Electronic Circuits – EE359A

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Lecture 18
Signal Generators and Waveform-shaping Circuits

Ch 17
Stability in feedback systems

Feedback system

Bounded input

Is output bounded?
Stability measures

![Diagram showing Bode plot for stability measures with gain margin and phase margin annotated on the graph.]
Using negative feedback system
to create a signal generator

\[ |A\beta(\omega_\pi)| \geq 1 \]
\[ \angle A\beta(\omega_\pi) = \pi \]
Basic oscillator structure
Basic oscillator structure

With positive feedback

\[ A_f(s) = \frac{A(s)}{1 - A(s)\beta(s)} \]
Basic oscillator structure

With positive feedback

\[ A_f(s) = \frac{A(s)}{1 - A(s)\beta(s)} \]

Loop gain

\[ -A(s)\beta(s) \]
Basic oscillator structure

With positive feedback

\[ A_f(s) = \frac{A(s)}{1 - A(s)\beta(s)} \]

Loop gain

\[ -A(s)\beta(s) \]

Define loop gain \( L(s) \)

\[ L(s) \equiv A(s)\beta(s) \]
Basic oscillator structure

With positive feedback

\[ A_f(s) = \frac{A(s)}{1 - A(s)\beta(s)} \]

Characteristic equation

\[ 1 - L(s) = 0 \]

Loop gain

\[ -A(s)\beta(s) \]

Define loop gain \( L(s) \)

\[ L(s) \equiv A(s)\beta(s) \]
Criteria for oscillation

For oscillation to occur at $\omega_o$

$$L(j\omega_o) \equiv A(j\omega_o)\beta(j\omega_o) = 1$$

The Barkhausen criteria:

At $\omega_o$, the loop gain has a magnitude 1 and the phase shift is 0 (for positive feedback)
Criteria for oscillation

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At $\omega_o$, the loop gain has a magnitude 1 and the phase shift is 0 (for positive feedback)

$$x_f = \beta x_o$$

$$Ax_f = x_o$$

$$A\beta x_o = x_o$$

$$A\beta = 1$$
Criteria for oscillation

For oscillation to occur at $\omega_o$

$$L(j\omega_o) \equiv A(j\omega_o)\beta(j\omega_o) = 1$$

The Barkhausen criteria:

At $\omega_o$, the loop gain has a magnitude 1 and the phase shift is 0 (for positive feedback)

$$x_f = \beta x_o$$

If gain is sufficient, frequency of oscillation is determined only by phase response

$$Ax_f = x_o$$

$$A\beta x_o = x_o$$

$$A\beta = 1$$
A steep phase response (\( \phi(\omega) \)) produces a stable oscillator.
Oscillator amplitude

\[ |L(j\omega_0)| < 1 \]

\[ |L(j\omega_0)| > 1 \]
Oscillator amplitude

$|L(j\omega_0)| = 1$

How do you stabilize the oscillator so the output level remains constant?

If the oscillator is adjustable, how is this possible across the full range?
Nonlinear oscillator amplitude control

\[
\text{Slope } \frac{(R_f/\|R_4\)}{R_1}
\]

\[
\text{Slope } -\frac{R_f}{R_1}
\]

\[
\text{Slope } -\frac{(R_f/\|R_3\)}{R_1}
\]
Nonlinear oscillator amplitude control

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\[ \text{Slope} = -\frac{(R_f/||R_3)}{R_1} \]
Basic oscillator structure

\[ A_f(s) = \frac{A(s)}{1 - A(s)\beta(s)} \]

Characteristic equation
\[ 1 - L(s) = 0 \]

Loop gain
\[ -A(s)\beta(s) \]

Define loop gain \( L(s) \)
\[ L(s) \equiv A(s)\beta(s) \]
Nonlinear oscillator amplitude control

\[ \text{Slope} = -\frac{(R_f // R_4)}{R_1} \]

\[ \text{Slope} = -\frac{R_f}{R_1} \]

\[ \text{Slope} = -\frac{(R_f // R_3)}{R_1} \]
Wein-Bridge oscillator (without amplitude stabilization)
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Wein-Bridge oscillator (without amplitude stabilization)

$$L(s) = A \beta(s)$$

$$A = 1 + \frac{R_2}{R_1}$$

$$\beta(s) = \frac{Z_p}{Z_p + Z_s}$$

$$L(s) = \left[ 1 + \frac{R_2}{R_1} \right] \frac{Z_p}{Z_p + Z_s}$$
Wein-Bridge oscillator
(without amplitude stabilization)

\[ A(s) = \frac{1}{1 + R_2 R_1 Z_p Z_s + Z_s} \]

\[ L(s) = \frac{1 + \frac{R_2}{R_1} \frac{Z_p}{Z_p + Z_s}}{1 + \frac{R_2}{R_1}} \]

\[ L(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{Z_s}{Z_p}} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{Z_s Y_p}{Z_p}} \]

\[ L(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \left( R + \frac{1}{sC} \right) \left( \frac{1}{R} + sC \right)} \]
Wein-Bridge oscillator (without amplitude stabilization)

\[ L(s) = \frac{1 + R_2/R_1}{1 + \left( R + \frac{1}{sC} \right) \left( \frac{1}{R} + sC \right)} \]

\[ L(s) = \frac{1 + R_2/R_1}{1 + \left( \frac{R}{R} + sCR + \frac{1}{sCR} + \frac{sC}{sC} \right)} \]

\[ L(j\omega) = \frac{1 + R_2/R_1}{3 + j\left( \omega CR - \frac{1}{\omega C_0 R} \right)} \]
Wein-Bridge oscillator
(without amplitude stabilization)

\[ L(j\omega) = \frac{1 + R_2/R_1}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)} \]

Oscillation at \( \omega_o \) if

\[ \omega_o CR = \frac{1}{\omega_o CR} \]

\[ \omega_o = \frac{1}{CR} \]
Wein-Bridge oscillator (without amplitude stabilization)

\[ L(j\omega) = \frac{1 + R_2/R_1}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)} \]

Oscillation if

\[ |L(j\omega)| = \frac{1 + R_2/R_1}{3} \]

\[ R_2/R_1 = 2 + \delta \]
Wein-Bridge oscillator
(with amplitude stabilization)
Wein-Bridge oscillator
(with amplitude stabilization)

\[ \omega_0 = \frac{1}{CR} \]

\[ \omega_0 = \frac{1}{(16 \cdot 10^{-9} \text{ F})(10 \cdot 10^3 \text{ } \Omega)} \]

\[ \omega_0 = 6250 \text{ rad/sec} \]

\[ f_0 \approx 1000 \text{ Hz} \]

\[ R_2/R_1 \geq 2 \]

\[ R_2/R_1 = 20.3/10 = 2.03 \]
Wein-Bridge oscillator
(with alternative stabilization)

D₁ and D₂ reduce $R_f$ at high amplitudes
Phase shift oscillator
Phase shift oscillator

Phase shift of each RC section must be 60° to generate a total phase shift of 180°

K must be large enough to compensate for the amplitude attenuation of the 3 RC sections at \( \omega_o \)
Quadrature oscillator
Quadrature oscillator

Limiting circuit

Integrator 1

Integrator 2

(Nominally $2R$)
Quadrature oscillator

\[ L(s) = \frac{1}{s^2 C^2 R^2} \]

\[ \omega_0 = \frac{1}{CR} \]
Quadrature oscillator

\[ \sin(\omega_0 t) \]

\[ \cos(\omega_0 t) \]
LC oscillator

Colpitts oscillator
LC oscillator

Hartley oscillator
LC oscillator

Colpitts oscillator

Hartley oscillator

Frequency determining element
LC oscillator

Colpitts oscillator

Gain stage

Hartley oscillator
LC oscillator

Colpitts oscillator

Hartley oscillator

Feedback voltage divider
LC oscillator

Colpitts oscillator

\[ \omega_0 = \frac{1}{\sqrt{L \left( \frac{C_1 C_2}{C_1 + C_2} \right)}} \]

Hartley oscillator

\[ \omega_0 = \frac{1}{\sqrt{(L_1 + L_2) C}} \]
Practical LC (Colpitts) oscillator
Piezoelectric oscillator

Quartz crystal schematic symbol
Piezoelectric oscillator

Quartz crystal schematic symbol  Equivalent circuit
Piezoelectric oscillator

Quartz crystal schematic symbol  Equivalent circuit  Reactance

Crystal reactance

Inductive  Capacitive

ω₀  ωₚ

0
Piezoelectric oscillator

\[ \omega_s = \frac{1}{\sqrt{LC_s}} \]

\[ \omega_p = \frac{1}{\sqrt{L \left( \frac{C_s C_p}{C_s + C_p} \right)}} \]

Crystal reactance

Series resonance
Inductive

Parallel resonance
Capacitive

Series resonance
Parallel resonance
Piezoelectric oscillator

\[ \omega_s = \frac{1}{\sqrt{LC_s}} \]

\[ \omega_p = \frac{1}{\sqrt{L \left( \frac{C_s C_p}{C_s + C_p} \right)}} \]

\( r << |Z_L| \)
Pierce crystal oscillator
Pierce crystal oscillator

Frequency determining elements
(but $C_S$ dominates)

DC bias circuit (near $V_{DD}/2$)

LPF to discourage harmonic/overtone oscillation

CMOS inverter (high gain amplifier)