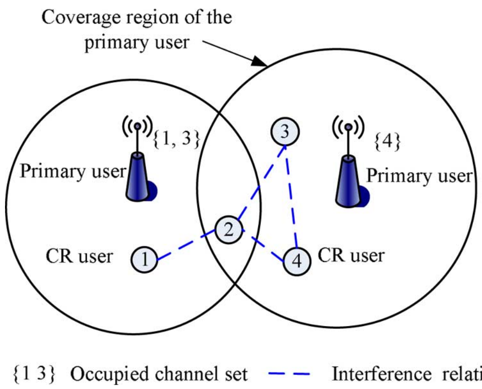

Fig. 1. Example of the considered CRN.

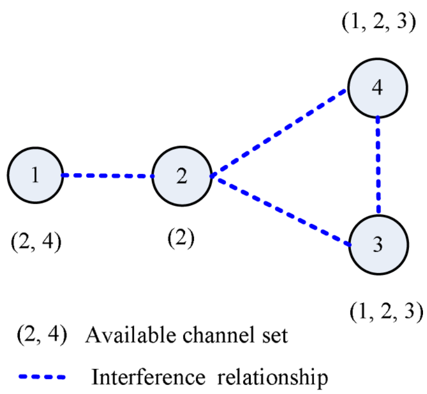

Fig. 2. Corresponding interference graph of the CRN example, where each small circle represents a CR transmitter–receiver pair.

Second, we characterize the limited range of transmission or interference in the CRN using an interference graph. The structure of the interference graph is determined by the distance between users. Specifically, each node on the interference graph represents a CR user. In addition, two CR users, say m and n, are connected by an edge if the distance between them is less than a predefined interference distance D_I. Denote the edge set as $\mathcal{E}, \mathcal{E} \subset N^2$. The connection topology is completely arbitrary (i.e., it might include loops). Furthermore, each user has information only about its local connection topology. Denote J_n as the set of connected (neighboring) users of user n, i.e.,

$$J_n = \{m \in \mathcal{N} : (n,m) \in \mathcal{E}\}. \tag{1}$$

For simplicity, it is assumed that the communication range is equal to the interference range. As a result, neighboring users can exchange information directly and interfere with each other when simultaneously transmitting on the same channel. For the example shown in Fig. 1, the corresponding interference graph is shown in Fig. 2. It is shown that the transmissions of the CR users are location dependent, e.g., User 1 interferes with User 2 when simultaneously transmitting on the same channel, whereas it never interfere with User 3.

B. Problem Formulation

It is assumed that all the CR users can sense all channels, but can only transmit on one channel due to hardware limitation; moreover, the spectrum sensing is assumed to be perfect.[1] We consider a collision channel model, i.e., a collision occurs when two or more neighboring CR users are transmitting simultaneously on the same channel. In this paper, the slotted Aloha transmission mechanism is considered.[2] Specifically, when a CR user decides to use a channel, it transmits with probability p in a slot, while being silent with probability $1-p$.

Let A_n be the available channel set of user n, i.e.,

$$A_n = \{m \in \mathcal{M} : C_{nm} = 1\}. \tag{2}$$

Suppose that user n chooses a channel $a_n \in A_n$ for transmission. Specifically, when there is no idle channel available for user n, which leads its available channel set to be empty, i.e., $A_n = \emptyset$, the user n does not choose any channel, i.e., $a_n = \emptyset$; otherwise, $a_n \neq \emptyset$. Then, for a user n with non-empty action a_n, the individual throughput is given by

$$g_n(a_1, \ldots, a_N) = p \prod_{k \in J_n} (1-p)^{f(a_n, a_k)} \tag{3}$$

where J_n is the neighbor set of n specified by (1), and $f(a_n, a_k)$ is the Kronecker delta function defined as

$$f(a_n, a_k) = \begin{cases} 1, & a_n = a_k \\ 0, & a_n \neq a_k. \end{cases} \tag{4}$$

On the other hand, any user m with empty action $a_m = \emptyset$ has to be silent and hence achieves zero throughput, i.e., $g_m = 0$.

According to (3), the network throughput is given by

$$U_0 = \sum_{n \in \mathcal{N}} g_n = p \sum_{n \in \mathcal{N}} \prod_{k \in J_n} (1-p)^{f(a_n, a_k)}. \tag{5}$$

Then, the first global objective is to find the optimal channel selection profile to maximize the network throughput, i.e.,

$$(P1): \quad \max U_0. \tag{6}$$

Although the current optimization technologies in CR systems are explicitly maximizing the network throughput, there are other alternative methods that implicitly maximize the network throughput, e.g., interference reduction [11], [35]. Motivated by this idea, we consider the problem of opportunistic spectrum access for CRNs from the perspective of network collision minimization.

For a CR user n with non-empty action a_n, the individual collision level is defined as follows:

$$s_n = \sum_{k \in J_n} f(a_n, a_k) \tag{7}$$

where $f(a_m, a_n)$ is the Kronecker delta function specified by (4). That is, the individual collision level of a user is defined as the number of neighboring CR users that compete for the same channel. According to (7), the individual throughput specified

[1]Since spectrum sensing is beyond the scope of this paper, we assume that spectrum sensing is perfect for simplicity of analysis. However, the results presented in this paper can easily be extended to the scenario with imperfect spectrum sensing.

[2]Slotted Aloha is considered for illustrations, and the discussion can easily be extended to other transmission models, e.g., various versions of carrier sensing multiple access (CSMA).

by (3) can be rewritten as $g_n(a_1, \dots, a_N) = p(1-p)^{s_n}$. Thus, lower value of s_n is desirable from the user-side since it leads to higher throughput. Also, lower aggregate collision level experienced by all CR users is more preferable for overall network performance. Accordingly, a quantitative characterization of the network collision level is given as follows:

$$I_0 = \frac{1}{2}\sum_{n\in\mathcal{N}} s_n = \frac{1}{2}\sum_{n\in\mathcal{N}}\sum_{k\in J_n} f(a_n, a_k). \tag{8}$$

In other words, the network collision level I_0 is defined as the total number of competing neighboring pairs which select the same channel.

The motivation behind the individual collision level concept is due to the fact that the individual achievable throughput of user n is a decreasing function of s_n. Moreover, the network collision level I_0 reflects the whole collision level of the network. Specifically, smaller I_0 implies less collision among users, and hence higher network throughput can be obtained. Thus, the second global objective is as follows:

$$(P2): \quad \min I_0. \tag{9}$$

It can be seen that both $P1$ and $P2$ are combinatorial optimization problems and are NP-hard [36]. They can be solved by using exhaustive search in a centralized manner, which obtains the optimal solutions but has untractable complexity. Heuristic methods are alternative approaches, but they can not obtain the optimal solutions. Thus, a distributed approach with low complexity that can obtain the optimal solutions is desirable for CRNs.

Remark 1: The channel selection profile that minimizes the network collision level can also achieve higher network throughput. In other words, there is an inherent connection between problems $P1$ and $P2$. However, exact characterization of this connection is difficult to obtain, due to its dependence on the network technology.

IV. Local Interaction Game Framework for Opportunistic Spectrum Access

Since there is no centralized controller available, the channel selections are self-determined by the CR users. This motivates us to formulate the problem of distributed channel selection in CRNs as a game. Moreover, in order to capture the local interactions among users, local interaction game is applied. However, we do not restrict ourselves to the topic of existence of NE as in most existing work; instead, we focus on achieving global optimization. First, we define action graph in the following, which is central to the local interaction game.

Definition 1 (Action Graph): We call $G_a = (\mathcal{N}, \mathcal{A}, \mathcal{E})$ an action graph where:

- $\mathcal{N}$ is the set of nodes.
- For each node $n \in \mathcal{N}$, let $a_n \in A_n$ be an action of node n, where A_n is the set of its available actions. Then, a pure strategy profile is a n-tuple $a = (a_1, \dots, a_n)$ and the set of action profiles is $\mathcal{A} = \bigotimes A_n, \forall n \in \mathcal{N}$, where $\bigotimes$ is the Cartesian product.
- $\mathcal{E}$ is the set of edges. We say node m is a neighbor of n if they are connected by an edge, i.e., $(m, n) \in \mathcal{E}$.

For notational convenience, we define $\mathcal{A}_{J_n} \equiv \bigotimes A_n, \forall n \in J_n$ as a set of action profiles of node n's neighbors, and an element of $\mathcal{A}_{J_n}$ as a_{J_n}. Also, we define $\mathcal{A}_{-n} \equiv \bigotimes A_n, \forall n \in \mathcal{N}\backslash n$ as a set of action profiles of all the nodes except n, and an element of $\mathcal{A}_{-n}$ as a_{-n}.

Based on the action graph, the local interaction game can be defined in the following.

Definition 2 (Local Interaction Game): A local interaction game is a tuple $\Gamma = (G_a, U)$ where:

- G_a is an action graph, in which each node corresponds to a player.
- U is the set of utility functions for the players. Specifically, the utility function of each player $n \in \mathcal{N}$ is given by $U_n(a_n, a_{J_n}) : a_n \otimes a_{J_n} \mapsto \Re$.

That is, the local interaction game is a kind of game, in which the utility of a player is only dependent on the action profile of its neighbors and itself. We now define Nash equilibrium (NE) in the following, which is the steady state of a noncooperative game.

Definition 3 (Nash Equilibrium): An action profile $a^* = (a_1^*, \dots, a_N^*)$ is a pure strategy NE if and only if no player can improve its utility by deviating unilaterally, i.e.,

$$U_n(a_n^*, a_{J_n}^*) \geq U_n(a_n, a_{J_n}^*) \qquad \forall n \in \mathcal{N}, \quad \forall a_n \in A_n, a_n \neq a_n^*. \tag{10}$$

Although a local interaction game fits the nature of local interactions in CRNs well, the efficiency of NE needs to be carefully considered. It is known that the efficiency of NE is mainly dependent on utility function. For a given utility function, generally multiple NEs exist; and in some cases, even the optimal NE of the game can not lead to the optimal network utilities, e.g., the maximum network throughput or minimum network collision level, not to mention other suboptimal NE points. Thus, in order to achieve global optimization, the following two points should be considered.

1) Design the utility function carefully, such that it guarantees the existence of NE and the optimal NE points coincide with the optimal solutions for network throughput maximization problem P_1 and network collision minimization problem P_2 respectively.
2) Develop efficient learning algorithms that can achieve the optimal NE but only require local information.

In the following, we propose two special cases of local interaction games to solve the distributed channel selection problem in CRNs. One is a local altruistic game which maximizes the network throughput, and the other is a local congestion game which minimizes the network collision level.

A. Local Altruistic Game for Network Throughput Maximization

1) Utility Function: In traditional game models, players always act selfishly. Then, a straightforward approach is to define the individual achievable throughput as the utility function for each player [9], [19], [24], [30], [31]. However, such approaches are selfish and can not guarantee to obtain the global optimization. To improve the efficiency of the games, we consider local altruistic behaviors among neighboring users, which is motivated by local cooperation in biographical systems [37], [38]. Specifically, we define the utility function as follows:

$$U1_n(a_n, a_{J_n}) = g_n(a_n, a_{J_n}) + \sum_{k \in J_n} g_k(a_k, a_{J_k}) \quad (11)$$

where $g_k(a_k, a_{J_k})$ is the individual achievable throughput of player k, $k \in \mathcal{N}$, as defined in (3). Note that the above defined utility function consists of two parts: the individual throughput of player n and the aggregate throughput of its neighbors. In other words, when a player makes a decision, it not only considers itself but also considers its neighbors. Then, the local altruistic game is expressed as follows:

$$(\mathcal{G}1): \max_{a_n \in A_n} U1_n(a_n, a_{J_n}) \quad \forall n \in \mathcal{N} \quad (12)$$

where A_n is the action set (i.e., the available channel set) of player n specified by (2).

Here, we present the utility functions of the players on the interference graph given in Fig. 2. Specifically, user 1 maximizes the aggregate throughput of users 1 and 2, user 2 maximizes the aggregate throughput of users 1, 2, 3, and 4, while both users 3 and 4 maximize the aggregate throughput of users 2, 3, and 4.

2) Analysis of NE: The properties of the proposed local altruistic game is characterized by the following theorem.

Theorem 1: $\mathcal{G}1$ is an exact potential game which has at least one pure strategy NE, and the optimal solution of the network throughput maximization problem $P1$ constitutes a pure strategy NE of $\mathcal{G}1$.

Proof: We construct the potential function as follows:

$$\Phi1(a_n, a_{-n}) = \sum_{n \in \mathcal{N}} g_n(a_n, a_{J_n}) \quad (13)$$

where $g_n(a_n, a_{J_n}) \equiv g_n(a_1, \ldots, a_n)$ is the individual achievable throughput of player n. Moreover, it is seen that the potential function is equal to the network throughput U_0 as defined in (5).

Suppose that an arbitrary player n unilaterally changes its channel selection from a_n to $\bar{a}_n$, then the change in individual utility function caused by this unilateral change is given by

$$U1_n(\bar{a}_n, a_{J_n}) - U1_n(a_n, a_{J_n}) = \left\{ g_n(\bar{a}_n, a_{J_n}) - g_n(a_n, a_{J_n}) + \sum_{i \in J_n} \left(g_i(a_i, \bar{a}_{J_i}) - g_i(a_i, a_{J_i})\right) \right\}. \quad (14)$$

On the other hand, the change in the potential function caused by this unilateral change is given by

$$\begin{aligned} &\Phi1(\bar{a}_n, a_{-n}) - \Phi1(a_n, a_{-n}) \\ &= \Big\{ g_n(\bar{a}_n, a_{J_n}) - g_n(a_n, a_{J_n}) \\ &\quad + \sum_{k \in J_n} \left(g_k(a_k, \bar{a}_{J_k}) - g_k(a_k, a_{J_k})\right) \\ &\quad + \sum_{k \in \{\mathcal{N} \backslash J_n\}, k \neq n} \left(g_k(a_k, \bar{a}_{J_k}) - g_k(a_k, a_{J_k})\right) \Big\} \quad (15) \end{aligned}$$

where $g_k(a_k, \bar{a}_{J_k})$ denotes the achievable throughput of player k after unilaterally changing the selection of player n, and $A \backslash B$ means that B is excluded from A. Since player n's action only affects the payoffs of its neighbors, then the following equation is known:

$$g_k(a_k, \bar{a}_{J_k}) - g_k(a_k, a_{J_k}) = 0, \quad \forall k \in \{\mathcal{N} \backslash J_n\}, k \neq n. \quad (16)$$

Then, from (14)–(16), we have the following equation:

$$\Phi1(\bar{a}_n, a_{-n}) - \Phi1(a_n, a_{-n}) = U1_n(\bar{a}_n, a_{J_n}) - U1_n(a_n, a_{J_n}). \quad (17)$$

It is seen from (17) that the change in individual utility function caused by any player's unilateral deviation is the same as the change in the potential function. Thus, according to the definition given in [39], it is known that $\mathcal{G}_1$ is an exact potential game with network throughput U_0 serving as the potential function.

Exact potential game belongs to potential games, which have been widely applied to wireless communication systems. Potential game exhibits several nice properties and the most important two are as follows.

- Every potential game has at least one pure strategy NE.
- Any global or local maxima of the potential function constitutes a pure strategy NE.

Based on the above properties, Theorem 1 is proved. ■

B. Local Congestion Game for Network Collision Minimization

1) Utility Function: In this subsection, we propose another local interaction game, from the perspective of minimizing network collision level. It is seen from (3) that the individual achievable throughput of a CR user is a decreasing function of the number of competing neighbors, $s_n = \sum_{k \in J_n} f(a_n, a_k)$. Therefore, minimizing s_n is equivalent to maximizing the individual throughput. This motivates us to define the utility function as follows:

$$U2_n(a_n, a_{J_n}) = -\sum_{k \in J_n} f(a_n, a_k) \quad (18)$$

where J_n is the neighboring user set of player n, and $f(a_n, a_k)$ is the Kronecker delta function specified by (4).

Then, the local congestion game is expressed as follows:

$$(\mathcal{G}2): \max_{a_n \in A_n} U2_n(a_n, a_{J_n}) \quad \forall \in \mathcal{N} \quad (19)$$

where A_n is the action set (i.e., the available channel set) of player n specified by (2).

The above defined utility function is motivated by the underlying idea of congestion games [41], in which the utility function is defined as a function of the number of the players who select the same action. Congestion game has been proved to be potential game, and has been well studied. However, note that the utility function of the proposed local congestion game is only dependent on its neighbors, whereas that of congestion game is dependent on all other players. This differentiates the proposed local congestion game from the traditional congestion game. That is, the local congestion game is not pure congestion game and actually a new game. Therefore, we need to reinvestigate its properties.

2) Analysis of NE: The property of the local congestion game is characterized by the following theorem.

Theorem 2: $\mathcal{G}2$ is an exact potential game which has at least one pure strategy NE, and the optimal solution of the network collision minimization problem $P2$ constitutes a pure strategy NE of $\mathcal{G}2$.

Proof: We construct the potential function as follows:

$$\Phi 2(a_n, a_{-n}) = -\frac{1}{2}\sum_{n\in\mathcal{N}}\sum_{k\in J_n} f(a_n, a_k) \tag{20}$$

which is exactly the negative value of the network collision level defined in (8).

First, let us define $I_n(a_n, a_{J_n})$ as the interference neighbor set which also selects the same channel with player n, i.e.,

$$\mathcal{I}_n(a_n, a_{J_n}) = \{k \in J_n : a_k = a_n\} \tag{21}$$

where J_n is the neighbor set of player n. Then, according to (4) and (7), we have

$$s_n = \sum_{k\in J_n} f(a_n, a_k) = |\mathcal{I}_n(a_n, a_{J_n})| \tag{22}$$

where $|A|$ is the cardinality of set A.

Now, suppose that an arbitrary player n unilaterally changes its channel selection from a_n to a_n^*, then the change in individual utility function caused by this unilateral change is given by

$$U2_n(a_n^*, a_{J_n}) - U2_n(a_n, a_{J_n}) = |I_n(a_n, a_{J_n})| - |I_n(a_n^*, a_{J_n})|. \tag{23}$$

On the other hand, the change in the potential function caused by this unilateral change is given by

$$\begin{aligned}
&\Phi 2(a_n^*, a_{-n}) - \Phi 2(a_n, a_{-n})\\
&= \frac{1}{2}\Bigg\{ |I_n(a_n, a_{J_n})| - |I_n(a_n^*, a_{J_n})|\\
&\quad + \sum_{k\in I_n(a_n, a_{J_n})} \left[|I_k(a_k, a_{J_k})| - |I_k(a_k, a_{J_k}^*)|\right]\\
&\quad + \sum_{k\in I_n(a_n^*, a_{J_n})} \left[|I_k(a_k, a_{J_k})| - |I_k(a_k, a_{J_k}^*)|\right]\\
&\quad + \sum_{k\in\mathcal{K}, k\neq n} \left[|I_k(a_k, a_{J_k})| - |I_k(a_k^*, a_{J_k})|\right]\Bigg\}
\end{aligned} \tag{24}$$

where $-|I_k(a_k, a_{J_k}^*)|$ denotes the utility function of player k after unilaterally changing the selection of player n and $\mathcal{K} = \mathcal{N}\backslash\{I_n(a_n, a_{J_n}) \cup I_n(a_n^*, a_{J_n})\}$, and $A\backslash B$ means that B is excluded from A. Since player n's action only affects the payoffs of its neighboring players, then the following equations are easily known:

$$|I_k(a_k, a_{J_k})| - |I_k(a_k, a_{J_k}^*)| = 1 \quad \forall k \in I_n(a_n, a_{J_n}) \tag{25}$$

$$|I_k(a_k, a_{J_k})| - |I_k(a_k, a_{J_k}^*)| = -1 \quad \forall k \in I_n(a_n^*, a_{J_n}) \tag{26}$$

$$|I_k(a_k, a_{J_k})| - |I_k(a_k, a_{J_k}^*)| = 0, \quad \forall k \in \mathcal{K}, k \neq n. \tag{27}$$

Based on –(27), we have

$$\Phi 2(a_n^*, a_{-n}) - \Phi 2(a_n, a_{-n}) = |I_n(a_n, a_{J_n})| - |I_n(a_n^*, a_{J_n})|. \tag{28}$$

From (23) and (28), it is seen that the change in individual utility function caused by any player's unilateral deviation is the same as the change in the potential function, i.e.,

$$\Phi 2(a_n^*, a_{-n}) - \Phi 2(a_n, a_{-n}) = U2_n(a_n^*, a_{J_n}) - U2_n(a_n, a_{J_n}). \tag{29}$$

Then, according to the definition of potential game presented in the last subsection, it is known that $\mathcal{G}_2$ is also an exact potential game with the negative value of the network collision level, $-I_0$, serving as the potential function. Thus, Theorem 2 follows. ■

Remark 2: It is noted from (20) that the potential function of the local congestion game is different form that of the traditional congestion game. Most importantly, the potential function of the local congestion game reflects the network collision level, which does not hold in the traditional congestion game.

C. Discussion of the Two Potential Games

With the proposed utility functions, we now have two potential games for the problem of distributed channel selection in CRN, i.e., the local altruistic game and the local congestion game. The most interesting feature of potential games is that if exactly one player is scheduled to change its strategy using best response dynamic in each iteration, then it always makes the potential function increase and finally converges to a NE in finite iterations. This is referred to as *finite improvement property (FIP)* [39].

Another important property of the proposed games is that the potential functions have physical meanings. Specifically, the potential function of the local altruistic game is equal to the network throughput, while that of the local congestion game is equal to the network collision level. Therefore, the optimal channel selection profiles that maximize the network throughput or minimize the network collision level can be achieved by finding the optimal NE points of the games.

According to Theorems 1 and 2, if there is an algorithm that can achieve the optimal NE points of the two games, the global optimum (i.e., network throughput maximization or network collision minimization) is achieved through distributed implementation. However, normally multiple NE points exist in $\mathcal{G}_1(\mathcal{G}_2)$, and most of them are suboptimal [18]. Thus, although there are a large number of learning algorithms available in the literature to achieve pure strategy NE of potential games, e.g., best response dynamic [39], [40], no-regret learning [11], [20] and fictitious play [42], [43], a main drawback is that they may converge to some suboptimal NE points. Thus, we seek for the learning algorithms that achieve the optimal NE points in the following.

V. Achieving Global Optimization With Local Information

The tasks of identifying and finding the optimal NE points of games are different, and the latter is generally much harder than the former [18]. In this section, we propose two learning algorithms to achieve the optimal NE points of the games. The first is based on an existing algorithm, called *spatial adaptive play (SAP)*, which is originally designed for investigating the stochastic stability of social networks [17]. However, SAP is not suitable for CRNs since it implicitly needs a global controller and its convergence speed is slow. We then propose the second improved learning algorithm, called *concurrent spatial adaptive play (C-SAP)*, which overcomes the drawbacks of SAP and is suitable for the CRNs.

A. Spatial Adaptive Play (SAP)

It is shown that in potential games, spatial adaptive play (SAP) which is a learning algorithm [17], [44], converges to a pure NE that maximizes the potential function with arbitrarily higher probability. To characterize SAP, we extend the game to a mixed strategy form. Let the mixed strategy for player n at iteration k be denoted by probability distribution $q_n(k) \in \Delta(A_n)$, where $\Delta(A_n)$ denotes the set of probability distributions over action set A_n. In SAP, exactly one player is randomly selected to update its selection according to the mixed strategy while all other players repeat their selections. This process is repeated until some stop criterion is met (see Algorithm 1).

Algorithm 1: Spatial adaptive play (SAP)

Step 1) Initially, set $k=0$ and let each CR user $n \in \mathcal{N}$ randomly select an available channel $a_n(0)$ from its available channel set A_n with equal probability.

Step 2) All the CR users exchange information with their neighbors.

Step 3) A CR users is randomly selected, say i.

Step 4) All other CR users repeat their selections, i.e., $a_{-i}(k+1) = a_{-i}(k)$. Meanwhile, with the information received from the neighbors, user i calculates the utility functions over its all available actions, i.e., $U_i(\bar{a}_i, a_{J_i}(k)), \forall \bar{a}_i \in A_i$. Then, it randomly chooses an action according to the mixed strategy $q_n(k+1) \in \Delta(A_i)$, where the a_nth component $q_i^{a_i}(k+1)$ of the mixed strategy is given as

$$q_i^{a_i}(k+1) = \frac{\exp\{\beta U_i(a_i, a_{J_i}(k))\}}{\sum_{\bar{a}_i \in A_i} \exp\{\beta U_i(\bar{a}_i, a_{J_i}(k))} \tag{30}$$

for some learning parameter $\beta > 0$. The utility function $U_i(a_i, a_{J_i}$ in the above equation for local altruistic game is $U1_i(a_i, a_{J_i})$ which is specified by (11), and that of local congestion game is $U2_i(a_i, a_{J_i})$ which is specified by (18).

Step 5) If the predefined maximum number of iteration steps is reached, stop; else go to Step 2.

Note that in Step 2 of SAP, different games require different exchange information. Specifically, in local altruistic game, the neighbors exchange the current channel selection $a_n(k)$ and the current individual achievable throughput $g_n(k)$; on the other hand, it only requires to exchange the current channel selection $a_n(k)$ in local congestion game.

Note that in Step 5 of SAP, the stop criterion is dependent on specific applications. For example, the following can also serve as the stop criterion:

1) If $\forall n \in \mathcal{N}$, the individual throughput g_n remains unchanged for a certain number of iterations.
2) For $\forall n \in \{k : A_k \neq \emptyset\}$, there exists a component of $p_n(k)$ which is sufficiently approaching one, say 0.99.

Denote the set of available selection profiles of all the CR users as $\mathcal{A}$, i.e., $\mathcal{A} = A_1 \otimes \cdots \otimes A_N$, then the asymptotic behavior of SAP is determined by the following theorems.

Theorem 3: In a potential game in which all players adhere to SAP, the unique stationary distribution $\mu(a) \in \Delta(\mathcal{A})$ of the joint action profiles, $\forall \beta > 0$, is given as

$$\mu(a) = \frac{\exp\{\beta\Phi(a)\}}{\sum_{s\in\mathcal{A}} \exp\{\beta\Phi(s)\}} \tag{31}$$

where $\Phi(\,)$ is the potential function of the games specified by (13) and (20), respectively.

Proof: The following proof follows the proof for Theorem 6.2 in [17].

First, let us denote the network state at the kth iteration by $a(k) = \{a_1(k), \ldots, a_N(k)\}$, where $a_n(k)$ is the action of player n. Note that a channel selection profile corresponds to a network state. Notably, $a(k)$ is a discrete time Markov process, which is irreducible and aperiodic. Therefore, it has an unique stationary distribution.

Second, we show that the unique stationary distribution must be (31) by verifying that the distribution (31) deduces to the balanced equations of the Markov process. Denote any two arbitrary network state by a and b, $a, b \in \mathcal{A}$, and the transmission probability from a to b by $P_{ab} = \Pr[a(k+1) = b | a(k) = a]$. Since only exactly one node is selected to update its selection in the proposed algorithm, there is at most one element that can be changed in the network states between any two successive iterations. Thus, there are only two nontrivial cases: 1) a and b differ by exactly one element, or 2) $a = b$.

Now, let us study the transition probability P_{ab}. Specifically, suppose that a and b differ by the ith element. Since node i has probability $1/N$ of being chosen in any given iteration, by omitting the iteration index k, it follows that

$$\mu(a)P_{ab} = \left(\frac{1}{N}\right)\left(\frac{\exp\{\beta\Phi(a)\}}{\sum_{s\in\mathcal{A}}\exp\{\beta\Phi(s)\}}\right) \times \left(\frac{\exp\{\beta U_i(b_i, b_{J_i}(k))\}}{\sum_{\bar{b}_i \in A_i} \exp\{\beta U_i(\bar{b}_i, b_{J_i}(k))}\right) \tag{32}$$

where $U_i(b_i, b_{J_i}(k))$ is the utility function of node i under the network state b, and the last item represents P_{ab}.

Letting

$$\begin{aligned}\lambda &= \frac{1/N}{\left(\sum_{s\in\mathcal{A}}\exp\{\beta\Phi(s)\}\right)\left(\sum_{\bar{b}_i\in A_i}\exp\{\beta U_i(\bar{b}_i, b_{J_i}(k))\right)} \\ &= \frac{1/N}{\left(\sum_{s\in\mathcal{A}}\exp\{\beta\Phi(s)\}\right)\left(\sum_{\bar{a}_i\in A_i}\exp\{\beta U_i(\bar{a}_i, a_{J_i}(k))\right)}\end{aligned} \tag{33}$$

where we use the fact that the network states a and b differ by the ith element, i.e., $a_{-i}(k) \equiv b_{-i}(k)$ and $a_{J_i}(k) \equiv b_{J_i}(k)$, we obtain

$$\mu(a)P_{ab} = \lambda \exp\{\beta\Phi(a) + U_i(b)\beta\}. \quad (34)$$

Due to symmetry, we also have

$$\mu(b)P_{ba} = \lambda \exp\{\beta\Phi(b) + U_i(a)\beta\}. \quad (35)$$

Considering that $\Phi(a) - \Phi(b) = U_i(a) - U_i(b)$, as specified by (17) and (29), respectively, (34) and (35) immediately yield the following balanced equation:

$$\mu(a)P_{ab} = \mu(b)P_{ba}. \quad (36)$$

The above analysis is for the case that a and b differ by exactly one element. In addition, (36) naturally holds when $a = b$.

Thus, we have

$$\sum_{a\in\mathcal{A}} \mu(a)P_{ab} = \sum_{a\in\mathcal{A}} \mu(b)P_{ba} = \mu(b)\sum_{a\in\mathcal{A}} P_{ba} = \mu(b) \quad (37)$$

which is exactly the balanced stationary equation of the Markov process. Then, we conclude that its stationary distribution must be (31) since SAP has unique distribution and the distribution (31) satisfies the balanced equations of its Markov process. Therefore, Theorem 3 is proved. ■

Theorem 4: With a sufficiently large β, SAP achieves the global optimum of problems P1 or P2 with an arbitrarily high probability.

Proof: For the two proposed games, the number of the optimal action profiles may be unique or multiple; moreover, this number is hard to obtain [18]. Even so, we will prove that SAP achieves the maximum potential function values of the games with an arbitrarily high probability. Specifically, the situation can be categorized as follows.

1) **Case I**: There is an unique optimal action profile that maximizes the potential function, which is denoted by a_{opt}, i.e.,

$$a_{\text{opt}} = \arg\max_{\bar{a}\in\mathcal{A}} \Phi(\bar{a}). \quad (38)$$

Then, the maximum potential function value is determined by $\Phi_{\max} = \Phi(a_{\text{opt}})$. When β becomes sufficiently large, i.e., $\beta \to \infty$, the following inequality holds:

$$\exp(\beta\Phi(a_{\text{opt}})) \gg \exp(\beta\Phi(\bar{a})), \quad \forall \bar{a} \in \mathcal{A}\backslash a_{\text{opt}} \quad (39)$$

Then, based on (31) and (39), the following can be obtained:

$$\lim_{\beta\to\infty} \mu(a_{\text{opt}}) = 1 \quad (40)$$

which substantiates that SAP converges to a_{opt} in probability. In other words, SAP achieves $\Phi_{\max}$ with arbitrarily high probability in this case.

2) **Case II**: There are multiple optimal action profiles indexed by $a_{\text{opt}1}, \ldots, a_{\text{opt}K}$, $K > 1$. Then, the maximum potential function value is determined by $\Phi_{\max} = \Phi(a_{\text{opt}1}) = \cdots = \Phi(a_{\text{opt}K})$. Also, based on (31) and (39), we have:

$$\lim_{\beta\to\infty} \sum_{k=1}^{K} \mu(a_{\text{opt}k}) = 1 \quad (41)$$

and

$$\lim_{\beta\to\infty} \mu(a_{\text{opt}k}) = \frac{1}{K}, \quad \forall k = 1, \ldots, K \quad (42)$$

That is, when β becomes large, the aggregate stationary distribution over all the optimal channel selection profiles asymptotically approaches one, and the stationary distribution over each profile are equal. In this case, SAP may oscillate around pairs of different optimal channel selection profiles but never converge. However, no matter whether the action profile converges or not, we conclude that it finally achieves the maximum potential function $\Phi_{\max}$ with arbitrarily high probability. The reason is that each profile $a_{\text{opt}k}$, $\forall k = 1, \ldots, K$, leads to the maximum potential function $\Phi_{\max}$ and the aggregate stationary distribution over all optimal profiles are one.

To summarize, the action profile converges in Cases I, while may oscillate in Cases II. Even so, it achieves the maximum potential function values for Cases I and II with arbitrarily high probability. Then, according to the property that the potential functions of the games coincide with the network utilities, i.e., the network throughput or the network collision level, Theorem 4 follows. ■

Theorem 4 validates the optimality of SAP. It is a desired learning algorithm because the optimal solutions for the network throughput maximization problem $P1$ and the network collision minimization problem $P2$ are achieved via just local information exchange between neighbors.

It should be pointed that the update of SAP, as specified by (30), was also used in reinforcement learning (RL) [45]. In fact, the update rule is referred to as *Boltzmann exploration strategy* [45], where actions with higher utilities have a greater chance of being selected than those with lower utilities. Such a random action selection provides with an opportunity to escape from local optimal points and finally achieves the global optimum.

This rule has been used for single-agent and multi-agent RL technologies extensively, and a comprehensive review can be found in [46]. Recently, an active research topic is to investigate the asymptotic behaviors by incorporating RL technologies with game theory [46]–[48]. The new knowledge in this paper is that the formulated games are local interaction games and potential games, which differentiates our work from previous work.

The learning parameter β in Step 4 of SAP balances the tradeoff between exploration and exploitation. Smaller β implies that the CR users are more willing to choose an suboptimal action to explore, whereas higher β implies that they are prone to choose the best response action. Specifically, $\beta = 0$ means that CR user i will select any action $a_i \in A_i$ with equal probability, while $\beta \to \infty$ means that it will select an action from its best response set, i.e., $a_i(k+1) \in \arg_{a_i\in A_i} \max U_i(a_i, a_{J_i}(k))$. Therefore, it is advisable that at the beginning phase, the value of β is set to small number and it keeps increasing as the learning algorithm iterates [49].

For practical implementation, the empirical frequency of the channel selection profile $a(k)$ asymptotically converges to the stationary distribution $\mu(a)$ given by (31), as the iteration number goes sufficiently large. As a result, the algorithm asymptotically converges to a global optimum as the iteration number goes sufficiently large, but may converge to a global or local optimum in finite iterations. Thus, there is a tradeoff between convergence iteration and optimality, and the selection of iteration number should be application-dependent in practice.

B. Concurrent Spatial Adaptive Play (C-SAP)

Although SAP achieves the global optimum, there are still two drawbacks: 1) it needs a global coordination mechanism to guarantee that only one player is scheduled to update its action in each iteration, e.g., random token mechanism given in [49], and 2) the convergence speed is slow since there is only one player updating its action in each iteration.

To overcome the above drawbacks, we seek for a method with which multiple players are selected in an autonomous fashion in each iteration, and then they concurrently update their actions. Specifically, by exploiting the nature of limited interference range in CRNs, we propose the concurrent spatial adaptive play (C-SAP). The procedure of C-SAP is similar to that of SAP, except that there are multiple CR users selected to update their actions in each iteration. Steps 3 and 4 in Algorithm 2 differentiate C-SAP from SAP.

Algorithm 2: Concurrent spatial adaptive play (C-SAP)

Step 1) Initially, set $k = 0$ and let each CR user $n \in \mathcal{N}$ randomly select an available channel $a_n(0)$ from its available channel set A_n with equal probability.

Step 2) All the CR users exchange information with their neighbors.

Step 3) Randomly select multiple CR users provided that they are not neighboring with each other.

Step 4) Each CR user, selected in Step 3, randomly chooses an action from its available channel set according to the mixed strategy specified by (30).

In Step 3 of C-SAP, multiple CR users are selected in an autonomous fashion rather than using a global coordination mechanism as in SAP. To achieve this, we assume that there is a common control channel (CCC) available and an 802.11 DCF-like contention mechanism can be applied. Specifically, each CR user $n \in \mathcal{N}$ executes the following steps:

1) Generate a backoff time τ_n according to uniform distribution in the interval $[0, \tau_{\max}]$ for some fixed parameter $\tau_{\max}$.
2) Upon expiry of the backoff timer, monitor the CCC and then send a updating request-to-send (URTS) message indicating that it is about to update its channel selection.
3) On hearing the URTS message, all neighboring CR users freeze their backoff timers and keep silent until the next iteration.

The comparison of SAP and C-SAP is shown in Fig. 3, and an illustration of the idea of the two learning algorithms is shown in Fig. 4. For detailed analysis of the above proposed autonomous mechanism, refer to [11] where a similar 802.11 DCF-based user selection scheme is proposed. The difference is as follows: in [11], exactly only one CR user is selected to update its channel selection at each iteration, while in this work, multiple non-neighboring CR users are selected.

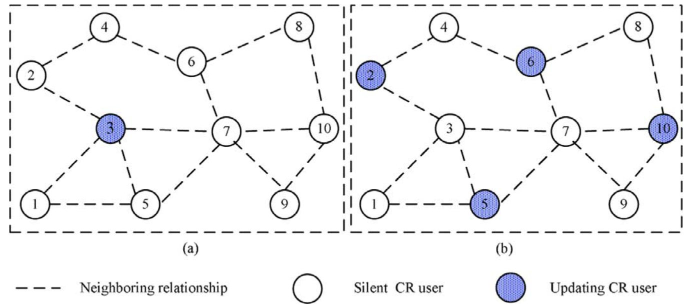

Fig. 3. Comparison of SAP and C-SAP. (a) Spatial adaptive play. (b) Concurrent spatial adaptive play.

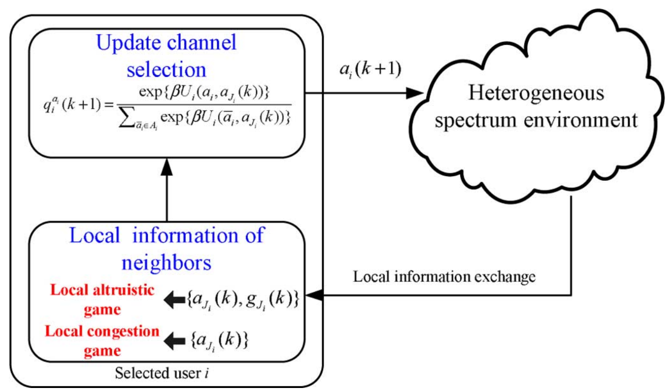

Fig. 4. Illustration of the two learning algorithms.

In C-SAP, the selected CR users in each iteration do not affect each other since they are not neighbors. Therefore, it can be viewed as the concurrent version of SAP. Hence, the unique stationary distribution $\mu(a) \in \Delta(\mathcal{A})$ of the joint action profile is also determined by (31). Then, according to Theorem 4, C-SAP also converges to the global optimum with arbitrary high probability. To summarize, the proposed C-SAP overcomes the drawbacks of SAP, and enjoys the following attractive features:

- it does not need global coordination mechanism; instead, it is implemented in an autonomous fashion;
- it rapidly converges to the global optimum since there are multiple non-neighboring CR users concurrently updating their channel selections.

For the above advantages, C-SAP is suitable in CRNs, and can be regarded as a significant improvement of SAP.

Remark 3: The most important characteristic of the proposed learning algorithms, including SAP and C-SAP, is that they achieve the global optimum of the local altruistic game (or the local congestion game) with arbitrary high probability via just local information exchange between neighbors. Here, arbitrarily high probability means that the convergence probability sufficiently approaches one. For example, suppose that there are multiple NE points in the local altruistic game, and they lead to the following network throughput, $U = \{8\ 8.5\ 8.5\ 9\}$.

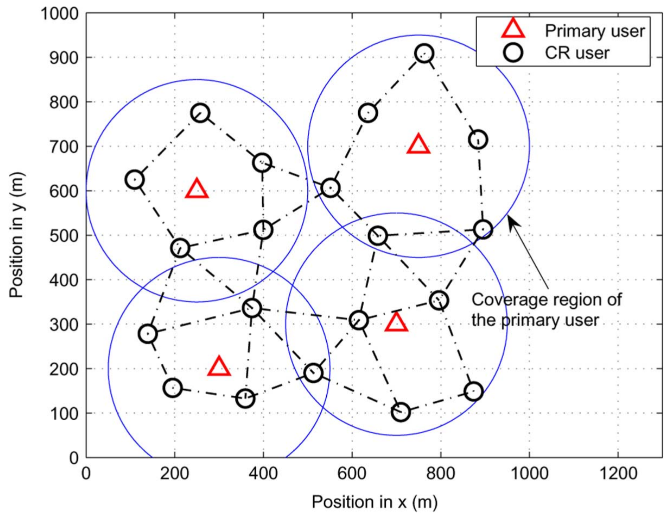

Fig. 5. Interference graph for a small CRN with four primary users and 20 CR users (the interference range is set to $D_I = 250$ m).

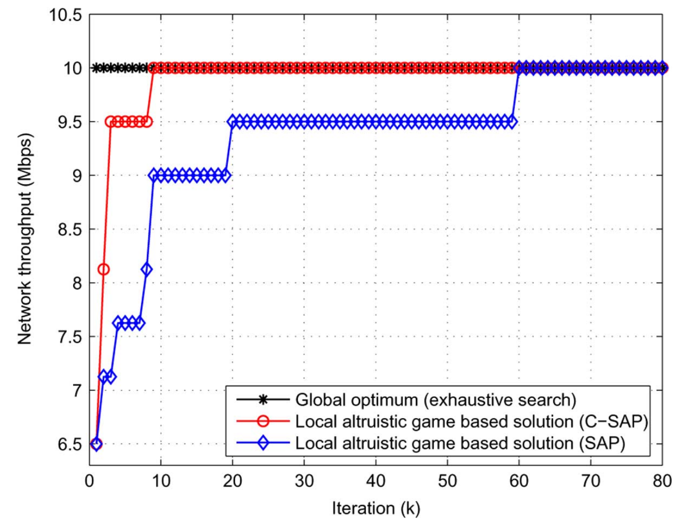

Fig. 6. Convergence behavior of the local altruistic game for an arbitrary spectrum opportunities (20 CR users).

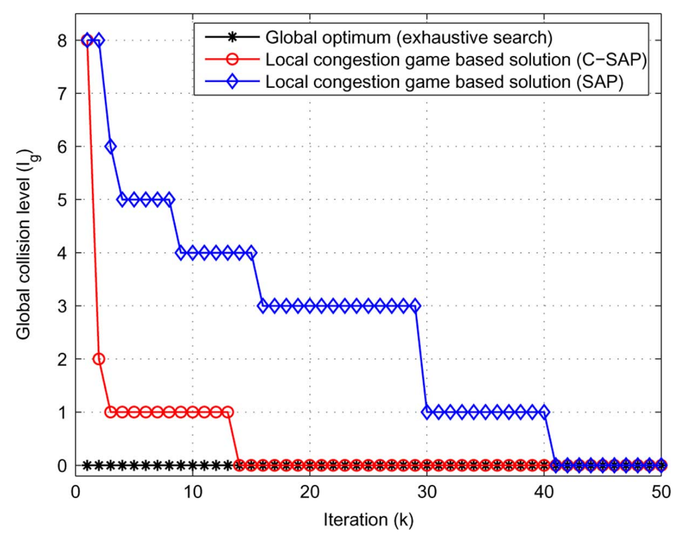

Fig. 7. Convergence behavior of the local congestion game for an arbitrary spectrum opportunities (20 CR users).

Denote P_c as the probability that the learning algorithms converge to the global maximum throughput $U_0 = \max\{U\} = 9$. Then, according to Theorem 3, it is known that $P_c = 0.9867$ for $\beta = 10$. Moreover, when the value of β increases, P_c sufficiently approaches one, e.g., $P_c = 0.9999$ for $\beta = 20$ and $P_c = 1 - 6.1 \times 10^{-7}$ for $\beta = 30$.

VI. Simulation Results

In the following simulation study, the CR users are randomly located in a square region. A CR user m interferes with n if the distance between them is less than a predefined interference range, D_I. Moreover, there are multiple licensed channels and multiple primary users. It is assumed that the licensed channels are independently occupied by the primary users with the same probability θ, $0 < \theta < 1$; and the spectrum opportunities are randomly generated according to the occupied probability. However, note that the spectrum opportunities vary slowly in time, or are at least static during the convergence. Moreover, all the channels are assumed to have the same transmission rate, $R = 1$ Mbps. To balance the tradeoff between exploration and exploitation, we choose $\beta = k$ in our simulation study, where k is the iteration step.

A. Convergence of the Learning Algorithms

1) Example of Small Networks: We consider a small CRN consisting of four primary users and 20 CR users, as shown in Fig. 5. The interference distance is set to $D_I = 250$. It is assumed that there are three licensed channels. Then, we have $\mathcal{N} = \{1, 2, \ldots, 20\}$ and $\mathcal{M} = \{1, 2, 3\}$. It can be seen that in this scenario, the maximum number of possible channel selection profiles is about 3×10^9.

For an arbitrary realization of the heterogeneous spectrum opportunities, the convergence behavior of the local altruistic game is shown in Fig. 6, in which the global optimum is obtained by using the exhaustive search method. For the local altruistic game based solution, both the C-SAP and SAP are applied. It is noted from the figure that C-SAP catches up with the global optimum and so does SAP. Moreover, it is also noted that C-SAP converges to the global optimum faster than SAP. The reason is that there are multiple non-neighboring CR users concurrently changing their channel selections in each iteration, as stated before.

Also, for the same realization of the heterogeneous spectrum opportunities, the convergence behavior of the local congestion game is shown in Fig. 7. It is noted that both the C-SAP and SAP converge to the global optimum, and the convergence speed of C-SAP is faster than that of SAP. The results presented in Figs. 6 and 7 validate the optimality of the proposed local altruistic game and local congestion game, in terms of maximizing network throughput or minimizing network collision level.

2) Example of Large Networks: It is seen that in the proposed game-based solutions, there is no need to collect information at a central controller and the global optimums are achieved via just local information exchange between neighbors. Therefore, the autonomous behavior and decentralized implementation make them suitable in large scale CRNs.

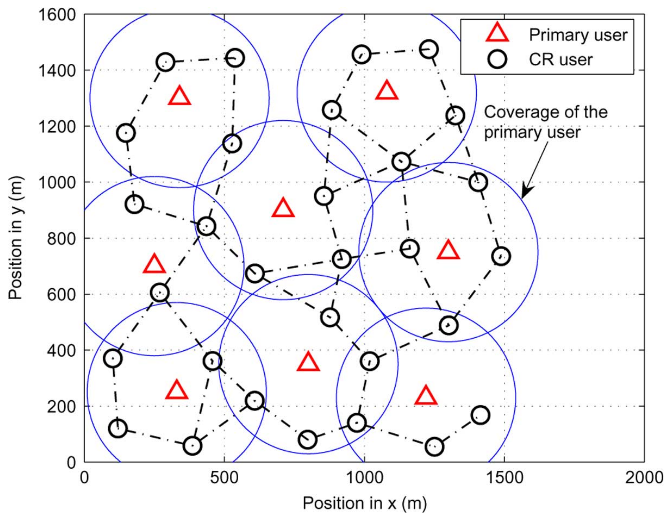

Fig. 8. Interference graph for a relative large CRN with eight primary users and 30 CR users (the interference range is set to $D_I = 250$ m).

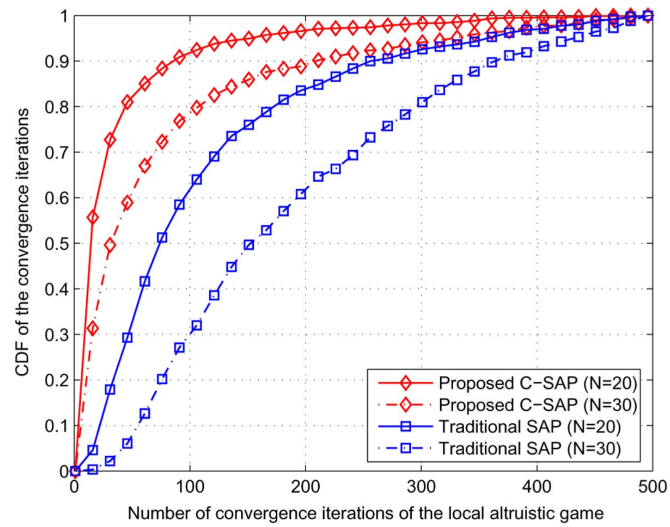

Fig. 9. Convergence speed of the local altruistic game for different network scales.

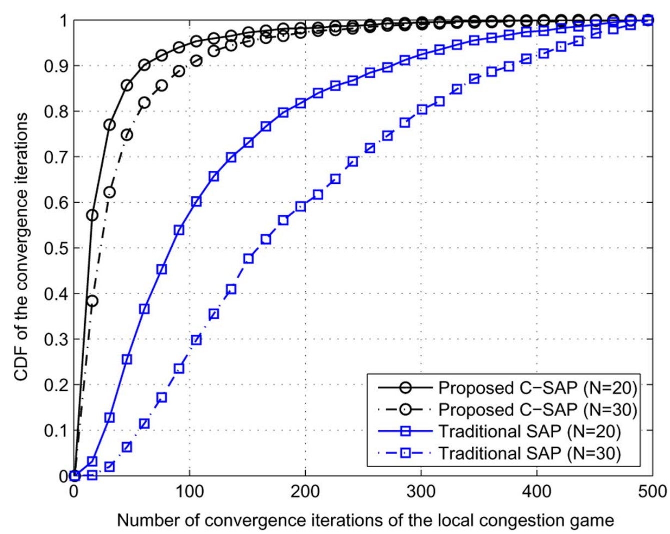

Fig. 10. Convergence speed of the local congestion game for different network scales.

We consider a relatively large CRN consisting of eight primary users and 30 CR users, as shown in Fig. 8. The interference range is also set to $D_I = 250$. It is also assumed that there are three licensed channels. Then, we have $\mathcal{N} = \{1, 2, \ldots, 30\}$ and $\mathcal{M} = \{1, 2, 3\}$. Note that in such a network, the maximum number of possible channel selection profiles is about 2×10^{14}. Simulation results show that using C-SAP or SAP, both local altruistic game and local congestion game converge to the global optimums for any spectrum opportunity and initial channel selection.

For both local altruistic game and local congestion game, the iterations needed to converge to the global optimums are random variables, which is inherently determined by the stochastic nature of the learning algorithms. Thus, we compare the convergence speeds towards the global optimum of C-SAP and SAP, from a statistical perspective. Specifically, the cumulative distribution function (cdf) of the iterations needed to converge to the global optimum of the local altruistic game is shown in Fig. 9, and that of the local congestion game is shown in Fig. 10. It is noted from the figures that for a given network scale (e.g., $N = 20$), the convergence speed of C-SAP is faster than that of the SAP as expected. Moreover, when the network scales up, the convergence speed of C-SAP decreases slightly, whereas that of SAP decreases significantly. The results show the advantage of C-SAP over SAP in CRNs.

B. Throughput Performance

It is noted that the neighboring relationship is determined by the interference range D_I. Specifically, larger D_I implies that a user will have more neighbors. As a result, the number of competing neighbors increases accordingly. In this subsection, we study the impact of the interference range D_I on the throughput performance of the proposed game theoretic solutions.

In this simulation study, we consider the CRN consisting of 20 CR users and four primary users. Moreover, there are three licensed channels. The C-SAP learning algorithm is applied to achieve the optimal NE. The maximum number of iterations is set to 200 and the results are obtained by simulating 10^4 trials for different spectrum opportunities and different initial channel selection profiles.

1) Scenario of Small Interfering Range: In the first scenario, the interference range is set to $D_I = 250$ m. The deployment of the simulated CRN is as shown in Fig. 5. The corresponding expected network throughput when varying the access probability, p, is shown in Fig. 11.

It is noted from the figure that the expected achievable network throughput increases almost linearly with p. The reason is that small interference range means that the CR users are located sparsely. Then, there are sufficient spectrum opportunities available, and hence neighboring users are spread over different channels. Therefore, the collision between neighbors becomes trivial, and the number of competing neighbors can be ignored. As a result, the expected achievable throughput of a user can be approximated by $g_n = \theta R p$ and the network throughput by $U_0 = N \theta R p$. Thus, the results in the figure follow. Also, it is

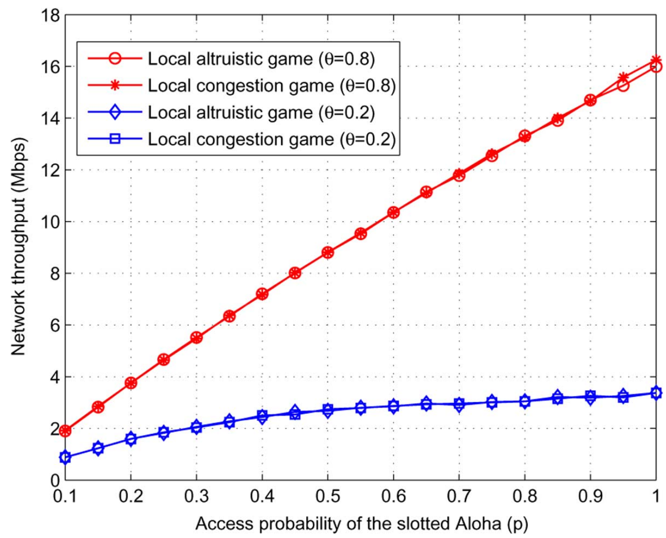

Fig. 11. Expected network throughput for small interference range ($D_I = 250$ m).

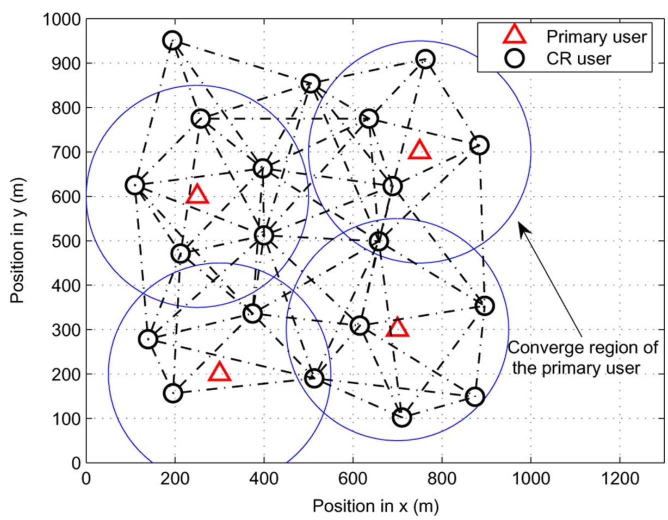

Fig. 12. Interference graph for a CRN with large interference range ($D_I = 400$ m).

noted from the figure that larger channel idle probability θ leads to higher network throughput as expected.

Moreover, it is noted from the figure that the obtained expected network throughput of the two game theoretic solutions is trivial. The reason is as follows: the optimal NE solutions for the games are that neighbors are spread over different channels, since there are sufficient spectrum opportunities available. Then, the optimal solutions for network throughput maximization $P1$ and network collision minimization problem $P2$ are the same in most cases. In other words, the connection between problem $P1$ and problem $P2$ is strong.

2) Scenario of Large Interference Range: In the second scenario, we consider a relatively large interference range. Specifically, the interference range is set to $D_I = 400$ m. The deployment of the simulated CRN is shown in Fig. 12. The corresponding expected network throughput when varying the access probability, p, is as shown in Fig. 13.

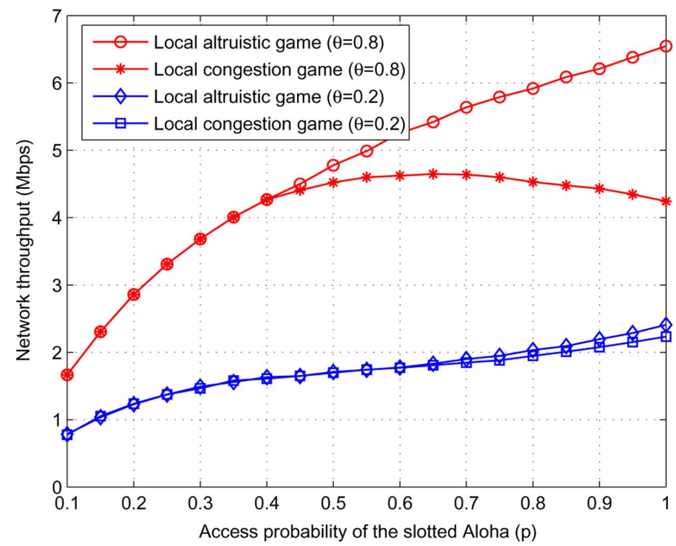

Fig. 13. Expected network throughput for large interference range ($D_I = 400$ m).

For the larger channel idle probability, i.e., $\theta = 0.8$, it is noted that the achievable throughput for both local altruistic game and local congestion game increases nonlinearly with p. The reason is that in this case, the spectrum opportunities become limited, and then the number of competing neighbors could not be ignored anymore. In particular, it is noted that the achievable network throughput for the local altruistic game keeps increasing when p increases. On the other hand, there is a peak in the achievable expected network throughput for the local congestion game (i.e., $p_{\max} \approx 0.65$) as can be expected in any Aloha transmission mechanism.

It is also noted from the figure that when the access probability is less than a value, i.e., $p \le 0.4$, the obtained network throughput of local congestion game is close to that of local altruistic game. However, as the access probability increases, i.e., $p > 0.4$, there is an increasing throughput gap. The reason is as follows: $(1-p)^{s_n}$ decreases significantly when p increases, which makes the connection between the network collision minimization problem $P2$ and the network throughput maximization problem $P1$ weak. In other words, there exists an channel selection profile that minimizes the network collision whereas it does not maximize the network throughput.

For a smaller channel idle probability, i.e., $\theta = 0.2$, it is noted from the figure that the achievable network throughput for local altruistic game is slightly greater than that of the local congestion game.

3) Result Analysis: From the above simulation results, it is seen that both two proposed game theoretic solutions are desirable in CRNs. Specifically:

- The local altruistic game always maximizes network throughput, for any interference range and any access probability p. Also, it should be mentioned that it needs to exchange relatively more information between neighbors.
- The local congestion game requires less exchange information and minimizes the network collision. For a network with small interference range, it maximizes the network

throughput as the local altruistic game. When the interference range increases, it still maximizes network throughput for a less access probability p and achieves higher network throughput for larger p.

Therefore, there is a tradeoff between network throughput and communication overhead, and the selection of a game model should be application-dependent in practice.

VII. Discussion

The most important characteristics of the proposed games are that they achieve global optimization with local information. The key idea is that the game models are carefully designed such that the utility functions are properly aligned with the global objective. We believe that the results presented in this paper provide a good understanding for distributed decision problems, and can be applied to other current research topics in CRNs, e.g., decentralized cooperative spectrum sensing and resource allocation. Also, it is noted that further studies are needed to address additional practical considerations.

1) This paper assumes perfect detection of primary users. However, interference from a secondary user to a primary suer due to misdetection is a significant issue in a practical cognitive radio network. We will investigate in the near future the impact of the secondary user interference (including aggregate interference from multiple secondary users).
2) In addition, various channel scenarios and parameters need to be examined to evaluate the impact of different communications and interferences ranges. Specifically, the channel transmission rate is not only associated with each channel, but also determined by the channel conditions. Moreover, the interference range in practical systems is generally larger than the communication range, which makes direct information exchange between some interfering users not feasible.
3) The algorithms proposed in this paper are more suitable for static networks rather than time-varying networks, as they admit the asymptotic optimality when the iteration number goes sufficiently large. We will investigate in the near future new algorithms for time-varying networks.

It is seen that in the proposed solutions, there is no need to collect information at a central point, which makes it suitable for large scale networks. However, it still needs to exchange some key information among neighboring users. For example, in the local altruistic game, the exchanged information include the current selected channel and the current individual achievable throughput; in the local congestion game, it only requires the current selected channel. For some resource-limited systems, this may lead to heavy communication overhead. To cope with this problem, the following can be promising methods: 1) new utility functions which require less information exchange between neighbors, and 2) efficient learning algorithms which can converge to the optimal NE more rapidly.

VIII. Conclusion

We proposed two special local interaction games: local altruistic game and local congestion game, to achieve global optimization with local information for distributed channel selections in cognitive radio networks. In the local altruistic game, each CR user considers the payoffs of itself as well as its neighbors rather than itself only as in general game models. In the local congestion game, each CR user minimizes the number of competing neighbors. It is shown that with the two games, global optimization in terms of network throughput maximization and network collision minimization are achieved with local information. Specifically, local altruistic game maximizes network throughput, which, however, needs more information exchanged between neighbors; on the other hand, local congestion game minimizes network collision level and achieves near-optimal throughput (optimal collision minimization), but requires less information exchange. Also, the concurrent spatial adaptive play (C-SAP), which is an extension of the existing spatial adaptive play (SAP), is proposed to achieve the global optimum both autonomously as well as rapidly. Future work will consider more practical system models and reduced communication overhead between neighbors.

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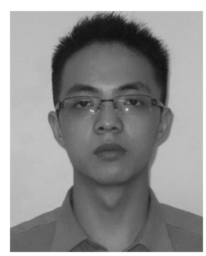

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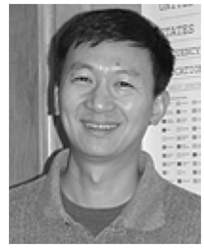

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