Electronic Circuits – EE359A

Bruce McNair

B206
bmcnair@stevens.edu
201-216-5549

Lecture 24
Second order LCR resonator

Basic resonator
Second order LCR resonator

Transfer function

\[
\frac{V_o}{I}
\]

\[
\frac{V_o}{V_i}
\]
Second order LCR resonator-poles

\[ \frac{V_o}{I} = \frac{1}{Y} = \frac{1}{\frac{1}{sL} + sC + \frac{1}{R}} \]

\[ = \frac{s}{C} \frac{s}{s^2 + \frac{s}{CR} + \frac{1}{LC}} \]

\[ \omega_0^2 = \frac{1}{LC} \]

\[ \frac{\omega_0}{Q} = \frac{1}{CR} \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ Q = \omega_0 CR \]
Second order LCR resonator-poles

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ Q = \omega_0 CR \]

\[ \frac{\omega_0}{2Q} = \frac{1}{2CR} \]
Second order LCR resonator-zeroes

Break any ground connection to inject $V_i$

Generic structure

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$
Second order LCR resonator-zeroes

Break any ground connection to inject $V_i$

Generic structure

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

Zeroes occur if $Z_2(s) = 0$, as long as $Z_1(s)$ is not also zero.
Second order LCR lowpass

Zero locations:

\[ sL \rightarrow \infty \]

\[ \frac{1}{sC + \frac{1}{R}} \rightarrow 0 \]
Second order LCR lowpass

Zero locations:

\[
\begin{align*}
sL & \to \infty \\
\frac{1}{sC + \frac{1}{R}} & \to 0
\end{align*}
\]
Second order LCR highpass

Zero locations:

\[ sC \rightarrow 0 \]

\[ sL \rightarrow 0 \]
Second order LCR highpass

Zero locations:

\[ sC \rightarrow 0 \]
\[ sL \rightarrow 0 \]
Practical limitations of LCR resonators

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ Q = \omega_0 CR \]
Consider a maximally flat LPF with $f_0 = 4$ kHz
$R = 5000 \, \Omega$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$
$$Q = \omega_0 CR$$
Practical limitations of LCR resonators

Consider a maximally flat LPF with $f_0 = 4$ kHz
$R = 5000 \, \Omega$

\[
Q = \frac{1}{\sqrt{2}}
\]

\[
C = \frac{Q}{\omega_0 R}
\]

$C = 5627 \, \text{pF}$

\[
\omega_0 = \frac{1}{\sqrt{LC}}
\]

\[
Q = \omega_0 CR
\]
Practical limitations of LCR resonators

Consider a maximally flat LPF with $f_0 = 4$ kHz, $R = 5000 \, \Omega$

$$Q = \frac{1}{\sqrt{2}}$$

$$C = \frac{Q}{\omega_0 R}$$

$$C = 5627 \, \text{pF}$$

$$L = \frac{1}{\omega_0^2 C}$$

$$L = 0.281 \, \text{H}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 CR$$
Alternatives to inductors
Alternatives to inductors

\[ Z_{in} \equiv \frac{V_1}{I_1} = sC_4R_1R_3R_5/R_2 \]
Alternatives to inductors

Impedance of inductor is $sL$

$Z_{in}$ looks like inductor with $L = C_4 R_1 R_3 R_5 / R_2$
Alternatives to inductors
Alternatives to inductors

LPF

sL
Alternatives to inductors

LPF

3 op-amps
1 capacitor
4-6 resistors

versus
1 inductor

V_o
C
R

V_i
Ki

A_1
-+

A_2
+-

R_1
R_2
R_3
C_4

R_5
Two integrator filter (high-pass example)

\[ \frac{V_{hp}}{V_i} = \frac{KS^2}{s^2 + \left( \frac{\omega_0}{Q} \right)s + \omega_0^2} \]

Biquadratic circuit (ratio of two quadratic polynomials)
Two integrator filter
(high-pass example)

\[ \frac{V_{hp}}{V_i} = \frac{Ks^2}{s^2 + \left( \frac{\omega_0}{Q} \right)s + \omega_0^2} \]

\[ V_{hp} + \frac{1}{Q} \left( \frac{\omega_0}{s} V_{hp} \right) + \left( \frac{\omega_0^2}{s^2} V_{hp} \right) = K V_i \]

\[ V_{hp} = K V_i - \frac{1}{Q} \left( \frac{\omega_0}{s} V_{hp} \right) - \left( \frac{\omega_0^2}{s^2} V_{hp} \right) \]
Two integrator filter (high-pass example)

\[
\frac{V_{hp}}{V_i} = \frac{Ks^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}
\]

\[
V_{hp} + \frac{1}{Q}\left(\frac{\omega_0}{s} V_{hp}\right) + \left(\frac{\omega_0^2}{s^2} V_{hp}\right) = KV_i
\]

\[
V_{hp} = KV_i - \frac{1}{Q}\left(\frac{\omega_0}{s} V_{hp}\right) - \left(\frac{\omega_0^2}{s^2} V_{hp}\right)
\]

Integrators
Two integrator filter
(high-pass example)

\[
V_{hp} = KV_i - \frac{1}{Q} \left( \frac{\omega_0}{s} V_{hp} \right) - \left( \frac{\omega_0^2}{s^2} V_{hp} \right)
\]
Two integrator filter
(high-pass example)

\[ V_{hp} = KV_i - \frac{1}{Q} \left( \frac{\omega_0}{s} V_{hp} \right) - \left( \frac{\omega_0^2}{s^2} V_{hp} \right) \]
Two integrator filter
(simultaneous LP, BP, HP)

\[
T_{hp}(s) = \frac{Ks^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]

\[
T_{bp}(s) = \frac{K \omega_0 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]

\[
T_{lp}(s) = \frac{K \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]
Alternate filter elements

Piezoelectric crystal filters

\[
\frac{B_{60}}{B_6} < 2
\]
Alternate filter elements

Mechanical filters
Alternate filter elements

Equivalent LC filter

Transmission line filters

Microstrip
Alternate filter elements

\[ \sum_{i=0}^{n-1} c_i x_i \]

**Input Samples** \( x(kT) \)

\( (\ldots, 0, 0, 0, 1, 0, 0, 0, \ldots) \)

an “impulse”

**Output Samples** \( y(kT) \)

\( (\ldots, 0, 0, 0, c_{n-1}, c_{n-2}, c_{n-3}, \ldots, c_2, c_1, c_0, 0, 0, 0, \ldots) \)

impulse response of filter

Digital filters (FIR)
Alternate filter elements

Digital filters (IIR)
Alternate filter elements

Digital filters (IIR)