A Selective-Relay Based Cooperative Spectrum Sensing Scheme without Dedicated Reporting Channels in Cognitive Radio Networks

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Abstract—Typically, each cooperative spectrum sensing process requires two phases: the primary user’s signal detection phase, in which all cognitive users attempt to detect the presence of the primary user within a certain observation window (called signal detection overhead); and the initial detection result reporting phase, in which the cognitive users forward their detection results to a fusion center. To avoid interfering with the primary user in the reporting phase, previous research assumed that there is a common control channel (also known as dedicated reporting channel) between the cognitive users and fusion center, which, however, requires extra channel resources and introduces an additional complexity due to the dedicated channel resource management. In this paper, we propose a selective-relay based cooperative spectrum sensing scheme, which is able to control and reduce the interference from cognitive reporting users to primary user without the dedicated channel. We analyze the interference impact on the primary user and show that the interference induced by the reporting users is controllable and can be reduced to satisfy a given outage probability requirement of the primary transmissions. In addition, we investigate the receiver operating characteristics (ROC) of the traditional cooperative sensing scheme (with dedicated reporting channel) and the proposed scheme (without dedicated reporting channel) by jointly considering the signal detection and reporting phases. It is proven that, given a target detection probability, a unique optimal signal detection overhead exists to minimize an asymptotic overall false alarm probability in high SNR regions. We illustrate that, compared to the traditional scheme, the selective-relay based cooperative sensing scheme can save the dedicated channel resources without sacrificing ROC performance. Numerical results also show that, under a guaranteed overall detection probability, an overall false alarm probability can be minimized through an optimization of the signal detection overhead.

Index Terms—Cognitive radio, cooperative spectrum sensing, receiver operating characteristics, data fusion.

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I. INTRODUCTION

C OGNITIVE radio (CR), built on software-defined radio, has been proposed as a means to improve the utilization of wireless spectrum resources [1] - [3]. Spectrum sensing is a core technology upon which the entire operation of cognitive radio rests [4], [5]. It enables unlicensed users (also referred to as secondary users or cognitive users) to communicate with each other over licensed bands by detecting spectrum holes [2]. In spectrum sensing, there are three broad categories of signal processing approaches: energy detection [6], matched filter detection [7], and feature detection [8], [9]. As has been discussed in [3], the energy detection can not differentiate signal types, which, however, has the advantage of simple implementation. Although the matched filter is an optimal detector in stationary Gaussian noise scenarios, it requires prior information of the primary user signal. As an alternative, the feature detector can differentiate the modulated signal from the interference and additive noise, which, however, comes at the expense of high computational complexities since it requires an extra training process to extract significant features.

In order to combat wireless fading effects, a collaborative spectrum sensing approach has been proposed in [10], where the detection results from multiple cognitive users are pooled and combined together at a fusion center by using a logic rule. Papers [11] and [12] applied cooperative diversity [13] - [16] to the primary user detection and showed that the sensing time can be reduced greatly through the cooperation between the cognitive users. Paper [17] has proposed a linear cooperative sensing framework based on the combination of local statistics from individual cognitive users. Simulation results have shown that a significant cooperative gain is achieved using the linear cooperation strategy. Furthermore, paper [18] has investigated the soft combination of the observed energies from different cognitive radio users and proposed an optimal soft combination scheme based on the Neyman-Pearson criterion. It is known that each cooperative sensing process requires two essential phases: the phase of primary user’s signal detection by cognitive users and the phase of initial detection result reporting from the cognitive users to the fusion center. Notice that the cognitive users will potentially interfere primary users when transmitting/reporting their initial detection results. To avoid this interference, all the pervious works [11], [12], [17], [18] assumed that there is a dedicated channel (or common control channel) between the cognitive users and fusion center.

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However, this approach requires extra channel resources and introduces an additional complexity due to the dedicated channel resource management.

Typically, the cooperative sensing process consists of the signal detection and the initial detection result reporting phases. However, the existing papers [10], [18] neglect the reporting phase by assuming a perfect transmission of initial spectrum sensing results over a dedicated channel without considering the noise and fading effects. Although the initial sensing results are of a few bits only in an information-theoretic sense, the cognitive users should scan the licensed channel periodically (e.g., in a millisecond scale), which will result in a non-negligible rate of the initial sensing result transmission. In addition, for the cooperative spectrum sensing process, the signal detection and reporting phases can not be designed and optimized in isolation since they could affect each other. For example, a cognitive user may not detect the presence of the primary user within a certain time duration and lead to making an incorrect detection, which may affect the final decision made at the fusion center and degrade the overall sensing performance. While increasing the time duration of the signal detection phase improves the individual detection performance of each cognitive user, it comes at the expense of a reduction in reporting performance since less time is now available for the reporting phase. This may also degrade the overall spectrum sensing performance at the fusion center. As a consequence, how to design and optimize the time durations for the signal detection and reporting phases is an important issue to be addressed.

The main contributions of this paper are described as follows. Firstly, we propose a selective-relay based cooperative spectrum sensing scheme without a dedicated channel, in which each cognitive user forwards/reports its initial detection result in a selective fashion. Secondly, we analyze the interference impact on the primary users and show that the interference induced by the selective-relay based cooperative sensing scheme is controllable and can be reduced to meet an arbitrary primary outage probability requirement. Thirdly, by jointly considering both the signal detection and reporting phases, we investigate the receiver operating characteristics of the traditional (with dedicated channel) and proposed (without dedicated channel) cooperative sensing schemes over Rayleigh fading channels. We show that compared to the traditional scheme, the selective-relay based cooperative sensing scheme can save the dedicated channel resources without sacrificing ROC performance. We also illustrate that under a guaranteed overall detection probability, the overall false alarm probability can be optimized through the allocation of time durations between the signal detection and reporting phases.

The remainder of this paper is organized as follows. Section II presents the system description and signal model for the selective-relay based cooperative sensing scheme, followed by the performance analysis in Section III, where both the interference impact on primary users and the receiver operating characteristics are analyzed for the proposed cooperative sensing scheme over Rayleigh fading channels. In Section IV, numerical evaluations are conducted to show ROC performance of the traditional and proposed cooperative spectrum sensing schemes. This section also illustrates the impact of time allocation between the signal detection and reporting phases on the ROC performance. Finally, we make some concluding remarks in Section V.

II. PROPOSED SELECTIVE-RELAY BASED COOPERATIVE SPECTRUM SENSING SCHEME IN COGNITIVE RADIO NETWORKS

A. System Description

As shown in Fig. 1, during each cooperative spectrum sensing process, there are two essential phases: 1) detection phase, where all cognitive users (CUs) attempt to detect the presence of a primary user (PU); and 2) reporting phase, where each CU relays its initial detection result to the fusion center (FC) such that FC can make a final decision on the presence of PU by using a given fusion rule, such as AND, OR and so on [10]. Throughout this paper, we will consider an AND-based and an OR-based fusion rules to combine all the initial detection results received at FC from CUs.

Fig. 2 depicts a slotted structure of the cooperative spectrum sensing, where the detection and reporting phases occupy $\alpha$ and $1-\alpha$ fractions, respectively, of one time slot, and $\alpha$ is referred to as signal detection overhead that can be varied to optimize the system performance. We assume here that the signal detection overhead is the same for all CUs. In the reporting phase, CUs forward their initial detection results to FC over the orthogonal sub-channels equally divided from the primary licensed channel in time domain, resulting in multiple sub-time slots. Clearly, all CUs will potentially interfere PU in the reporting phase. In order to mitigate this interference as much as possible, we propose a selective-relay based cooperative sensing scheme, where each CU forwards its initial detection result in a selective fashion depending on if the absence of PU is detected or not. Specifically, if a CU detected the absence of PU in its detection phase, it will transmit an indicator signal to FC, which is encoded
by a cyclic redundancy code (CRC); otherwise, nothing is transmitted from the CU to avoid interfering the primary user. If an indicator signal was transmitted and no outage event of the indicator transmission occurred, it is assumed that the CRC checking performed at FU would be successful; otherwise, the CRC checking will fail, implying no indicator signal transmitted. Therefore, if the CRC checking is successful at FC over $i$-th orthogonal sub-channel, FC will consider the absence of PU as the initial result detected by CU$_i$; otherwise, it will consider the presence of PU as the CU$_i$’s initial detection result. Accordingly, in the proposed scheme, a CU will interfere the primary transmissions only if it fails to detect the presence of the primary user when PU is active. As will be shown in Section III-A, this interference can be controlled and reduced.

In addition, if a CU malfunctions (e.g., due to out of battery), it will not sense and transmit an indicator signal to FC. However, FC will assume that the presence of PU is detected by this CU, which will impair the performance of the selective relay based cooperative sensing scheme. To address this issue, we consider that FC may periodically broadcast a request control packet and these CUs, which are able to assist FC sense the licensed spectrum, will transmit an acknowledgement. In this paper, we assume that FC has a perfect knowledge of which CUs will participate in the cooperative sensing process.

### B. Signal Model

In this subsection, we focus on the signal modeling for the proposed cooperative spectrum sensing scheme. Each transmission link between any two nodes as shown in Fig. 1 is modeled as Rayleigh fading and, moreover, the fading is viewed as constant during one whole time slot. The additive white Gaussian noise (AWGN) at all receivers has the same power spectral density $N_0$.

Besides, let $P_p$ and $P_s$ denote the transmit powers of PU and CU, respectively. For notational convenience, let $H_p$ denote whether PU is active or not, namely $H_p = H_1$ represents the presence of PU and $H_p = H_0$ represents its absence. Throughout this paper, we assume that the primary user status (i.e., presence or absence) does not change during one time slot. Note that this assumption is applicable to most of the existing medium access protocols, even for a random access protocol. This is because that many random access protocols are based on a time slot structure (e.g., slotted ALOHA, slotted CSMA, and so on), which are more efficient than the corresponding non-slotted protocols. For such slotted random access protocols, the primary user is present in a slot-by-slot manner.

During the detection phase (i.e., the first phase) of time slot $k$, the signal received at CU$_i$ is expressed as

$$y_i(1) = \sqrt{P_p}h_{pi}\theta(1) + n_1(i), \quad i = 1, 2, \cdots, M$$  

(1)

where the index 1 represents the first phase of time slot $k$, the time slot index $k$ is dropped for the notational convenience, and $M$ stands for the number of CUs. Moreover, $h_{pi}$ is the fading coefficient of the channel from PU to CU$_i$, $n_1(i)$ is an additive white Gaussian noise with zero mean and variance $N_0$, and $\theta(1)$ is defined as

$$\theta(1) = \begin{cases} 0, & H_p = H_0 \\ x_p(1), & H_p = H_1 \end{cases}$$  

(2)

where $x_p(1)$ is the transmit signal of PU in the first phase of time slot $k$. Based on the received signal as given in Eq. (1), each CU decides whether PU is active or not, and the corresponding decision is referred to as an initial detection result as denoted by $\hat{H}_i(1)$. It is noted that, throughout this paper, the energy detector [6], [7], [21] is used to evaluate the spectrum sensing performance. Thus, using an energy detection approach, the initial detection result $\hat{H}_i(1)$ is given by

$$\hat{H}_i(1) = \begin{cases} H_0, & T[y_i(1)] < \lambda_i \\ H_1, & T[y_i(1)] > \lambda_i \end{cases}$$  

(3)

where $T[y_i(1)]$ is the output statistic of the energy detector of CU$_i$ as given by

$$T[y_i(1)] = \frac{1}{N} \sum_{n=1}^{N} |y_i^{(n)}(1)|^2$$  

(4)

where $|y_i^{(n)}(1)|^2$ is the energy of the $n$-th sample of the signal received at CU$_i$, $N = \alpha T f_s$ is the number of samples, $T$ and $f_s$ are the time slot length and sampling frequency, respectively. In the subsequent reporting phase, each CU forwards a signal $\beta_i$ to FC over an orthogonal sub-channel and the corresponding received signal at FC can written as

$$y_c(2) = \sqrt{P_s}h_{ic}\beta_i + \sqrt{P_p}h_{pc}\theta(2) + n_c(2)$$  

(5)

where the index 2 stands for the second phase (i.e, reporting phases), $h_{ic}$ and $h_{pc}$ are, respectively, the fading coefficients of the channel from CU$_i$ to FC and that from PU to FC, and $\beta_i$ and $\theta(2)$ are defined as

$$\beta_i = \begin{cases} x_i, & \hat{H}_i(1) = H_0 \\ 0, & \hat{H}_i(1) = H_1 \end{cases}$$  

(6)

where $x_i$ is an indicator signal that is encoded by a CRC code, and

$$\theta(2) = \begin{cases} 0, & H_p = H_0 \\ x_p(2), & H_p = H_1 \end{cases}$$  

(7)

where $x_p(2)$ is the transmit signal of PU in the second phase of time slot $k$. Hence, from Eq. (5), FC attempts to decode the signal $\beta_i$ and perform CRC checking. As known in [13], [15] and [20], if the channel capacity is below a required data rate, an outage event is said to occur and the decoder fails to recover the original signal no matter what decoding algorithm is adopted. In this case, the CRC checking is assumed to fail and FC will consider that no indicator signal is transmitted from CU$_i$, i.e., the corresponding initial detection result received at FC from CU$_i$ is given by $\hat{H}_i(2) = H_1$; otherwise, $\hat{H}_i(2) = H_0$. Accordingly, we obtain

$$\hat{H}_i(2) = \begin{cases} H_1, & \Theta_{ic}(2) = 1 \\ H_0, & \Theta_{ic}(2) = 0 \end{cases}$$  

(8)

where $\Theta_{ic}(2) = 1$ denotes that an outage event of the initial detection result transmission from CU$_i$ to FC occurs as defined
in Eq. (9), and \( \Theta_{ic}(2) = 0 \) represents the other case. In an information-theoretic sense [13] - [15], [20], the outage event \( \Theta_{ic}(2) = 1 \) can be described from Eq. (5) as
\[
\Theta_{ic}(2) = 1: \frac{(1 - \alpha)}{M} \log_2(1 + \frac{|h_{ic}|^2 \gamma_s \beta_i^2}{|h_{ic}|^2 \gamma_p \theta(2)^2 + 1}) < \frac{1}{BT}
\]
where \( \gamma_s = P_s/N_0, \gamma_p = P_p/N_0, \beta_i \) and \( \theta(2) \) are, respectively, defined in Eq. (6) and Eq. (7), \( B \) and \( T \) are the frequency bandwidth and time duration of time slot \( k \), respectively. In general, we can understand the preceding equation as follows. The spectrum sensing is performed periodically over each time slot, which results in the data rate of initial decision result transmission as \( 1/(BT) \). However, such transmission process is completed during the reporting phase, i.e., \( 1 - \alpha \) fraction of the whole time slot, which implies that only \( 1 - \alpha \) degree of freedom of the channel is occupied by the reporting phase. Therefore, the reporting phase capacity should be scaled by \( 1 - \alpha \). From Eq. (9), one can see that the outage event \( \Theta_{ic}(2) = 1 \) occurs under two scenarios: 1) \( \beta_i = 0 \) when \( H_i(1) = H_1 \), which means that no indicator signal is transmitted from CU; and 2) a relatively small \( |h_{ic}|^2 \) value (i.e., a deep channel fading), which results in the channel capacity from CU to FC to be below a required data rate \( 1/(BT) \). Finally, FC combines all \( H_i(2) \) through a given fusion rule, leading to its final decision, \( H_c \). Throughout this paper, we consider two logic rules, i.e., “AND” and “OR”, to combine initial detection results. Given an “AND” rule, the final decision \( H_c \) can be expressed as
\[
\hat{H}_c = \bigotimes \hat{H}_i(2)
\]
where \( \bigotimes \) represents the logic “AND” operation. Using an “OR” as fusion rule, we can write the final decision as
\[
\hat{H}_c = \bigoplus \hat{H}_i(2)
\]
where \( \bigoplus \) stands for the logic “OR”.

III. PERFORMANCE ANALYSIS OF THE SELECTIVE-RELAY BASED COOPERATIVE SPECTRUM SENSING SCHEME

This section begins with the investigation of the interference impact on the primary user and shows that the interference induced by the proposed selective-relay based cooperative sensing scheme is controllable. Then, we analyze the ROC performance of the traditional and proposed schemes over Rayleigh fading channels.

A. Interference Analysis

Clearly, in the proposed selective-relay based cooperative spectrum sensing scheme, a CU will interfere PU only when it fails to detect the presence of PU given that PU is active. For simplicity in analytical derivations, we assume that, once miss detection happens at a CU in a time slot, it causes interference to PU during the whole slot, which is viewed as an interference upper bound. By constraining this interference to a required level, it will be safer to satisfy a quality-of-service (QoS) requirement of the primary transmissions. Therefore, considering the proposed cooperative sensing scheme, such an upper bound on the interference received at a primary receiver from the cognitive users is given by
\[
I_{upper} = \frac{1}{M} \sum_{i=1}^{M} P_s(1 - P_{d,i})|h_{id}|^2
\]
where the factor \( 1/M \) is due to the fact that each CU occupies \( 1/M \) fraction of the primary licensed channel, \( P_{d,i} \), is the probability of individual detection of the presence of PU at CU, and \( h_{id} \) is the fading coefficient of the channel from CU to the primary destination. Suppose that the primary user is sending data traffic to the primary destination with the transmit power \( P_t \) and date rate \( R_{p} \). Thus, the instantaneous signal-to-interference ratio (SIR) received at the primary destination can be expressed as
\[
SIR = \frac{P_p|h_{pd}|^2}{\frac{1}{M} \sum_{i=1}^{M} P_s(1 - P_{d,i})|h_{id}|^2}
\]
where \( h_{pd} \) is the fading coefficient of the channel from the primary user to primary destination. In interference-limited systems, an outage event of the primary traffic transmission is said to occur when the received SIR falls below a predefined threshold \( SIR_{thr} \). Following [13] - [15], the threshold \( SIR_{thr} \) relates to the data rate of primary transmissions \( R_{p} \) as given by \( SIR_{thr} = 2^{R_{p}} - 1 \). Accordingly, the outage probability of primary transmissions (also called primary outage probability) is given by
\[
P_{out} = Pr(SIR < SIR_{thr})
\]
Note that random variables \( |h_{pd}|^2 \) and \( |h_{id}|^2 \) follow the exponential distributions with parameters \( 1/\sigma^2_{pd} \) and \( 1/\sigma^2_{id} \), respectively. Substituting SIR from Eq. (13) into Eq. (14) and following [19], the primary outage probability can be derived as
\[
P_{out} = 1 - \sum_{i=1}^{M} P_p \sigma^2_{pd} M + P_p \sigma^2_{id}(1 - P_{d,i})SIR_{thr} \prod_{j=1, j \neq i}^{M} \frac{\sigma^2_{id}(1 - P_{d,j}) - \sigma^2_{id}(1 - P_{d,j})}{\sigma^2_{id}(1 - P_{d,j})}
\]
which is valid only for \( \sigma^2_{id}(1 - P_{d,i}) \neq \sigma^2_{id}(1 - P_{d,j}) \) when \( i \neq j \). For the case of \( \sigma^2_{id}(1 - P_{d,i}) = \sigma^2_{id}(1 - P_{d,j}) = \cdot \cdot \cdot = \sigma^2_{id}(1 - P_{d,M,i}) \), Eq. (14) can be derived as
\[
P_{out} = 1 - \frac{P_p \sigma^2_{pd} M}{[P_p \sigma^2_{pd}(1 - P_{d,i})SIR_{thr} + P_p \sigma^2_{pd} M]^M}.
\]
Throughout this paper, in order to satisfy QoS requirement of primary transmissions, the primary outage probability is guaranteed to be below a threshold, \( P_{out,thr} \). Considering the case of \( \sigma^2_{id}(1 - P_{d,i}) = \sigma^2_{id}(1 - P_{d,j}) = \cdot \cdot \cdot = \sigma^2_{id}(1 - P_{d,M,i}) \), the individual detection probability, \( P_{d,i} \), with the constraint of a given primary QoS requirement, \( P_{out,thr} \), is given as follows,
\[
P_{d,i} \geq 1 - \frac{\gamma_p P_t M [1 - (1 - P_{out,thr})]^{1/M}}{\gamma_s \sigma^2_{id}(1 - P_{out,thr})^{1/M} (2^{R_{p}} - 1)}.
\]
In obtaining the preceding equation, we have used \( SIR_{thr} = 2^{R_{p}} - 1 \). Alternatively, given the primary QoS requirement
Poutthr and individual detection probability Pd,i, we can limit the transmit power Pd,i from Eq. (17) as
\[
\gamma_s \leq \frac{\gamma_p \sigma^2_{pd} M [1 - (1 - Poutthr)^{1/M}]}{\sigma^2_{id} (1 - Poutthr)^{1/M} (1 - Pd,i)(2H_p - 1)}. \quad (18)
\]
From Eqs. (17) and (18), one can conclude that the interference induced by the proposed selective-relay based cooperative sensing scheme is controllable and can be reduced to satisfy an arbitrarily given primary outage probability requirement by adjusting either the individual detection probability or the transmit power of cognitive users.

B. ROC Analysis

Now, we analyze the receiver operating characteristics (ROC) performance of the cooperative spectrum sensing over Rayleigh fading channels.

1) Traditional Cooperative Sensing with a Dedicated Reporting Channel: For the purpose of performance comparison, let us first consider the traditional cooperative sensing with a dedicated reporting channel, in which the initial detection results of CUs (encoded by a CRC code) are always forwarded to the fusion center over a dedicated channel. Then, FC will decode the received signals and combine the successfully decoded outcomes only, i.e., only the successfully decoded outcomes are used for fusion. For convenience, those CUs whose initial detection results are received and decoded successfully at FC constitute a set C. Accordingly, the sample space of all such possible sets is given by \( \{ C \in \emptyset \cup C_m, \ m = 1, 2, \cdots, 2^M - 1 \} \), where \( C_m \) is a non-empty subcollection of the M cognitive users. Without loss of generality, let \( C = \emptyset \) represent the case that all the initial detection results from CUs fail to decode at FC and \( C = C_m \) correspond to the other case.

- Case \( C = \emptyset \): FC fails to decode all the initial detection results from CUs, which can be described as
\[
\log_2(1 + |h_{ic}|^2 \gamma_s^T) < \frac{1}{B_dT_d}, \quad i = 1, 2, \cdots, M \quad (19)
\]
where \( \gamma_s^T \) is the transmit power of CUs considering the traditional cooperative sensing channel and \( B_dT_d \) is the bandwidth-time-product of the dedicated channel. Therefore, given that case \( C = \emptyset \) has occurred, FC will discard all the received initial results from CUs and nothing is used for fusion. From the viewpoint of protecting the primary user, FC determines that PU is active in this case, i.e.,
\[
\hat{H}_c(C = \emptyset) = H_1. \quad (20)
\]
Although the occurrence of case \( C = \emptyset \) will greatly degrade the spectrum sensing performance, the corresponding occurrence probability will be very small.

- Case \( C = C_m \): FC successfully decodes these initial spectrum sensing results from the CUs in set \( C_m \), i.e.,
\[
\log_2(1 + |h_{ic}|^2 \gamma_s^T) > \frac{1}{B_dT_d}, \quad i \in C_m
\]
\[
\log_2(1 + |h_{jc}|^2 \gamma_s^T) < \frac{1}{B_dT_d}, \quad j \in \bar{C}_m
\]

where \( \bar{C}_m = \mathcal{R} - C_m \) is the complementary set of \( C_m \). In the given case \( C = C_m \) and an “AND” fusion rule, the final spectrum sensing result fused at FC is given by
\[
\hat{H}_c(C = C_m) = \bigotimes_{i \in C_m} \hat{H}_i(1) \quad (22)
\]
where \( \hat{H}_i(1) \) is the initial spectrum sensing result of CU \( i \) in the set \( C_m \). Similarly, if an “OR” rule is used at FC for fusion, the final sensing result \( \hat{H}_c(C = C_m) \) is expressed as
\[
\hat{H}_c(C = C_m) = \bigoplus_{i \in C_m} \hat{H}_i(1). \quad (23)
\]

Accordingly, following Eqs. (20) and (22), the probability of overall detection of the presence of PU at FC, referred to as overall detection probability, for the “AND” based traditional cooperative sensing scheme as denoted by \( Pd_{trad}^{AND} \) is calculated as Eq. (24) at the top of following page, where \( Pd_{i,1} = \Pr(\hat{H}_i(1) = H_1|H_p = H_1) \) indicates the probability of individual detection of the presence of PU at CU \( i \) (called individual detection probability) and, moreover, the first term \( \Pr(C = \emptyset) \) in last equation of Eq. (24) arises from \( \Pr(\hat{H}_c = H_1|H_p = H_1, C = \emptyset) = 1 \) (due to \( \hat{H}_c = H_1 \) given \( C = \emptyset \)) and \( \Pr(C = \emptyset|H_p = H_1) = \Pr(C = \emptyset) \) that is resulted from the event \( C = \emptyset \) independent of \( H_p = H_1 \), since the transmission of initial detection results will not be affected by the primary user due to the fact the traditional cooperative sensing scheme utilizes a dedicated reporting channel for reporting the initial detection results to the fusion center. We can also calculate the probability of overall false alarm of the presence of PU at FC (referred to as overall false alarm probability) for the “AND” based traditional cooperative sensing scheme as Eq. (25), where \( Pf_{i,1} = \Pr(\hat{H}_i(1) = H_1|H_p = H_0) \) indicates the probability of individual false alarm of the presence of PU at CU \( i \), called individual false alarm probability. Similarly, from Eqs. (20) and (23), the probabilities of overall detection and false alarm of the presence of PU for the “OR” based traditional cooperative sensing scheme are given by
\[
Pd_{trad}^{OR} = \Pr(C = \emptyset)
\]
\[
+ \sum_{m=1}^{2^M-1} \Pr(C = C_m)[1 - \prod_{i \in C_m} (1 - Pd_{i,1})] \quad (26)
\]
and
\[
Pf_{trad}^{OR} = \Pr(C = \emptyset)
\]
\[
+ \sum_{m=1}^{2^M-1} \Pr(C = C_m)[1 - \prod_{i \in C_m} (1 - Pd_{i,1})]. \quad (27)
\]

Using the results of Appendix A, we can obtain
\[
Pf_{i,1} = \begin{cases} \text{Pd}_{i,1}, & \text{Pd}_{i,1} = Q(-\sqrt{N}) \\ \text{Pd}_{i,1} - Q(-1\text{Pd}_{i,1}) + \frac{1}{\sigma_{\kappa_i}^2} \exp(\xi_i), & \text{otherwise} \end{cases}
\]
where \( \kappa_i = \gamma_p Q^{-1}(\text{Pd}_{i,1}) + \sqrt{N}\gamma_p, \ \xi_i = \frac{Q^{-1}(\text{Pd}_{i,1})}{\sigma_{\kappa_i}^2} + \frac{1}{2\sigma_{\kappa_i}^2}, \) and the number of samples should satisfy \( N \geq Q^{-2}(\text{Pd}_{i,1})^2 \). Notice that random variables \( |h_{ic}|^2 \) follows an exponential
where

\[
\Delta = \frac{2^{M-1}}{\gamma} \sum_{m=1}^{2^M-1} \Pr\{\hat{H}_c = H_1|H_p = H_1, C = m\} \Pr\{C = m|H_p = H_1\}
\]

and

\[
\Pr\{C = m\} = \prod_{i \in C_m} \text{Pr}\{C = m\} \prod_{i \in C_m} \text{Pr}\{C = m\}
\]

from Eq. (24) - (27) are calculated from Eq. (19) as

\[
\text{Pr}(C = 0) = \prod_{i=1}^{M} \left[1 - \exp\left(-\frac{\Delta}{\sigma_{ic}^2}\right)\right]
\]

where \(\Delta = \frac{2^{1/(M-1)} - 1}{\gamma^T_c}\). Similarly, from Eq. (21), \(\text{Pr}(C = C_m)\) is given by

\[
\Pr(C = C_m) = \prod_{i \in C_m} \exp\left(-\frac{\Delta}{\sigma_{ic}^2}\right) \prod_{j \in C_m} \left[1 - \exp\left(-\frac{\Delta}{\sigma_{jc}^2}\right)\right].
\]

2) Proposed Cooperative Sensing without a Dedicated Reporting Channel: Now, we start the ROC analysis for the selective-relay based cooperative sensing scheme by considering two kinds of logic fusion rules, i.e., “AND” and “OR”. For the “AND” based proposed cooperative scheme, the probability of overall detection of the presence of primary user at fusion center is calculated as

\[
P_{\text{d,proposed}} = \Pr\{\hat{H}_c = H_1|H_p = H_1\} = \prod_{i=1}^{M} \text{Pd}_{c,i}
\]

where \(\text{Pd}_{c,i} = \Pr\{\hat{H}_i(2) = H_1|H_p = H_1\}\). Also, from Eq. (10), the probability of overall false alarm of the presence of primary user at FC is given by

\[
P_{\text{f,proposed}} = \Pr\{\hat{H}_c = H_1|H_p = H_0\} = \prod_{i=1}^{M} \text{Pf}_{c,i}
\]

where \(\text{Pf}_{c,i} = \Pr\{\hat{H}_i(2) = H_1|H_p = H_0\}\). Considering an “OR” logic fusion rule used, we can similarly obtain the probabilities of overall detection and false alarm of the presence of primary user from Eq. (11) as

\[
P_{\text{d,OR}} = 1 - \prod_{i=1}^{M} (1 - \text{Pd}_{c,i})
\]

and

\[
P_{\text{f,OR}} = 1 - \prod_{i=1}^{M} (1 - \text{Pf}_{c,i}).
\]

By using Eq. (8), \(\text{Pd}_{c,i}\) can be rewritten as

\[
\text{Pd}_{c,i} = 1 - \prod_{i=1}^{M} \left(1 - \text{Pd}_{c,i}\right)
\]

\[
\times \Pr\left\{\frac{(1 - \alpha)}{M} \log_2 \left(1 + \frac{|h_{ic}|^2 \gamma_s}{|h_{pc}|^2 \gamma_p + 1}\right) > \frac{1}{BT}\right\}
\]

where \(\text{Pd}_{c,i}\) is the probability of individual detection of the presence of PU at CU. Notice that random variables \(|h_{ic}|^2\) and \(|h_{pc}|^2\) follow exponential distribution with parameters \(1/\sigma_{ic}^2\) and \(1/\sigma_{pc}^2\), respectively, and are independent from each other. Hence, performing the probability integral, Eq. (36) can be further rewritten as

\[
\text{Pd}_{c,i} = 1 - \frac{\sigma_{ic}^2 (1 - \text{Pd}_{c,1})}{\sigma_{pc}^2 \gamma_p \Lambda + \sigma_{ic}^2} \exp\left(-\frac{\Lambda}{\sigma_{ic}^2}\right)
\]

where \(\Lambda = \frac{2^M}{(1 - \alpha) BT} - 1/\gamma_s\). Following the same procedures as in deriving \(\text{Pd}_{c,i}\), we can calculate \(\text{Pf}_{c,i}\) as follows
Pf_{c,i} = 1 - Pr\{H_i(2) = H_0|H_p = H_0\}
= 1 - Pr\{(\Theta_{c,i}(2) = 0|H_p = H_0\}
= 1 - (1 - Pf_{c,i}) Pr\left\{\frac{(1 - \alpha)}{M} \log_2(1 + |h_{ic}|^2\gamma_s) > \frac{1}{BT}\right\}
= 1 - (1 - Pf_{c,i}) \exp(-\Lambda \sigma_{ic}) \quad (38)

where Pf_{c,i} is the probability of individual false alarm of the presence of PU at CU_i. Notice that the relationship between the individual detection probability Pd_{c,i} and the individual false alarm probability Pf_{c,i} is given by Eq. (28).

**Theorem 1:** Considering the “AND” fusion rule and each CU with the same detection performance, a unique optimal signal detection overhead (0 < \alpha < 1) exists to minimize an asymptotic overall false alarm probability in high SNR regions given a target detection probability.

**Proof:** See Appendix B.

As is known, if the fusion center fails to detect the presence of the primary user given that PU is active, it will notify a cognitive source node to start traffic transmissions, which would interfere the primary user. Accordingly, the overall detection probability shall be set to a required threshold by the cognitive system. Given a target value Pd_{thr} and assuming each CU with the same detection performance, we can obtain solutions Pd_{c,i} from Eqs. (31) and (33) as Pd_{c,i} = (Pd_{thr})^{1/M} and 1 - (1 - Pd_{thr})^{1/M}, respectively. Although the solutions are not optimal for all scenarios, they have the advantage of simple implementation and do not need any additional resource for the channel state information feedback (from CUs to FC) to find an optimal solution. This is attractive especially for cognitive radio networks, since cognitive radio is supposed to reuse the unoccupied licensed spectrum (also called white space) without dedicated channel (or, with very limited dedicated channel resources). Using this result and following Eq. (37), the individual detection probability is given by

$$Pd_{c,i} = 1 - \frac{(1 - Pd_{thr})(\sigma_{pc}^2 + \sigma_{ic}^2)}{\sigma_{ic}^2} \exp\left(-\frac{\Lambda}{\sigma_{ic}}\right). \quad (39)$$

As discussed before, to limit the interference induced in the phase of initial detection result reporting from CUs to FC, the individual detection probability is constrained to the primary outage probability requirement, Pout_{thr}, as given by Eq. (17). Meanwhile, the individual detection probability should satisfy Eq. (39) to guarantee that the overall detection probability is above a threshold value Pd_{thr}. Therefore, given a requirement pair of (Pout_{thr}, Pd_{c,thr}), the individual detection probability Pd_{c,i} is determined by

$$Pd_{c,i} = 1 - \min\left\{\frac{\gamma_s \sigma_{id}^2 M [1 - (1 - Pout_{thr})^{1/M}]}{\gamma_s \sigma_{id}^2 (1 - Pout_{thr})^{1/M} (2^{R_p} - 1)} + \frac{(1 - Pd_{c,i})(\sigma_{pc}^2 + \gamma_s \sigma_{id}^2)}{\sigma_{ic}^2} \exp\left(-\frac{\Lambda}{\sigma_{ic}}\right)\right\} \quad (40)$$

Using Eq. (40), we illustrate in Table 1 the required individual detection probability Pd_{c,i} under the different requirement pairs (Pout_{thr}, Pd_{c,thr}) with \gamma_p = 5 dB, R_p = 1 bit/s/Hz, \gamma_s = -5 dB, M = 1, T = 25 ms, B = 50 kHz, f_s = 100 kHz, \gamma_f = 10 dB, B_J/T_d = 1000, \alpha = 0.2, \sigma_{pc}^2 = \sigma_{id}^2 = 0.2, and \sigma_{id}^2 = \sigma_{pd}^2 = \sigma_{ic}^2 = 1.

**Table I**

<table>
<thead>
<tr>
<th>(Pout_{thr}, Pd_{c,thr})</th>
<th>Pd_{c,i}</th>
<th>(Pout_{thr}, Pd_{c,thr})</th>
<th>Pd_{c,i}</th>
</tr>
</thead>
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<td>(1e-3, 0.950)</td>
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</tr>
<tr>
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<td>0.9899</td>
<td>(5e-3, 0.990)</td>
<td>0.9899</td>
</tr>
<tr>
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<td>(5e-3, 0.990)</td>
<td>0.9894</td>
</tr>
<tr>
<td>(1e-3, 0.999)</td>
<td>0.9995</td>
<td>(5e-3, 0.990)</td>
<td>0.9992</td>
</tr>
<tr>
<td>(1e-3, 0.999)</td>
<td>0.9990</td>
<td>(5e-3, 0.990)</td>
<td>0.9998</td>
</tr>
</tbody>
</table>

**Fig. 3.** The overall false alarm probability versus the overall detection probability of the logic “OR” and “AND” based traditional and proposed cooperative spectrum sensing schemes with M = 2, \gamma_p = 5 dB, R_p = 1 bit/s/Hz, Pout_{thr} = 0.01, T = 25 ms, B = 50 kHz, f_s = 100 kHz, \gamma_f = 10 dB, B_J/T_d = 1000, \alpha = 0.2, \sigma_{pc}^2 = \sigma_{id}^2 = 0.2, and \sigma_{id}^2 = \sigma_{pd}^2 = \sigma_{ic}^2 = 1.

IV. NUMERICAL RESULTS AND ANALYSIS

In this section, we first show the ROC curves of the traditional and proposed cooperative sensing schemes with logic “AND” and “OR” rules by using Eqs. (24) - (27) and (31) - (34). Fig. 3 illustrates the overall detection probability versus the overall false alarm probability for the traditional and proposed schemes, where the two ROC curve pairs correspond to the logic “AND” and “OR” rules, respectively. As shown in Fig. 3, the ROC performances of “AND” based traditional and proposed cooperative sensing schemes are, respectively, better than that of “OR” based cases. From Fig. 3, one can also see that in the low detection probability regions, the overall false alarm probabilities of the proposed scheme are larger than that of the traditional scheme, no matter which fusion rule is used. Moreover, as the overall detection probability...
The overall false alarm probability of the proposed scheme do not decrease as expected. It even increases surprisingly as shown from the ROC curve of the proposed scheme with the logic “AND” fusion rule. This is because that when the overall detection probability is overly small, the secondary transmit power allowed as given by Eq. (18) should be very low due to the primary outage probability requirement, which will result in an unreliable reporting of the initial spectrum sensing results from cognitive users to the fusion center and thus increases the overall false alarm probability. Such an adverse impact is more noticeable, as the number of cognitive users increases, since the secondary transmit power allowed will decrease with an increasing number of cognitive users. On the other hand, in the higher detection probability regions of Fig. 3, one can observe that the ROC performance of the proposed scheme is nearly identical to the traditional scheme, especially when using the logic “AND” fusion. Notice that in practical cognitive radio systems, the overall detection probability shall be guaranteed to be above a relatively large value (e.g., \( \text{PdT} \geq 0.9 \)) for protecting the primary users. In this sense, the proposed selective-relay based cooperative sensing scheme can save the dedicated channel resources without sacrificing ROC performance, which confirms the advantage of the proposed scheme.

In Fig. 4, we show the overall false alarm probability versus the overall detection probability of the traditional and proposed cooperative sensing schemes under different primary outage probability requirements. All cases in Fig. 4 demonstrate that in the relatively high detection probability regions (i.e., \( \text{PdT} \geq 0.9 \)), the ROC performance of the proposed scheme is nearly identical to the traditional dedicated channel based cooperative sensing scheme. Moreover, as the primary outage probability requirement loosens, the ROC curve of the proposed selective-relay based cooperative sensing scheme becomes closer to that of the traditional scheme. In addition, in low detection probability regions, one can see that as the overall detection probability decreases toward zero, the overall false alarm probabilities of the proposed scheme increase unexpectedly, as shown from the ROC curves of the proposed scheme for \( \text{Pout}_{tr} = 0.005 \) and \( \text{Pout}_{tr} = 0.01 \). This is because that when the overall detection probability is overly small, the secondary transmit power allowed as given by Eq. (18) should be very low, which will result in an unreliable initial spectrum sensing results reporting from cognitive users to the fusion center and thus increases the overall false alarm probability. Such an adverse impact becomes dominant and results in an increasing overall false alarm probability, as the primary outage probability requirement becomes more stringent.

Fig. 5 illustrates the overall false alarm probability versus the signal detection overhead of the selective-relay based cooperative sensing scheme for different number of cognitive users, where the requirement pair is specified to be \( (\text{Pout}_{tr}, \text{Pd}_{c, \text{thr}}) = (10^{-3}, 0.99) \). From Fig. 5, one can see that there always exists an optimal signal detection overhead to minimize the overall false alarm probability under a guaranteed overall detection probability \( \text{Pd}_{c, \text{thr}} = 0.99 \), i.e., a minimum false alarm probability can be achieved through an optimal allocation of the time durations between the signal detection and reporting phases. As observed in Fig. 5, the optimal value of the signal detection overhead decreases with an increasing number of cognitive users. This is due to the fact that, as the number of cognitive users increases, each subchannel assigned to a cognitive user for its initial detection result reporting is allocated with less bandwidth resources and thus a longer time duration is needed to meet a required transmission quality for the reporting phase, resulting in the decrease of the optimal signal detection overhead.

In Fig. 6, we show the overall false alarm probability versus the signal detection overhead for different transmit SNR \( \gamma_s \) with the requirement pair \( (\text{Pout}_{tr}, \text{Pd}_{c, \text{thr}}) = (10^{-3}, 0.99) \). From Fig. 6, one can see that a minimum false alarm probability can be obtained through an optimization of the signal detection overhead and, moreover, a significant performance
improvement is achieved with the optimal signal detection overhead. Besides, one can observe that, as the transmit SNR $\gamma_s$ increases, the optimal signal detection overhead increases. This is because that, with an increasing transmit SNR $\gamma_s$, a higher transmit power is used for the initial spectrum sensing result reporting and thus a shorter time duration is required for the reporting phase, leading to the increase of the optimal signal detection overhead.

V. CONCLUSION

In this paper, we have proposed a selective-relay based cooperative sensing scheme without the dedicated reporting channel and presented a comprehensive analysis of the proposed scheme by jointly considering both the signal detection and reporting phases. Closed-form expressions of the probabilities of overall detection and false alarm of the presence of the primary user are derived for the traditional and proposed cooperative sensing schemes over Rayleigh fading channels. Compared with the traditional cooperative sensing scheme, the proposed selective-relay based cooperative spectrum sensing scheme can save the dedicated channel resources without sacrificing ROC performance. In addition, we have shown that an optimal signal detection overhead exists to minimize the overall false alarm probability under a guaranteed overall detection probability.

APPENDIX A

CALCULATION OF EQ. (28)

Without loss of generality, we consider that the primary signal $x_p$ follows a complex symmetric Gaussian distribution. According to the proposition 2 in [21], for a large number $N$, random variable $T[y_i(1)]$ given $H_p = H_1$ follows a Gaussian distribution with $(|h_{pi}|^2\gamma_p + 1)N_0$ and variance $(|h_{pi}|^2\gamma_p + 1)^2N_0^2/N$, where $\gamma_p = P_p/N_0$ and $h_{pi}$ is a fading coefficient of the channel from PU to CU $i$ at time slot $k$. Hence, given the fading coefficient $h_{pi}$, the probability of individual detection of the presence of PU, $P_{d_i,1}$, at time slot $k$ is calculated from Eq. (4) as

$$P_{d_i,1} = Q\left(\frac{\lambda_i\sqrt{N}}{N_0(|h_{pi}|^2\gamma_p + 1)\sqrt{N}}\right) \tag{A.1}$$

where $Q(\cdot)$ is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{y^2}{2}\right)dy. \tag{A.2}$$

Similarly, considering the central limit theorem, for a large number $N$, random variable $T[y_i(1)]$ given $H_p = H_0$ follows a Gaussian distribution with mean $N_0$ and variance $N_0^2/N$. Therefore, the probability of individual false alarm of the presence of PU, $P_{f_i,1}$, at time slot $k$ is given by

$$P_{f_i,1} = Q\left(\frac{\lambda_i}{N_0} - \frac{1}{\sqrt{N}}\right) \tag{A.3}$$

As discussed before, if a CU fails to detect the presence of the primary user given that PU is active, it would interfere the primary user. In order to guarantee the PU’s quality of service, each individual detection probability $P_{d_i,1}$ should be set to a target value. Therefore, for given target detection probability $P_{d_i,1}$ and fading coefficient $h_{pi}$, $P_{f_i,1}$ is expressed, following Eq. (A.1) and Eq. (A.3), as

$$P_{f_i,1} = Q\left(\kappa|h_{pi}|^2 + Q^{-1}(P_{d_i,1})\right) \tag{A.4}$$

where $\kappa = \gamma_pQ^{-1}(P_{d_i,1}) + \sqrt{N}\gamma_p$ and $Q^{-1}(\cdot)$ is an inverse $Q(\cdot)$ function. Notice that random variable $X = |h_{pi}|^2$ follows an exponential distribution with parameter $1/\sigma_{pi}^2$. Hence, an average probability of false alarm of the presence of PU, $P_{f_i,1}$, can be calculated from Eq. (A.4) as

$$P_{f_i,1} = \int_0^\infty Q\left(\kappa x + Q^{-1}(P_{d_i,1})\right)\frac{1}{\sigma_{pi}^2} \exp\left(-\frac{x}{\sigma_{pi}^2}\right)dx$$

$$= \int_{\Xi} \frac{1}{\sigma_{pi}^2} \exp\left(-\frac{x}{\sigma_{pi}^2}\right)\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right)dy \tag{A.5}$$

where $\Xi = \{(x, y)|0 < x < \infty, \kappa x + Q^{-1}(P_{d_i,1}) < y < \infty\}$. Integrating Eq. (A.5) first with respect to $x$, then with respect to $y$, we obtain Eq. (A.6), where $\kappa_i = \gamma_pQ^{-1}(P_{d_i,1}) + \sqrt{N}\gamma_p$ and $\xi_i = \frac{Q^{-1}(P_{d_i,1})}{\sigma_{pi}^2} + \frac{1}{2\sigma_{pi}^4}$. According to detection theory, for any reasonable detector, the false alarm probability is always smaller than or equal to the detection probability, or else it is worse than tossing a coin. Therefore, the number of samples $N$ should satisfy

$$N \geq [Q^{-1}(P_{d_i,1})]^2 \tag{A.7}$$

which is due to the fact that from central limit theorem, the number of samples should be sufficiently large so that the output statistic $T[y_i(1)]$ of the energy detector can be approximated to a Gaussian distribution. Combining Eq. (A.6) and Eq. (A.7) yields

$$P_{f_i,1} = \begin{cases} P_{d_i,1}, & P_{d_i,1} = Q(-\sqrt{N}) \\ P_{d_i,1} - Q(Q^{-1}(P_{d_i,1}) + \frac{1}{\sigma_{pi}^2})\exp(\xi_i), & \text{otherwise} \end{cases} \tag{A.8}$$

This is Eq. (28).
APPENDIX B
PROOF OF THEOREM 1

Considering each CU with the same detection performance and following Eq. (32), we can rewrite the overall false alarm probability expression of the proposed selective-relay based spectrum sensing scheme with “AND” fusion rules as

\[
P_f^{\text{proposed}} = \left( P_{f,c,i} \right)^M
\]  

(B.1)

The second-order derivative of Eq. (B.1) with respective to the signal detection overhead \( \alpha \) is given by

\[
\frac{d^2(P_f^{\text{AND}})}{d\alpha^2} = M(M - 1)(P_{f,c,i})^{M-2} \left[ \frac{d(P_{f,c,i})}{d\alpha} \right]^2 \\
\times \left[ 1 - \delta(M - 1) \right] \\
+ M(P_{f,c,i})^{M-1} \frac{d^2(P_{f,c,i})}{d\alpha^2}
\]

(B.2)

where \( \delta(M - 1) = 1 \) for \( M = 1 \), otherwise \( \delta(M - 1) = 0 \). From Eq. (B.2), one can easily obtain \( M(M - 1)(P_{f,c,i})^{M-2} \left[ \frac{d(P_{f,c,i})}{d\alpha} \right]^2 [1 - \delta(M - 1)] \geq 0 \). Hence, if \( \frac{d^2(P_{f,c,i})}{d\alpha^2} > 0 \), the second-order derivative \( \frac{d^2(P_{f,c,i})}{d\alpha^2} \) is positive and thus a unique optimal signal detection overhead \( \alpha \) exists to minimize \( P_f^{\text{proposed}} \). In the following, we prove \( \frac{d^2(P_{f,c,i})}{d\alpha^2} > 0 \) in high SNR regions. Considering \( \gamma_s \to \infty \) and using Taylor series, we can expand Eq. (38) as

\[
P_{f,c,i} = 1 - (1 - P_d) \left[ 1 - \frac{\Lambda}{\sigma_{ic}^2} + O \left( \frac{1}{\gamma_s} \right) \right]
\]

\[
\approx P_{f,c,i} + (1 - P_d) \frac{\Lambda}{\sigma_{ic}^2}
\]  

(B.3)

where \( \Lambda = [2M/(1-\alpha)^B_0] - 1 \) \( \gamma_s \). Similarly, letting \( \gamma_p \to \infty \) and applying Taylor approximation to Eq. (28) yield

\[
P_{f,c,i} \approx P_{d,i} - P_{d,i,1} \left[ 1 - \frac{Q^{-1}(P_{d,i,1})}{\sigma_{pi}^2 \kappa_i} \right] = -P_{d,i,1}Q^{-1}(P_{d,i,1}) \frac{\Lambda}{\sigma_{ic}^2} \]  

(B.4)

where \( \kappa_i = \gamma_p Q^{-1}(P_{d,i,1}) + \sqrt{N} \gamma_p \) and \( N = \alpha T_j \). In obtaining (B.4), we have ignored the term \( 1/(2\sigma_{pi}^2 \kappa_i^2) \), since it is a higher-order infinitesimal compared to the term \( Q^{-1}(P_d)/(\sigma_{pi}^2 \kappa_i^2) \) for \( \gamma_p \to \infty \). Substituting Eq. (B.4) into Eq. (B.3) gives

\[
P_{f,c,i} = -P_{d,i,1}Q^{-1}(P_{d,i,1}) \frac{\Lambda}{\sigma_{ic}^2} \]  

\[
\approx \frac{\Lambda}{\sigma_{ic}^2} - P_{d,i,1}Q^{-1}(P_{d,i,1}) \frac{\Lambda}{\sigma_{ic}^2}
\]  

(B.5)

Following Eq. (B.5), we can obtain \( \frac{d^2(P_{f,c,i})}{d\alpha^2} \) as

\[
\frac{d^2(P_{f,c,i})}{d\alpha^2} = \frac{1}{\sigma_{ic}^2} \frac{d^2P_f}{d\alpha^2} - 2P_{d,i,1}Q^{-1}(P_{d,i,1}) \left( \frac{d\kappa_i}{d\alpha} \right)^2 \\
+ P_{d,i,1}Q^{-1}(P_{d,i,1}) \frac{d^2\kappa_i}{d\alpha^2} \]  

(B.6)

where \( \frac{d^2P_f}{d\alpha^2} \) and \( \frac{d^2\kappa_i}{d\alpha^2} \) are given by

\[
\frac{d^2P_f}{d\alpha^2} = \frac{2M/(1-\alpha)^B_0}{\sigma_{pi}^2 \kappa_i^2} \frac{1}{\gamma_s} \frac{2}{(1-\alpha)^3} + \frac{M\ln2}{(1-\alpha)^4 B_0} \]

(B.7)

and

\[
\frac{d^2\kappa_i}{d\alpha^2} = -\frac{\sqrt{\kappa_i} \gamma_p}{4} \frac{1}{\gamma_s} \frac{d\kappa_i}{d\alpha} < 0
\]

(B.8)

In general, a required detection probability \( P_{d,i,1} \) should be above 0.5 for the primary user protection, implying \( Q^{-1}(P_{d,i,1}) < 0 \). Thus, using \( Q^{-1}(P_{d,i,1}) < 0 \) and substituting Eqs. (B.7) and (B.8) into Eq. (B.6), one can easily conclude \( \frac{d^2(P_{f,c,i})}{d\alpha^2} > 0 \), resulting in \( \frac{d^2(P_f^{\text{proposed}})}{d\alpha^2} > 0 \). Therefore, a unique optimal signal detection overhead \( \alpha \) exists and the proof of Theorem 1 is completed.

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