Electronic Circuits – EE359A

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Lecture 22
Filters and Tuned Amplifiers

Ch 16
Two-port model of filter

Filter circuit

\[ T(s) = \frac{V_o(s)}{V_i(s)} \]

General response:
Two-port model of filter

General response:

\[ T(s) = \frac{V_o(s)}{V_i(s)} \]

Substituting \( s = j\omega \) and using polar representation:

\[ T(j\omega) = |T(j\omega)|e^{j\phi(\omega)} \]
Two-port model of filter

General response:

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Substituting \( s = j\omega \) and using polar representation:

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Gain/Attenuation in dB:

\[ G(\omega) = 20\log \left( |T(j\omega)| \right) \quad A(\omega) = -20\log \left( |T(j\omega)| \right) \]
Ideal filter characteristics (Low-pass)
Ideal filter characteristics
(High-pass)
Ideal filter characteristics (Band-pass)
Ideal filter characteristics
(Band-stop)
Practical limitations
(Low-pass)

<table>
<thead>
<tr>
<th>( T )</th>
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\( \omega_p \)  \hspace{2cm}  \omega

Passband  \hspace{2cm}  Stopband
Practical limitations (Low-pass)

- Zero-width transition band
- Infinite attenuation in stop-band

Diagram showing: $|T|$ vs $\omega$

- Passband
- Stopband

$\omega_p$
Practical limitations (Low-pass)

- Zero-width transition band
- Infinite attenuation in stop-band
- Infinite complexity
- Infinite time delay
Practical limitations
(Low-pass) – impulse response

\[ x(t) = \delta(t) \]

\[ y(t) = \text{sinc}(t) = \frac{\sin(t)}{t} \]
Practical limitations
(Low-pass) – impulse response

\[ x(t) = \delta(t) \]

\[ y(t) = \text{sinc}(t) = \frac{\sin(t)}{t} \]

Response precedes input!!
Example Low-pass specification

- Pass-band edge
- Stop-band edge
- Pass-band variation
- Transition band
- Minimum stop-band attenuation

$|T|$, dB

$\omega_p$, $\omega_s$, $\omega_{\ell_1}$, $\omega_{\ell_2}$
Example Band-pass specification

- **Pass-band variation**
- **Lower stop-band edge**
- **Pass-band edges**
- **Upper stop-band edge**
- **Minimum stop-band attenuation**
Typical Low-pass specification

Often no constraints on filter curve
Typical Low-pass specification

Often no constraints on filter curve
Might be monotonic
Typical Low-pass specification

Often no constraints on filter curve
May have passband ripple
Typical Low-pass specification

Often no constraints on filter curve
May have stop band ripple
Typical Low-pass specification

Often no constraints on filter curve
May have both passband and stopband ripple
Typical Low-pass specification

Many different approximations to the ideal filter response
Describing $T(s)$

$$T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_1 s + a_0}{b_N s^N + b_{N-1} s^{N-1} + \cdots + b_1 s + b_0}$$
Describing $T(s)$

\[ T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_1 s + a_0}{s^N + b_{N-1} s^{N-1} + \cdots + b_1 s + b_0} \]

$N$ – filter order (number of poles)

For stability, $M \leq N$

Why?
Describing $T(s)$

$$T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_1 s + a_0}{s^N + b_{N-1} s^{N-1} + \cdots + b_1 s + b_0}$$

Transfer function zeroes

$$T(s) = \frac{a_M (s - z_1)(s - z_2)\cdots(s - z_M)}{(s - p_1)(s - p_2)\cdots(s - p_N)}$$

Transfer function poles
Pole-zero diagram of $T(s)$

- Zeroes
- Peaks due to poles

$|T|$, dB

- $A_{\text{max}}$
- $A_{\text{min}}$

Passband

Stopband

Transition band

$s$ plane

- $\infty$
- $j\omega$

- $0$
- $\omega_p$
- $\omega_s$
- $\omega_t$
- $\omega_{t1}$
- $\omega_{t2}$
- $\omega_{\ell1}$
- $\omega_{\ell2}$
- $-\omega_p$
- $-\omega_{\ell1}$
- $-\omega_{\ell2}$
Band-pass filter as a translated
Low-pass filter
Filter designs – approximations to ideal response

![Diagram of filter designs with Ideal filter and Butterworth response](image)
Filter designs – approximations to ideal response
Butterworth response

\[ |T(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2}} \left( \frac{\omega}{\omega_p} \right)^{2N} \]

\[ |T(j\omega_p)| = \frac{1}{\sqrt{1 + \varepsilon^2}} \]

Passband variation
Filter designs – approximations to ideal response

Butterworth response

\[
|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2}} \left( \frac{\omega}{\omega_p} \right)^{2N}
\]

\[
|T(j\omega_p)| = \frac{1}{\sqrt{1 + \epsilon^2}}
\]

Maximally flat passband

Poor transition band response

Passband variation
Filter designs – approximations to ideal response

Butterworth response

Better response with increased filter order
Filter designs – approximations to ideal response

Butterworth response

\[ T(s) = \frac{K \omega_0^N}{(s - p_1)(s - p_2)(s - p_3)\cdots(s - p_N)} \]

\[ p_k = -\omega_p \sin \left( \frac{2k - 1}{N} \frac{\pi}{2} \right) \left( \frac{1}{\varepsilon} \right)^{\frac{1}{N}} + j\omega_p \cos \left( \frac{2k - 1}{N} \frac{\pi}{2} \right) \left( \frac{1}{\varepsilon} \right)^{\frac{1}{N}} \]

for \( k = 1, 2, \ldots, N \)
Filter designs – approximations to ideal response
Butterworth response

Tradeoffs of Butterworth

+ Maximally flat passband

+ Good phase shift characteristics
  (more on this later)

- Poor attenuation in stopband

-Large N (complex filter) needed for reasonable stopband attenuation, rolloff
Designing a filter

\[ T(s) = \frac{a_M (s - z_1)(s - z_2)(s - z_3) \cdots (s - z_{M-1})(s - z_M)}{(s - p_1)(s - p_2)(s - p_3) \cdots (s - p_{N-1})(s - p_N)} \]

\[ T(s) = \frac{a_M (s - z_1)(s - z_2)(s - z_3) \cdots (s - z_{M-1})(s - z_M)}{(s - p_1)(s - p_2)(s - p_3) \cdots (s - p_{N-1})(s - p_N)} \]

\[ T(s) = (\prod (0 \text{ or more first order sections})) (\prod (0 \text{ or more second order sections})) \]
First order section

\[ T(s) = \frac{a_1s + a_0}{s + \omega_0} \]

Pole at \( s = -\omega_0 \)

Zero at \( s = -a_0/a_1 \)
**First order sections**

| Filter Type and $T(s)$ | s-Plane Singularities | Bode Plot for $|T|$ | Passive Realization | Op Amp–RC Realization |
|------------------------|-----------------------|-------------------|--------------------|-----------------------|
| **(a) Low pass (LP)**  | ![Diagram](image1.png) | ![Diagram](image2.png) | ![Diagram](image3.png) | ![Diagram](image4.png) |
| $T(s) = \frac{a_0}{s + a_0}$ | $\sigma$ O at $\infty$ | $20 \log \frac{a_0}{\omega_0} \text{ dB}$ | $CR = \frac{1}{\omega_0}$ DC gain = 1 | $CR = \frac{1}{\omega_0}$ DC gain = $-\frac{R_2}{R_1}$ |
| **(b) High pass (HP)** | ![Diagram](image5.png) | ![Diagram](image6.png) | ![Diagram](image7.png) | ![Diagram](image8.png) |
| $T(s) = \frac{a_1 s}{s + a_0}$ | $\sigma$ O at $\infty$ | $20 \log a_1 \text{ dB}$ | $CR = \frac{1}{\omega_0}$ High-frequency gain = 1 | $CR = \frac{1}{\omega_0}$ High-frequency gain = $-\frac{R_2}{R_1}$ |
| **(c) General** | ![Diagram](image9.png) | ![Diagram](image10.png) | ![Diagram](image11.png) | ![Diagram](image12.png) |
| $T(s) = \frac{a_1 s + a_0}{s + a_0}$ | $\sigma$ O at $\infty$ | $20 \log \frac{a_0}{a_1} \text{ dB}$ | $C_1 R_1 = \frac{a_1}{a_0}$ DC gain = $\frac{R_2}{R_1 + R_2}$ HF gain = $\frac{C_1}{C_1 + C_2}$ | $C_1 R_1 = \frac{a_1}{a_0}$ DC gain = $\frac{R_2}{R_1}$ HF gain = $-\frac{C_1}{C_2}$ |
# First order sections

| $T(s)$ | Singularities | $|T|$ and $\phi$ | Passive Realization | Op Amp–RC Realization |
|--------|---------------|------------------|---------------------|----------------------|
| All pass (AP) | $T(s) = -\frac{s - \omega_0}{s + \omega_0}$ | $|T|$, dB | [Diagram of $|T|$ and $\phi$] | [Diagram of Op Amp–RC Realization] |

where $\omega_0 > 0$.

- $|T|$ dB: $20 \log |a_1|$
- $\phi$: $0^\circ$ at $\omega = 0$, $-90^\circ$ at $\omega = \omega_0$, $-180^\circ$ at $\omega = \infty$

**Passive Realization**

- $CR = 1/\omega_0$
- Flat gain $(a_1) = 0.5$

**Op Amp–RC Realization**

- $CR = 1/\omega_0$
- Flat gain $(a_1) = 1$
Second order section

\[ T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2} \]

\[ p_1, p_2 = -\frac{\omega_0}{Q} \pm \sqrt{1 - \frac{1}{4Q^2}} \]
Second order section

\[ T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2} \]

\[ p_1, p_2 = -\frac{\omega_0}{Q} \pm \sqrt{1 - \frac{1}{4Q^2}} \]

Numerator zeroes determine filter response: LP, HP, BP, BS, AP
## Second order sections

| Filter Type and \( T(s) \) | \( s \)-Plane Singularities | \(|T|\) |
|-----------------------------|-----------------------------|--------|
| **(a) Low pass (LP)**      | \[ T(s) = \frac{a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \] | \[ |T| = \frac{|a_0|Q}{\omega_0 \sqrt{1 - \frac{1}{4Q^2}}} \] |
| DC gain = \[ \frac{a_0}{\omega_0} \] | \[ \omega_0 \] | \[ \omega_{\text{max}} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} \] |

| **(b) High pass (HP)**     | \[ T(s) = \frac{\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \] | \[ |T| = \frac{|a_2|Q}{\sqrt{1 - \frac{1}{4Q^2}}} \] |
| High-frequency gain = \( a_2 \) | \[ \omega_{\text{max}} = \frac{\omega_0}{\sqrt{1 - \frac{1}{2Q^2}}} \] |

| **(c) Bandpass (BP)**     | \[ T(s) = \frac{a_1 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \] | \[ \omega_{\text{max}} - \frac{(a_1 Q / \omega_0)}{\omega_0} \] |
| Center-frequency gain = \[ \frac{a_1 Q}{a_0} \] | \[ \omega_0 \sqrt{1 + \frac{1}{4Q^2}} + \frac{\omega_0^2}{2Q} \] | \[ \omega_1 \omega_2 = \omega_0 \] |

\[ \omega_1, \omega_2 = \frac{\omega_0}{\sqrt{1 + \frac{1}{4Q^2}} + \frac{\omega_0^2}{2Q}} \]

\( \omega_0 \) = \( \omega_0 \sqrt{1 + \frac{1}{4Q^2}} + \frac{\omega_0^2}{2Q} \)

\( \omega_1 \omega_2 = \omega_0 \)
Second order sections

| Filter Type and $T(s)$ | $s$-Plane Singularities | $|T|$ |
|------------------------|-------------------------|------|
| (d) Notch | $T(s) = a_2 \frac{s^2 + a_0^2}{s^2 + s \frac{a_0}{Q} + a_0^2}$ | $|T|$ |
| DC gain | $\frac{a_0}{Q}$ | $\sqrt{2}$ |
| High-frequency gain $= a_2$ | $\omega_n$ | $\omega_n$ |
| | $\alpha_0$ | $\alpha_0$ |
| | $\omega_n$ | $\omega_n$ |
| | $\omega_j$ | $\omega_j$ |
| | $\omega_j(Q)$ | $\omega_j(Q)$ |

| (e) Low-pass notch (LPN) | $T(s) = a_2 \frac{s^2 + a_0^2}{s^2 + s \frac{a_0}{Q} + a_0^2}$ | $|T|$ |
| DC gain | $\frac{a_0}{Q}$ | $\frac{a_0}{Q}$ |
| High-frequency gain $= a_2$ | $\omega_n$ | $\omega_n$ |
| | $\omega_n^2$ | $\omega_n^2$ |
| | $\alpha_0$ | $\alpha_0$ |
| | $\omega_n$ | $\omega_n$ |
| | $\alpha_0(Q)$ | $\alpha_0(Q)$ |

| (f) High-pass notch (HPN) | $T(s) = a_2 \frac{s^2 + a_0^2}{s^2 + s \frac{a_0}{Q} + a_0^2}$ | $|T|$ |
| DC gain | $\frac{a_0}{Q}$ | $\frac{a_0}{Q}$ |
| High-frequency gain $= a_2$ | $\omega_n$ | $\omega_n$ |
| | $\omega_n^2$ | $\omega_n^2$ |
| | $\alpha_0$ | $\alpha_0$ |
| | $\omega_n$ | $\omega_n$ |
| | $\alpha_0(Q)$ | $\alpha_0(Q)$ |
Second order sections

\[ T(s) = \frac{s^2 - s \alpha_0 + \alpha_0^2}{s^2 + s \alpha_2 + \alpha_0^2} \]

Flat gain = \( \alpha_2 \)
Second order LCR resonator

Basic resonator
Second order LCR resonator

Current drive

Voltage drive

Transfer function

\[
\frac{V_o}{I} \quad \frac{V_o}{V_i}
\]